First Semester Examination
2018/2019 Academic Session

December 2018/January 2019

EEE228 – SIGNAL AND SYSTEM
(Isyarat dan Sistem)

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **ELEVEN (11)** pages and **TWELVE (12)** pages of printed appendix material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEBELAS (11**) muka surat dan **DUA BELAS(12)** muka surat lampiran yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: This question paper consists of **FOUR (4)** questions. Answer **ALL** questions. All questions carry the same marks.

[***Arahan:*** *Kertas soalan ini mengandungi* ***EMPAT (4)*** *soalan. Jawab* **SEMUA** *soalan. Semua soalan membawa jumlah markah yang sama.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

1. (a) Consider the system in Figure 1(a). Determine whether the system is:-

Pertimbangkan sistem dalam Rajah 1(a). Tentukan samada sistem tersebut:-

(i) Memoryless
Tanpa memori

(ii) Causal
Kausal

(iii) Linear
Lelurus

(iv) Time-invariant
Tidak varian masa

(v) Stable
Stabil

(15 marks/*markah*)

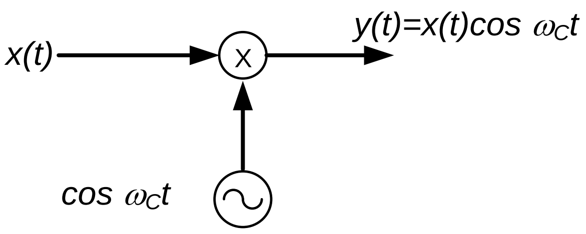


Figure 1(a)

Rajah 1(a)

(b) A continuous-time signal, $f(t)$ is shown in Figure 1(b).

Suatu isyarat berterusan masa $f(t)$ *ditunjukkan dalam Rajah 1(b).*

(i) Define the mathematical expression for the signal, for all values of t.

Perihalkan ungkapan matematik bagi isyarat tersebut, untuk semua nilai t*.*

(10 marks/*markah*)

(ii) Express the signal in terms of singularity functions only, for all values of t.

Nyatakan isyarat tersebut dalam terma fungsi unit sahaja, untuk semua nilai t*.*

(10 marks/*markah*)

(iii) From the signal shown in Figure 1(b), plot $g(t) = 3f(-2t + 2)$.

Dari isyarat ditunjukkan dalam Rajah 1(b), lakarkan $g(t) = 3f(-2t + 2)$*.*

(10 marks/*markah*)

(iv) Verify your results in (iii) by evaluating at least 4 points in time.

Tentusahkan keputusan dalam (iii) dengan menilai sekurangnya 4 titik masa.

(10 marks/*markah*)

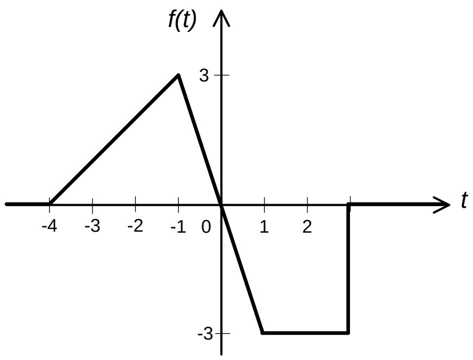


Figure 1(b)

Rajah 1(b)

(c) A discrete-time signal is shown in Figure 1(c).

Suatu isyarat diskret masa ditunjukkan dalam Rajah 1(c).

(i) Sketch the even and odd components of the signal.

Lakarkan komponen genap dan ganjil bagi isyarat tersebut.

