First Semester Examination
Academic Session 2018/2019

December 2018/January 2019

EEE232 – COMPLEX ANALYSIS
(Analisis Kompleks)

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **ELEVEN (11)** pages and **TWO (2)** pages of printed appendix material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEBELAS (11**) muka surat dan **DUA(2)** muka surat lampiran yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: This question paper consists of **FOUR (4)** questions. Answer **ALL** questions. All questions carry the same marks.

[***Arahan:*** *Kertas soalan ini mengandungi **EMPAT (4)** soalan. Jawab **SEMUA** soalan. Semua soalan membawa jumlah markah yang sama.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

1. (a) Solve the quadratic equations z^2 - (1-j4)z - (5-j) = 0 using De Moivre's Theorem
Selesaikan persamaan kuadratik z^2 - (1-j4)z - (5-j) = 0 menggunakan Teorem De Moivre

(35 marks/*markah*)

(b) Expand the trigonometric function in terms of sines and/or cosines of multiple angles: $\sin^3\theta\cos^3\theta$
Kembangkan fungsi trigonometri dari segi sine dan / atau kosina gandaan sudut: $\sin^3\theta\cos^3\theta$

(30 marks/*markah*)

(c) Find all the solutions to the equation *cosh z = 1/4*, where z = x +jy
Cari semua penyelesaian untuk persamaan cosh z = 1/4, di mana z = x +jy

(35 marks/*markah*)

2. (a) Determine whether the following series converges or diverges using Root Test:
Tentukan sama ada siri yang berikut menumpu atau mencapah dengan menggunakan Ujian Punca:

(i)

$$\sum_{n=0}^{\infty}\frac{n^n}{2^n}$$

(ii)

$$\sum_{n=0}^{\infty}\left(\frac{n+1}{2n+1}\right)^n$$

(10 marks/*markah*)

(b) Determine whether the following series converges or diverges using Ratio Test:
Tentukan sama ada siri yang berikut menumpu atau mencapah dengan menggunakan Ujian Nisbah:

(i)

$$\sum_{n=0}^{\infty} \frac{n^2 2^n}{n!}$$

(ii)

$$\sum_{n=0}^{\infty} \frac{9^n}{(-2)^{n+1} n}$$

(10 marks/*markah*)

(c) Find the center and the radius of convergence for the following power series using Ratio Test:
Tentukan pusat dan jejari ketumpuan untuk siri kuasa yang dinyatakan di bawah dengan menggunakan Ujian Nisbah:

(i)

$$\sum_{n=0}^{\infty} \frac{(z+3)^2}{(n+1)4^n}$$

(ii)

$$\sum_{n=0}^{\infty} \frac{(z-1)^2}{3^n (n+1)}$$

(20 marks/*markah*)

(d) Solve the following using the Residue Theorem:

Selesaikan yang berikut dengan menggunakan Teorem Baki:

(i)

$$\oint \frac{1}{(z-1)^2\,(z-3)}\,dz \qquad C = \{z\colon |z| = 5\}$$

(ii)

$$\oint \frac{1-2z}{z(z-1)(z-3)}\,dz \qquad C = \{z\colon |z| = 2\}$$

(60 marks/*markah*)

3. (a) If for every *z* in a set *S*, a unique value w is associated, the *w* is said to be a function of *z* and is denoted by

$$w = f(z)$$

Since *w* is complex, it is written as

$$w = f(z) = u(x,y) + jv(x,y)$$

Jika untuk setiap z di dalam satu set S, satu nilai unik w diberikan maka w digelar sebagai satu fungsi bagi z dan dinyatakan sebagai

$$w = f(z)$$

Oleh kerana w adalah komplek, ia ditulis sebagai

$$w = f(z) = u(x,y) + jv(x,y)$$

(i) What are $u(x,y)$ and $v(x,y)$ represented?

Apakah yang diwakili oleh $u(x,y)$ *dan* $v(x,y)$

(10 marks/*markah*)

(ii) If $f(z) = 3z^2 - 3jz$, find $u(x,y)$ and $v(x,y)$

Jika $f(z) = 3z^2 - 3jz$, *cari* $u(x,y)$ dan $v(x,y)$

(10 marks/*markah*)

(b) A function $f(z)$ is said to be differentiable at a point z_o if the limit

$$f'(z_o) = \lim_{\Delta z \to 0} \frac{f(Z_o + \Delta z) - f(Z_o)}{\Delta Z}$$

exist. The limit $f'(z_o)$ is known as the derivative of $f(z)$ at z_o .

