# INDIVIDUAL AND ENSEMBLE PATTERN CLASSIFICATION MODELS USING ENHANCED FUZZY MIN-MAX NEURAL NETWORKS 

by

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## LIST OF ABBREVIATIONS

| ACS | Acute Coronary Syndrome |
| :---: | :---: |
| AMGS | Adaptive Merging and Growing Strategy |
| ANN | Artificial Neural Network |
| ANR | Automatic Noise Reduction |
| ARC | Adaptive Resolution Classifier |
| ART | Adaptive Resonance Theory |
| BBA | Bucket Brigade Algorithm |
| CBS | Certified Belief in Strength |
| CF | Confidence Factor |
| CI | Computational Intelligence |
| CNs | Classifying Neurons |
| CPTS | Correct Predicted Test Samples |
| CS | Clustering-and-Selection |
| DCFMN | Data-Core-Based Fuzzy Min-Max Neural Network |
| DDA | Dynamic Decay Adjustment |
| DEIR | Distributed Embedded Intelligence Room |
| DT | Decision Templates |
| DWM | Dynamic Weighted Majority |
| EFMM | Enhanced FMM |
| FAM | Fuzzy ARTMAP |
| FMCN | Fuzzy Neural Network with Compensatory Neuron |
| FMM | Fuzzy min-max |
| GA | Genetic Algorithm |


| GFMM | General Fuzzy Min-Max |
| :---: | :---: |
| GRFMN | General Reflex Fuzzy Min-Max Neural Network |
| HA | Hyperbox Accuracy |
| ICPTS | Incorrect Predicted Test Samples |
| JADE | Java Agent DEvelopment |
| KEEL | Knowledge Extraction based on Evolutionary Learning |
| kNN | k-Nearest Neighbor |
| LP | Linear Programming |
| MACS | Multi-Agent Classifier System |
| MANNFIS | Multi-Agent Neural Net Fuzzy Inference System |
| MAS | Multi Agent System |
| MFMM | Modified FMM |
| MFSs | Multiple Feature Subsets |
| MLP | Multi-Layer Perceptron |
| OLNs | Overlapping Neurons |
| PARC | Pruned Adaptive Resolution Classifier |
| PWBTS | Performance Weight Based on Tough Samples |
| RVM | Relevance Vector Machines |
| SVM | Support Vector Machines |
| TACF | Threshold-Adjusted Classification Filter |
| TNC | Trust-Negotiation-Communication |
| UCI | University of California, Irvine |
| WAVE | Weight-Adjusted Voting for Ensembles of classifiers |
| WFMM | Weighted FMM network |

# MODEL PENGECAMAN CORAK INDIVIDU DAN GABUNGAN DENGAN MENGGUNAKAN RANGKAIAN NEURAL MIN-MAX KABUR YANG DIPERTINGKATKAN 


#### Abstract

ABSTRAK

Klasifikasi corak adalah salah satu daripada komponen utama untuk rekabentuk dan pembangunan sistem pengecaman corak berkomputer. Tertumpu kepada model kecerdasan berkomputer, tesis ini menerangkan secara mendalam kajian-kajian menerusi dua pendekatan yang berkemungkinan bagi tujuan merekabentuk model klasifikasi corak yang kukuh dan anjal serta berprestasi tinggi. Pertamanya, dengan meningkatkan prestasi pembelajaran rangkaian neural-kabur dan keduanya dengan merangka model gabungan bagi menggabungkan ramalan daripada pelbagai rangkaian neural-kabur menggunakan rangka kerja berasaskan ejen. Disebabkan terdapat beberapa ciri penting termasuk keupayaan pembelajaran secara berperingkat dan wujudnya sempadan keputusan tak linear dengan 'hyperboxes', rangkaian MinMax Kabur (FMM) dipilih sebagai asas bagi merekabentuk model klasifikasi corak yang boleh digunakan dalam kajian ini. Dua varian FMM yang dipertingkatkan, iaitu EFMM dan EFMM2, telah dicadangkan bagi menangani beberapa kelemahan yang terdapat dalam algoritma pembelajaran FMM asal. Untuk EFMM, tiga kaedah heuristik diperkenalkan bagi meningkatkan perkembangan 'hyperbox', ujian pertindihan, dan proses-proses penguncupan. Kerumitan rangkaian dan isu-isu toleransi hingar diambil kira dalam EFMM2. Di samping itu, rangka kerja berasaskan ejen digunapakai sebagai model gabungan yang kukuh bagi menempatkan rangkaian berganda berasaskan EFMM. Satu kaedah pengukuran berguna yang dikenali sebagai