(10 marks/*markah*)

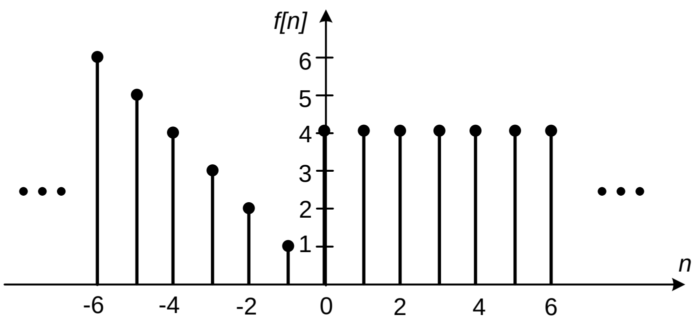


Figure 1(c)

Rajah 1(c)

(ii) Sketch and label the signal:

Lakar dan label isyarat:

$$g[n] = f[-n-3] - r[n-1] - 4u[n-2] - u[-n-6].$$

(25 marks/ *markah*)

(iii) The block diagram of continuous-time systems are shown in Figure 1(d). Express the output, $y[n]$ as the function of the input and the system transformations.

Gambarajah blok bagi sistem masa berterusan ditunjukkan dalam Rajah 1(d). Nyatakan keluaran, $y[n]$ dalam fungsi masukan dan transformasi sistem.

(10 marks/*markah*)

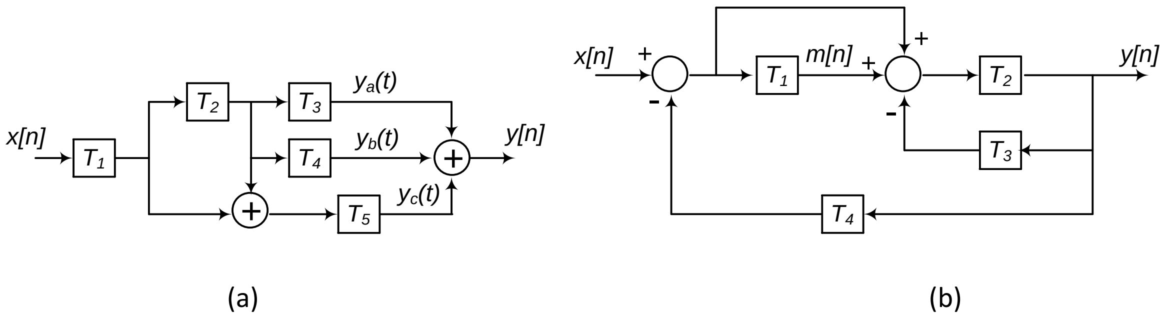
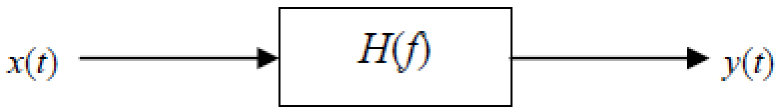


Figure 1(d)

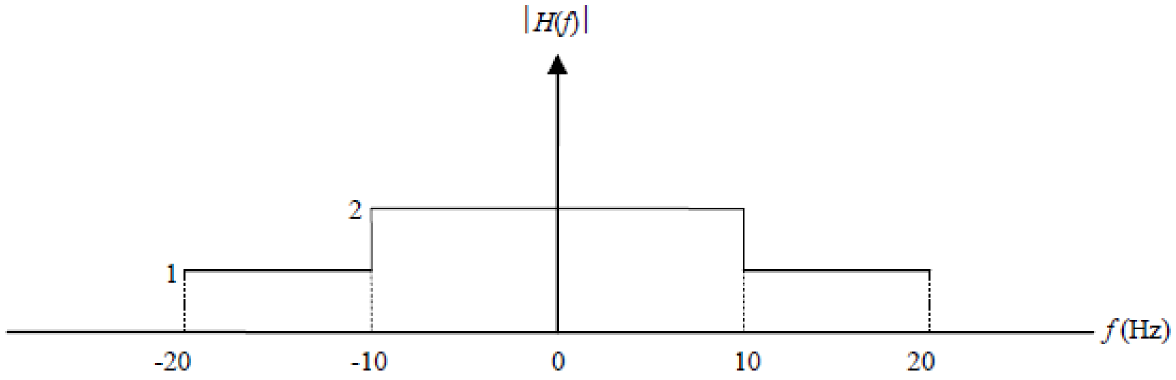
Rajah 1(d)

2. (a) Consider the linear time invariant (LTI) system shown in Figure 2(a) with input $x(t)$, output $y(t)$, and transfer function $H(f)$. The amplitude response and phase shift of $H(f)$ are shown in Figure 2(b) and Figure 2(c).