Satu fungsi $f(z)$ *dikatakan boleh diterbitkan pada satu titik* z_o *jika had*

$$f'(z_o) = \lim_{\Delta z \to 0} \frac{f(Z_o + \Delta z) - f(Z_o)}{\Delta Z}$$

wujud. *Had bagi* $f'(z_o)$ *diberi sebagai terbitan bagi of* $f(z)$ *pada* z_o.

Given $(z) = 3z^2 + 4jz + 15 - 3j$, then

Diberi f$(z) = 3z^2 + 4jz + 15 - 3j$, *maka*

(i) find the derivative of $f(z)$ at $z = 3$ by finding its limit.

cari terbitan bagi $f(z)$ pada $z = 3$ *dengan mencari hadnya*.

(10 marks/*markah*)

(ii) find the derivative of $f(z)$ at $z = 3$ by using rule of differentiation.

cari terbitan bagi $f(z)$ *pada* $z = 3$ *dengan menggunakan petua terbitan.*

(10 marks/*markah*)

(c) For the following functions, determine the region where the Cauchy – Riemann equations are satisfied. Then, determine the region of analyticity.
Untuk fungsi-fungsi yang berikut, tentukan rantau di mana persamaan Cauchy – Riemann adalah dipenuhi. Kemudian, tentukan rantau analitik.

(i) $f(z) = e^{x}\,(\cos y + j\sin y)$

(10 marks/*markah*)

(ii) $f(z) = e^{y}\,(\cos x + j\sin x)$

(10 marks/*markah*)

(d) Subsequently, by the observation and calculation in (b) and (c), explain Figure 3(d).

Seterusnya, dengan pemerhatian dan pengiraan dalam (b), dan (c), terangkan Rajah 3(d).

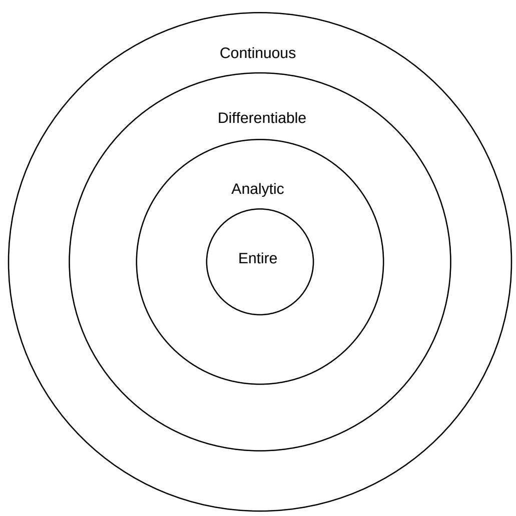

Figure 3(d)
Rajah 3(d)

(10 marks/*markah*)

(e) $f(z) = u + jv$ is an analytic function. Given that:

$$u = 2\sin x \cdot \sinh y + e^{x}\,(x\cos y - y\sin y) + x^3 - 3xy^2 + y$$

$f(z) = u + jv$ *adalah satu fungsi analitik*. *Diberi bahawa*:

$$u = 2\sin x \cdot \sinh y + e^{x}\,(x\cos y - y\sin y) + x^3 - 3xy^2 + y$$

(i) Find u_x and u_y

Cari u_x *dan* u_y

(10 marks/*markah*)

(ii) Find $f'(z)$ by using Milne –Thompson method by replacing $x\ by\ z$ and $y = 0$.

Cari $f'(z)$ *dengan menggunakan kaedah Milne-Thompson dengan menggantikan* x *dengan* z *dan* $y = 0$.

(10 marks/markah)

(iii) Finally, find $f'(z)$ by integrating (ii) with respect to *z.*

Akhirnya, cari $f'(z)$ *dengan mengkamirkan (ii) terhadap z.*

(10 marks/markah)

4. (a) By using the method of definite integration of analytic funtion as below:

Dengan menggunakan kaedah kamiran tentu bagi fungsi analitik seperti dibawah:

$$\int_{Z_0}^{Z_1} f(z)dz = F(Z_1) - F(Z_0), \quad F'(z) = f(z)$$

calculate the integration of the following funtions.

kira kamiran bagi fungsi-fungsi berikut:

(i)

$$\int_{0}^{2+j} z^2 \ dz$$

(10 marks/*markah*)

(ii)

$$\int_{-\pi_j}^{\pi_j} \cos z \ dz$$

(10 marks/*markah*)

(iii)

$$\int_{0}^{2\pi} e^{-jt} \ j\, e^{jt} \ dt$$

(10 marks/*markah*)