Certified Belief in Strength (CBS) telah dibangunkan dan dimasukkan ke dalam model gabungan bagi mengeksploitasi prestasi ramalan rangkaian berasaskan EFMM yang berbeza. Model yang terhasil ini dipanggil 'Multi-Agent Classifier System with Certified Belief in Strength (MACS-CBS)'. Prestasi kedua-dua model berasaskan kedua-dua EFMM iaitu yang tunggal dan gabungan dikaji secara sistematik menggunakan satu siri kajian penanda aras, yang mana keputusan tersebut di analisis dan dibincangkan. Dari sudut penilaian empirikal, model EFMM dan model EFMM2 yang dibangunkan menunjukkan peningkatan prestasi jika dibandingkan dengan rangkaian-rangkaian FMM asal. Keputusan kajian juga menunjukkan bahawa prestasi kedua-dua model adalah setanding atau lebih baik daripada kebanyakan sistem pembelajaran mesin yang telah dilaporkan dalam kajian ilmiah. Tambahan pula, dua aplikasi sebenar yang melibatkan permasalahan dalam bidang perubatan dan industri telah digunakan bagi tujuan penilaian. Keputusan positif yang diperolehi menunjukkan potensi dan keberkesanan model gabungan berasaskan EFMM sekiranya dibandingkan dengan FMM dan model berkaitan yang lain berdasarkan kajian ilmiah dalam menyelesaikan masalah klasifikasi corak dalam persekitaran sebenar.

# INDIVIDUAL AND ENSEMBLE PATTERN CLASSIFICATION MODELS USING ENHANCED FUZZY MIN-MAX NEURAL NETWORKS 


#### Abstract

Pattern classification is one of the major components for the design and development of a computerized pattern recognition system. Focused on computational intelligence models, this thesis describes in-depth investigations on two possible directions to design robust and flexible pattern classification models with high performance. Firstly is by enhancing the learning algorithm of a neural-fuzzy network; and secondly by devising an ensemble model to combine the predictions from multiple neural-fuzzy networks using an agent-based framework. Owing to a number of salient features which include the ability of learning incrementally and establishing nonlinear decision boundary with hyperboxes, the Fuzzy Min-Max (FMM) network is selected as the backbone for designing useful and usable pattern classification models in this research. Two enhanced FMM variants, i.e. EFMM and EFMM2, are proposed to address a number of limitations in the original FMM learning algorithm. In EFMM, three heuristic rules are introduced to improve the hyperbox expansion, overlap test, and contraction processes. The network complexity and noise tolerance issues are undertaken in EFMM2. In addition, an agent-based framework is capitalized as a robust ensemble model to house multiple EFMM-based networks. A useful trust measurement method known as Certified Belief in Strength (CBS) is developed and incorporated into the ensemble model for exploiting the predictive performances of different EFMM-based networks. The resulting model is known as


a Multi-Agent Classifier System with Certified Belief in Strength (MACS-CBS). The usefulness of both individual and ensemble EFMM-based models is evaluated systematically using a series of benchmark studies, with the results analyzed and discussed. From the empirical evaluation, the proposed EFMM and EFMM2 models show improved performances as compared with those from the original FMM network. The results are also either comparable with or better than those from many other machine learning systems reported in the literature. Furthermore, two realworld medical and industrial problems are used for evaluation. The outcomes positively demonstrate the potential and efficacy of the proposed ensemble EFMMbased model, as compared with FMM and other related models in the literature, in undertaking pattern classification problems in the real environments.

## CHAPTER 1

## INTRODUCTION

### 1.1 Background

Since ancient times, humans are considered as the best pattern recognizer in most instances (Jain et al., 2000). Our brain receives patterns from sensing organs and processes them to become useful information, and subsequently allows us to make appropriate decisions based on the received patterns. The development in sciences and technologies has led humans to research into models and techniques to emulate the functionality of the human brain. One of the attempts is to understand the process and action of pattern recognition by the human brain, and subsequently develop computerized systems to imitate this pattern recognition capability. However, for a computerized system to function as a useful pattern recognizer, it needs to be equipped with robust algorithms in order to be able to extract meaningful features from events or objects, and classify them into different categories. Nowadays, pattern recognition has attracted the attention of many researchers from different fields, and is becoming one of the most important characteristics of intelligent behaviours. In general, the design and development of a computerized pattern recognition system comprises four major components (Rosenfeld and Wechsler, 2000), i.e.:

- Data Acquisition and Collection: it is the procedure for finding patterns from physical conditions (events or objects) and changing them from analog to digital values that a computer can process. In other words, a data acquisition
element of a computerized pattern recognition system is considered as the equivalent to the sensing organs in humans.
- Feature Extraction and Representation: It is the procedure for transforming raw data (digit values) into a set of specific attributes or features. The input features are then processed by using some form of mathematical function to provide informative and representative measurements for the raw data.
- Similarity Detection and Pattern Classification: It is the procedure for categorizing and assigning the input features into one of the target clusters (for unsupervised learning) or target classes (for supervised learning) by applying some form of decision rule.
- Performance Evaluation: It is the procedure for applying mathematical measurements to estimate the effectiveness of the computerized pattern recognition system quantitatively, normally based on a set of new and unseen data samples.