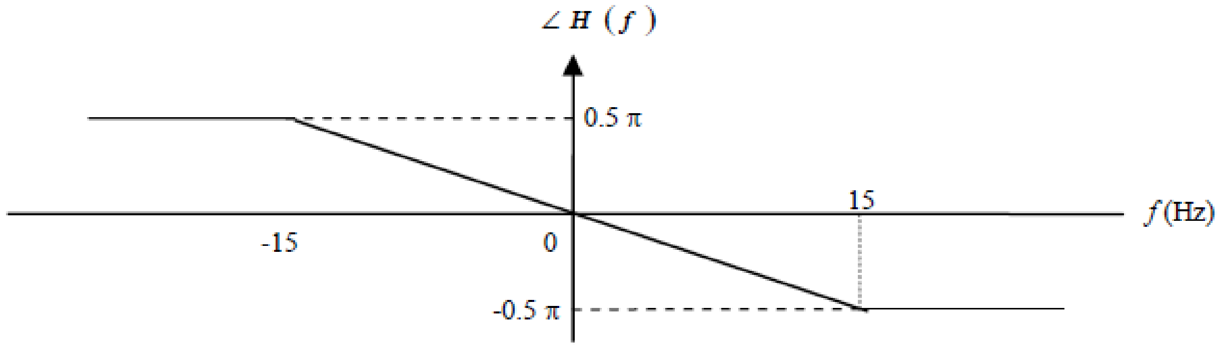
Pertimbangkan sistem lelurus tak varian masa (LTI) yang ditunjukkan dalam Rajah 2(a) dengan masukan x(t), keluaran y(t), and fungsi pindahan H(f). Amplitud tindak balas dan anjakan fasa bagi H(f) adalah ditunjukkan dalam Rajah 2 (b) dan 2 (c).



(a)



(b)



(c)

Figure 2
Rajah 2

Sketch the output **amplitude** response for **each** of the following inputs:
*Lakarkan **amplitud** tindak balas bagi keluaran untuk **setiap** masukan yang berikut:*

(i) $x_1(t) = \cos 10\pi t + \cos 12\pi t$

(ii) $x_2(t) = \cos 10\pi t + \cos 26\pi t$

(iii) $x_3(t) = \cos 32\pi t + \cos 34\pi t$

(30 marks/*markah*)

(b) A system will be classified as distortionless, if it introduces the same attenuation and time delay to all spectral components of the input. Based on your answer in 2 (a), determine in which cases the transmission is distortionless. Please justify your answer.

Sebuah sistem boleh diklasifikasikan sebagai tanpa herotan, jika ia memperkenalkan pelemahan dan kelewatan yang sama kepada semua komponen spektra pada masukan. Berdasarkan jawapan anda di dalam 2 (a), dapatkan dalam situasi-situasi bagaimanakah penghantaran itu tanpa herotan. Sila beri justifikasi bagi jawapan anda.

(30 marks/*markah*)

(c) *x(t)* is the input to an LTI system with a unit impulse response , where

x(t) adalah masukan pada sistem LTI dengan tindak balas satu unit, di mana

$$x(t) = u(t)$$
and
$$h(t) = 2u(t-5)$$

Determine and sketch the convolution of the two signals.
Dapatkan dan lakarkan pelingkaran bagi kedua-dua isyarat tersebut.

(40 marks/*markah*)

3. (a) Explain the difference between Fourier series and Fourier Transform with graphical figures.

Terangkan perbezaan di antara Siri Fourier dan Jelmaan Fourier dengan rajah-rajah grafik.

(10 marks/*markah*)

(b) (i) By using the given Table 1 in Appendix, determine the exponential Fourier Series of *x(t)* shown in Figure 3(b). Sketch the corresponding amplitude exponential Fourier Spectrum if *a = 1* until fourth harmonic, n = 4.