(b) (i) For each of the following, illustrate a sketch of
Untuk setiap yang berikut, gambarkan satu lakaran bagi

- Simple close path
  Laluan tertutup mudah
- *Not simple close path*
  Laluan tertutup tak mudah
- *Simply connected domain.*
  Domain tergabung mudah
- *Multiply connected domain*
  Domain tergabung banyak

(20 marks/*markah*)

(ii) From Cauchy's Integral Theorem, if *f(z)* is analytic in a simply connected domain D, then for every simple closed path *c* in *D*:
Dari teori kamiran Cauchy, jika f(z) adalah analitik dalam dominan tergabung mudah, D, maka bagi setiap laluan tertutup mudah c dalam D:

$$\oint_c f(z)\, dz = 0$$

For each of the following calculate
Untuk setiap yang berikut, kira

- $\oint_c \; e^z \, dz$
  where e^z is an entire function
  dimana e^z adalah fungsi seluruh

(10marks/*markah*)

- $\oint_c \frac{1}{\cos z}\, dz$

  where c is a unit circle

  dimana c adalah satu unit bulatan.

(10 marks/*markah*)

- $\oint_c \frac{1}{z}\, dz$

  where c lies in the annulus $\frac{1}{2} < 1z1 < \frac{3}{4}$

  dimana c adalah berada pada annulus $\frac{1}{2} < 1z1 < \frac{3}{4}$

(10 marks/*markah*)

Does each of them follow the Cauchy’s Integral Theorem? Why?

Adalah setiap mereka mengikut teori Kamiran Cauchy? Mengapa?

(c) By using Cauchy’s Integral Formule below:

Dengan menggunakan Formula Kamiran Cauchy di bawah:

$$f(z_0 = \frac{1}{2\pi j} \oint_c \frac{f(z)}{z - z_0}\, dz$$

Calculate

Kira

$$\oint_c \frac{z^2 + 1}{(z+1)(z-1)}\, dz$$

for

u*ntuk*

(i) *C* is circle of a (as in Figure 3(c))

C ialah bulatan a (seperti dalam Rajah 3 (c))

(10marks/*markah*)

(ii) *C* is circle of b (as in figure 3 (c))

C ialah bulatan b (seperti dalam Rajah 3 (c))

(10marks/*markah*)

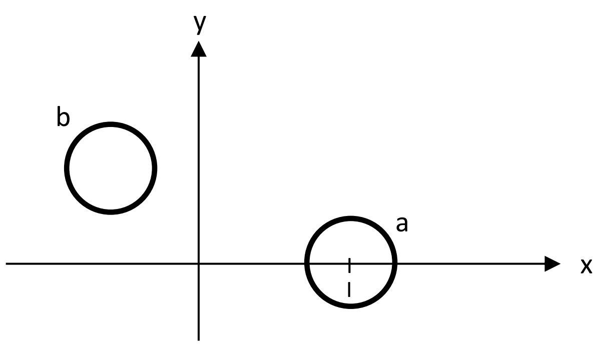

Figure 3(c)

Rajah 3(c)

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APPENDIX
LAMPIRAN

$$cosh^2x - sinh^2x = 1$$

$$cosh^2x - cosh^2x + sinh^2x$$

$$sinh^2x = 2\sinh \quad x\cosh \quad x$$

$$\cosh x = \frac{e^x + e^{-2}}{2}$$

$$\sinh x = \frac{e^x + e^{-x}}{2}$$

$$\cosh \ \mathrm{j}x = \frac{e^{jx} + e^{-jx}}{2} = \cos x$$

$$\sinh \ \mathrm{j}x = \frac{e^{jx} + e^{-jx}}{2} = j\sin \quad x$$

$$j\tanh x = \tan j\ x$$

APPENDIX

LAMPIRAN

Maclaurin's series

$$f(z) = \sum_{n=0}^{\alpha} \frac{f^{(n)}(0)}{n!} \; z^n$$

Taylor's series

$$f(z) = \sum_{n=0}^{\alpha} \frac{f^{(n)}(z_0)}{n!} \; (z - z_0)^n$$

Residue of $f(z)$ at z_0

$$\text{Res}\,[f(z), z_0] = \frac{1}{(m-1)!} \lim_{z \to z_0} \left[\frac{d^{m-1}}{dz^{m-1}} \; (z - z_0)^m \; f(z) \right]$$

Residue Theorem

$$\oint_C \; f(z)dz = 2\pi j \sum_{k=1}^{n} \text{Res}\, f(z)$$