Dengan menggunakan Jadual 1 di Lampiran, dapatkan Siri Fourier eksponen bagi x(t) yang ditunjukkan dalam Rajah 3(b). Lakarkan amplitud spektrum Fourier eksponen jika a = 1 sehingga harmonik ke 4, n = 4.

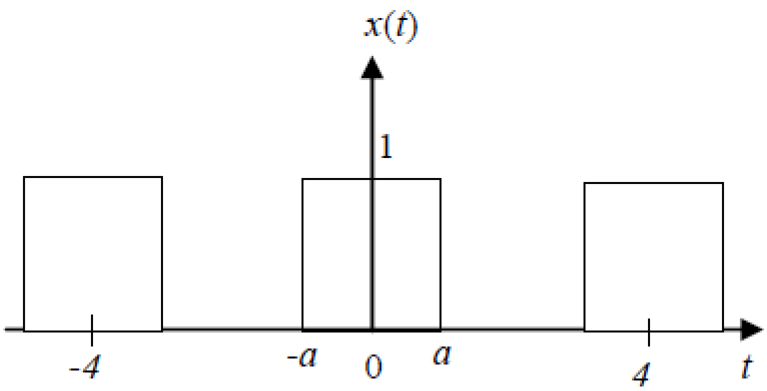


Figure 3(a)

Rajah 3(a)

(25 marks/*markah*)

(ii) Give one observation, what would happen to Fourier spectrum if ***a*** is decreased by a factor of 2. Support your answer by sketching the output spectra.

*Berikan satu pemerhatian, apa akan terjadi kepada spektrum Fourier jika **a** adalah dikurangkan dengan satu faktor sebanyak 2. Sokong jawapan anda dengan melakarkan spektra bagi keluaran.*

(15 marks/*markah*)

(c) (i) Compute the 4-point DFT (Discrete Fourier Transform) and IDFT (Inverse Discrete Fourier Transform) for the waveform shown in Figure 3(c).

Kirakan 4-titik DFT dan IDFT bagi gelombang yang ditunjukkan dalam Rajah 3(c).

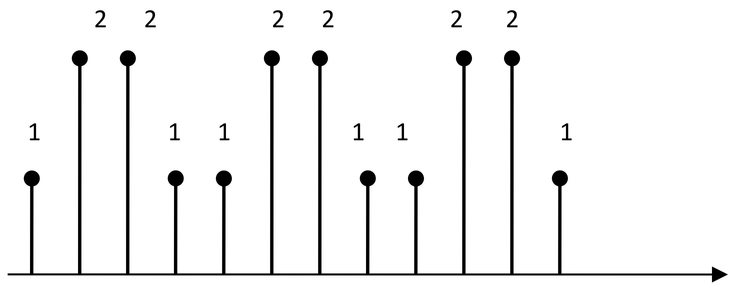


Figure 3(c)

Rajah 3(c)

Given that:

Diberi:

$$F_r = \sum_{k=0}^{N_0-1} f[k]e^{-j\Omega_0 rk} \qquad ,\Omega_0 = \frac{2\pi}{N_0}$$

$$f[k] = \frac{1}{N_0}\sum_{r=0}^{N_0-1} F_r e^{j\Omega_0 rk}$$

(35 marks/markah)

(ii) Find the Z-transform of the signal

Cari jelmaan-Z bagi isyarat

$$x(n) = \left(\frac{1}{2}\right)^n u(n) * \left(\frac{1}{3}\right)^n u(n)$$

(15 marks/*markah*)

4. (a) Given a function as below,

Diberi fungsi seperti berikut,

$$X(z) = \frac{3 + 2z^{-1} + z^{-2}}{1 - 3z^{-1} + 2z^{-2}}$$

Find the inverse z-transform of it using a power series expansion and find the first four terms of x[n]

Cari jelmaan-z songsang fungsi tersebut dan dapatkan empat ungkapan pertama bagi x[n]

(35 marks/markah)

(b) Find the Fourier transform of the following time-domain signals:

Cari jelmaan Fourier bagi isyarat domain masa berikut:

(i) $e^{at}u(-t)$

(ii) $Asin(\omega_1 t) + Bcos(\omega_2 t)$

(iii) Signal x(t) in Figure 4(b)

Isyarat x(t) dalam Rajah 4(b)

(30 marks/*markah*)

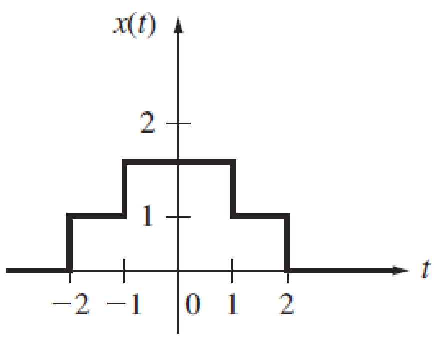


Figure 4(b)

Rajah 4(b)

(c) Consider a stable linear time invariant (LTI) system that is characterized by the differential equation.

Pertimbangkan sistem lelurus tak varian masa (LTI) yang dicirikan oleh persamaan perbezaan berikut.

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)$$

By using Fourier Transform:

Dengan menggunakan Jelmaan Fourier:

(i) Find the Fourier Transform of each expression in the equation.

Cari Jelmaan Fourier bagi setiap ungkapan dalam persamaan tersebut.

(5 marks/*markah*)

(ii) Find the frequency response, H(ω).

Cari sambutan frekuensi, H(ω).

(10 marks/*markah*)

(iii) Find the corresponding impulse response, h(t).

Cari sambutan impuls yang berkaitan, h(t).

(10 marks/*markah*)

(iv) Given that the input, $x(t) = e^{-t}u(t)$, find the output response, y(t).

Diberi input, $(t) = e^{-t}u(t)$ *, cari sambutan output, y(t).*

(10 marks/*markah*)

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APPENDIX

LAMPIRAN

Mathematical Formulas

This appendix – by no means exhaustive – serves as a handy reference. It does contain all the formulas needed to solve circuit problems in this examination book.

Quadratic Formula

The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric Identities

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\sec x = \frac{1}{\cos x}, \csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x}, \cot x = \frac{1}{\tan x}$$

$$\sin(x \pm 90°) = \pm \cos x$$

$$\cos(x \pm 90°) = \mp \sin x$$

$$\sin(x \pm 180°) = -\sin x$$

$$\cos(x \pm 180°) = -\cos x$$

$$\cos^2 x + \sin^2 x = 1$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{(law of sines)}$$

$$a^2 = b^2 + c^2 - 2bc\cos A \quad \text{(law of cosines)}$$

$$\frac{\tan\frac{1}{2}(A - B)}{\tan\frac{1}{2}(A + B)} = \frac{a - b}{a + b} \quad \text{(law of tangents)}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$2\sin x \sin y = \cos(x - y) - \cos(x + y)$$

$$2\sin x \cos y = \sin(x + y) + \sin(x - y)$$

$$2\cos x \cos y = \cos(x + y) + \cos(x - y)$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$
$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$
$$K_1 \cos x + K_2 \sin x = \sqrt{K_1^2 + K_2^2} \cos\left(x + \tan^{-1} - \frac{K_2}{K_1}\right)$$
$$e^{\pm jx} = \cos x \pm j \sin x \quad \text{(Euler's identity)}$$
$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$
$$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$$
$$1 \text{ rad} = 57.296°$$

Hyperbolic Functions

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$
$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$
$$\tanh x = \frac{\sinh x}{\cosh x}$$
$$\coth x = \frac{1}{\tanh x}$$
$$\operatorname{csch} x = \frac{1}{\sinh x}$$
$$\operatorname{sech} x = \frac{1}{\cosh x}$$
$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$
$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

Derivatives

If $U = U(x)$, $V = V(x)$, and $a =$ constant,

$$\frac{d}{dx}(aU) = a\frac{dU}{dx}$$
$$\frac{d}{dx}(UV) = U\frac{dV}{dx} + V\frac{dU}{dx}$$
$$\frac{d}{dx}\left(\frac{U}{V}\right) = \frac{\left(V\frac{dU}{dx} - U\frac{dV}{dx}\right)}{V^2}$$
$$\frac{d}{dx}(aU^n) = naU^{n-1}$$
$$\frac{d}{dx}(a^U) = a^U \ln a \frac{dU}{dx}$$
$$\frac{d}{dx}(e^U) = e^U\frac{dU}{dx}$$

$$\frac{d}{dx}(\sin U) = \cos U \frac{dU}{dx}$$
$$\frac{d}{dx}(\cos U) = -\sin U \frac{dU}{dx}$$

Indefinite Integrals

If $U = U(x)$, $V = V(x)$, and $a =$ constant,

$$\int a\,dx = ax + C$$
$$\int U\,dV = UV - \int V\,dU \quad \text{(integration by parts)}$$
$$\int U^n\,dU = \frac{U^{n+1}}{n+1} + C, \quad n \neq 1$$
$$\int \frac{dU}{U} = \ln U + C$$
$$\int a^U\,dU = \frac{a^U}{\ln a} + C, \quad a > 0, a \neq 1$$
$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$$
$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1) + C$$
$$\int x^2e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2) + C$$
$$\int \ln x\,dx = x\ln x - x + C$$
$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$$
$$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$$
$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$
$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$
$$\int x\sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax) + C$$
$$\int x\cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax) + C$$
$$\int x^2\sin ax\,dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax) + C$$
$$\int x^2\cos ax\,dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax) + C$$
$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + C$$

$$\int \sin ax \sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$\int \sin ax \cos bx\,dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

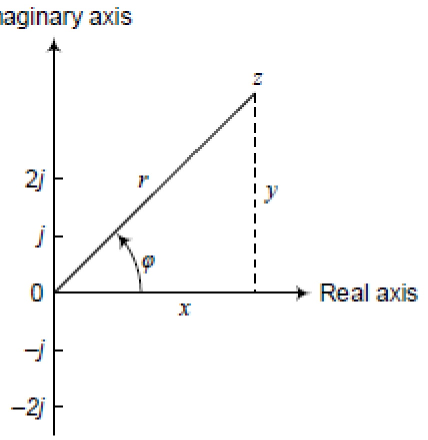
$$\int \cos ax \cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

$$\int \frac{x^2dx}{a^2+x^2} = x - a\tan^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{(a^2+x^2)^2} = \frac{1}{2a^2}\left(\frac{x}{x^2+a^2} + \frac{1}{a}\tan^{-1}\frac{x}{a}\right) + C$$

Phasor & Complex Number



Complex number in rectangular form:

$$z = x + jy$$

$$r = \sqrt{x^2+y^2}$$

$$\varphi = \tan^{-1}\frac{y}{x}$$

$$z = r(\cos\varphi + j\sin\varphi)$$

$$\frac{1}{j} = -j \text{ and } j = 1\angle 90^\circ$$

Complex number in polar form:

$$z = r\angle\varphi$$

Complex number in exponential form:

$$z = re^{j\varphi}$$

Sinusoid ↔ phasor transformation:

$$V_m \cos(\omega t + \varphi) \leftrightarrow V_m \angle \varphi$$

$$V_m \sin(\omega t + \varphi) \leftrightarrow V_m \angle (\varphi - 90°)$$

$$I_m \cos(\omega t + \theta) \leftrightarrow I_m \angle \theta$$

$$I_m \sin(\omega t + \theta) \leftrightarrow I_m \angle (\theta - 90°)$$

Mathematic operation of complex number:

Addition	$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
Subtraction	$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
Multiplication	$z_1 z_2 = r_1 r_2 \angle (\varphi_1 + \varphi_2)$
Division	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle (\varphi_1 - \varphi_2)$
Reciprocal	$\frac{1}{z} = \frac{1}{r} \angle - \varphi$
Square-root	$\sqrt{z} = \sqrt{r} \angle (\varphi / 2)$
Complex conjugate	$z^* = x - jy = r \angle - \varphi = r e^{-j\varphi}$

- Trigonometric Fourier Series:

$$f(t) = a_o + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

Where the Fourier Coefficients are:

$$a_0 = \frac{1}{T}\int_0^T f(t)dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t dt$$

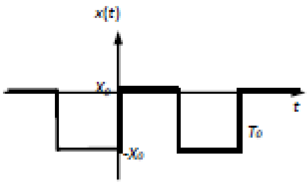
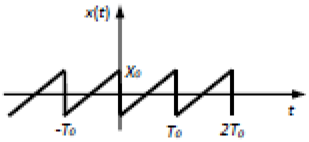
- Spectrum frequency form can be obtained from this equation:

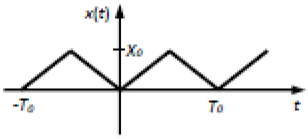
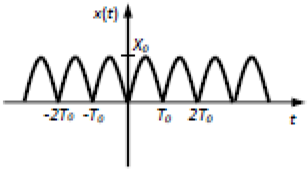
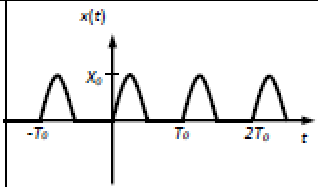
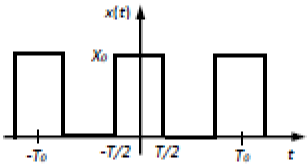
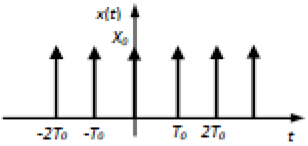
$$f(t) = a_0 + \sum_{1}^{\infty}(a_n^{\ 2} + b_n^{\ 2})^{\frac{1}{2}} \cos(n\omega t + \theta_n)$$

Where $\theta_n = \tan^{-1}(-\frac{b_n}{a_n})$

JADUAL 1: PEKALI COMPLEX FOURIER

TABLE 1: COMPLEX FOURIER COEFFICIENTS

Name	Waveform	C_0	$C_n, n \neq 0$	Comments
1. Square Wave		0	$-j\frac{2X_0}{\pi n}$	$C_n = 0,$ n even
2. Sawtooth		$\frac{X_0}{2}$	$j\frac{X_0}{2\pi n}$	

3. Triangular Wave		$\frac{X_0}{2}$	$\frac{-2X_0}{(\pi n)^2}$	$C_n = 0$, n even
4. Full-wave rectified		$\frac{2X_0}{\pi}$	$\frac{-2X_0}{\pi(4n^2-1)}$	
5. Half-wave rectified		$\frac{X_0}{\pi}$	$\frac{-X_0}{\pi(n^2-1)}$	$C_n = 0$, n odd, except for $C_1 = -j\frac{X_0}{4}$, and $C_{-1} = j\frac{X_0}{4}$
6. Rectangular Wave		$\frac{TX_0}{T_0}$	$\frac{TX_0}{T_0}\text{sinc}\frac{Tn\omega_0}{2}$	$\frac{Tn\omega_0}{2} = \frac{\pi Tn}{T_0}$
7. Impulse Train		$\frac{X_0}{T_0}$	$\frac{X_0}{T_0}$	

- General Exponential Fourier Series $$x(t) = C_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} C_n e^{jn\omega_0 t}$$
- Parseval's Theorem

$$P_{ave} = C_0^2 + \frac{1}{2}\sum_{1}^{\infty}\left(a_n^2 + b_n^2\right)$$

$$= C_0^2 + 2\sum_{1}^{\infty}\left|C_n\right|^2$$

$$P_{ave} = \sum_{-\infty}^{\infty}\left|C_n\right|^2$$

(For exponential Fourier Series)

A Short Table of Fourier Transforms

	$f(t)$	$F(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table 4.2

Fourier Transform Operations

Operation	$f(t)$	$F(\omega)$
Addition	$f_1(t) + f_2(t)$	$F_1(\omega) + F_2(\omega)$
Scalar multiplication	$kf(t)$	$kF(\omega)$
Symmetry	$F(t)$	$2\pi f(-\omega)$
Scaling (a real)	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Frequency shift (ω_0 real)	$f(t)e^{j\omega_0 t}$	$F(\omega - \omega_0)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi}F_1(\omega) * F_2(\omega)$
Time differentiation	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(x)\,dx$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$

Table 11.1: (Unilateral) z-Transform Pairs

	$f[k]$	$F[z]$
1	$\delta[k-j]$	z^{-j}
2	$u[k]$	$\frac{z}{z-1}$
3	$ku[k]$	$\frac{z}{(z-1)^2}$
4	$k^2u[k]$	$\frac{z(z+1)}{(z-1)^3}$
5	$k^3u[k]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^{k-1}u[k-1]$	$\frac{1}{z-\gamma}$
7	$\gamma^k u[k]$	$\frac{z}{z-\gamma}$
8	$k\gamma^k u[k]$	$\frac{\gamma z}{(z-\gamma)^2}$
9	$k^2\gamma^k u[k]$	$\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\frac{k(k-1)(k-2)\cdots(k-m+1)}{\gamma^m m!}\gamma^k u[k]$	$\frac{z}{(z-\gamma)^{m+1}}$
11a	$\lvert\gamma\rvert^k \cos\beta k\, u[k]$	$\frac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
11b	$\lvert\gamma\rvert^k \sin\beta k\, u[k]$	$\frac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12a	$r\lvert\gamma\rvert^k\cos(\beta k+\theta)u[k]$	$\frac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12b	$r\lvert\gamma\rvert^k\cos(\beta k+\theta)u[k]$ $\quad \gamma=\lvert\gamma\rvert e^{j\beta}$	$\frac{(0.5re^{j\theta})z}{z-\gamma}+\frac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r\lvert\gamma\rvert^k\cos(\beta k+\theta)u[k]$ $r=\sqrt{\frac{A^2\lvert\gamma\rvert^2+B^2-2AaB}{\lvert\gamma\rvert^2-a^2}}$ $\beta=\cos^{-1}\frac{-a}{\lvert\gamma\rvert},\ \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{\lvert\gamma\rvert^2-a^2}}$	$\frac{z(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$

Table 11.2

Z- Transform Operations

Operation	$f[k]$	$F[z]$
Addition	$f_1[k] + f_2[k]$	$F_1[z] + F_2[z]$
Scalar multiplication	$af[k]$	$aF[z]$
Right-shift	$f[k-m]u[k-m]$	$\frac{1}{z^m}F[z]$
	$f[k-m]u[k]$	$\frac{1}{z^m}F[z] + \frac{1}{z^m}\sum_{k=1}^{m} f[-k]z^k$
	$f[k-1]u[k]$	$\frac{1}{z}F[z] + f[-1]$
	$f[k-2]u[k]$	$\frac{1}{z^2}F[z] + \frac{1}{z}f[-1] + f[-2]$
	$f[k-3]u[k]$	$\frac{1}{z^3}F[z] + \frac{1}{z^2}f[-1] + \frac{1}{z}f[-2] + f[-3]$
Left-shift	$f[k+m]u[k]$	$z^m F[z] - z^m \sum_{k=0}^{m-1} f[k]z^{-k}$
	$f[k+1]u[k]$	$zF[z] - zf[0]$
	$f[k+2]u[k]$	$z^2F[z] - z^2f[0] - zf[1]$
	$f[k+3]u[k]$	$z^3F[z] - z^3f[0] - z^2f[1] - zf[2]$
Multiplication by γ^k	$\gamma^k f[k]u[k]$	$F\left[\frac{z}{\gamma}\right]$
Multiplication by k	$kf[k]u[k]$	$-z\frac{d}{dz}F[z]$
Time Convolution	$f_1[k] * f_2[k]$	$F_1[z]F_2[z]$
Frequency Convolution	$f_1[k]f_2[k]$	$\frac{1}{2\pi j}\oint F_1[u]F_2\left[\frac{z}{u}\right]u^{-1}\,du$
Initial value	$f[0]$	$\lim_{z\to\infty} F[z]$
Final value	$\lim_{N\to\infty} f[N]$	$\lim_{z\to 1}(z-1)F[z]$ poles of $(z-1)F[z]$ inside the unit circle.