The focus of this thesis is on the pattern classification aspect of the overall design and development of a computerized pattern recognition system.

Many methods have been developed for pattern classification. One of the earliest methods for pattern classification was statistical approaches, which were started by designing classical linear discrimination methods proposed in Fisher (1936) and Rao (1948). Then, the Bayesian decision method became one of the most popular statistical approaches for pattern classification (Devijver and Kittler, 1982; Duda and Hart, 1973). However, statistical approaches have difficulties in handling contextual/structural information in patterns, as indicated in Pal and Pal (2002). This
problem was then tackled by using syntactic approaches related to the theory of formal language for pattern classification (Hopcroft and Ullman, 1979). While syntactic approaches work fine for idealized patterns, they are inefficient in handling noisy and distorted patterns (Pal and Pal, 2002). On the other hand, the use of classification trees constitutes another useful method for pattern classification (Breiman et al., 1984). Nevertheless, the method faces the same inefficiency problem as syntactic approaches in dealing with noisy, distorted patterns (Pal and Pal, 2002).

Recently, Computational Intelligence (CI) (Bezdek, 1994) models have emerged as one of the useful methods for pattern classification. CI is a new field that capitalizes on interdisciplinary theories and principles for designing and developing computerized intelligent systems (Jain et al., 2008). In the following sections, a definition of CI is provided. The motivations for developing an ensemble of CI-based systems are given. Then, the research objectives and scope are explained, which is followed by the research methodology. Finally, an overview of the organization of this thesis is presented.

### 1.2 Computational Intelligence

Imitating human behaviours is the main driving force that inspires researchers to develop CI-based systems. Bezdek (1994) introduced one of the earliest definitions for CI , as follows:
"...A system is computationally intelligent when it: deals with only numerical (low-level) data, has pattern recognition components, does not use knowledge in the AI sense; and additionally when it
(begins to) exhibit i) computational adaptivity, ii) computational fault tolerance, iii) speed approaching human-like tumaround and iv) error rates that approximate human performance."

CI is an emerging field which comprise of a highly interdisciplinary framework that can solve various problems with the use of computers to perform numerical calculations (Rutkowski, 2008). A number of paradigms exist under the umbrella of CI models (Rutkowski, 2008), which include neural networks (Oong and Isa, 2011), fuzzy logic (Chen et al., 2012), evolutionary algorithms (Michalewicz, 1996), rough sets (Pawlak, 1992), and probabilistic methods (Pagan and Ullah, 1999). There are a lot of successful applications of CI-based systems in different areas, which include industrial (West and West, 2000), web intelligence (Cercone et al., 2002), management (Meesad and Yen, 2003), finance and economics (Isasi et al., 2007), medical decision (Papageorgiou, 2009), future power systems (Vale et al., 2011), as well as education (Kim and Cho, 2013).

The focus of this thesis is on Artificial Neural Network (ANN) and other complementary paradigms for developing useful pattern classification systems. An ANN is a mathematical representation model inspired from the biological neural network in the human brain. One of the earlier attempts to understand the organizing principles of the brain was implemented in McCulloch and Pitts (1943). The aim was to imitate the biological neural structure by formulating a mathematical model of biological neurons. The attempt then brought about the development of ANNs. To date, there are a number of different ANN architectures, which include the MultiLayer Perceptron (MLP) networks (Rumelhart and Zipser, 1986), Hopfield network
(Hopfield, 1982; Hopfield, 1984), and Radial Basis Function (RBF) network (Lowe and Broomhead, 1988).

### 1.3 Problems and Motivations

ANNs have emerged as one of the popular methods in tackling pattern classification problems. ANNs are useful for handling noisy data collected from real environments. The learning property of ANNs gives them the ability to recognize different types of input patterns. Most of the ANN learning strategies are related to batch or offline learning (Puttige and Anavatti, 2007). However, one of the problems of batch learning in ANN models, such as MLP and RBF, is catastrophic forgetting (Robins, 1993; Ratcliff, 1990; McCloskey and Cohen, 1989). The phenomenon of catastrophic forgetting is concerned with the inability of an ANN to remember what it has previously learned when new information is learned by the ANN (Polikar et al., 2001; Polikar et al., 2000).

There have been many attempts from researchers to solve this catastrophic forgetting problem. One of these attempts was by McCloskey and Cohen (1989), where they used the back-propagation ANN to understand the catastrophic forgetting problem. They found that the network created a new solution based only on the most recent information, when it was given multiple materials to learn (McCloskey and Cohen, 1989). This is obviously different from the functionality of the human brain.

On the other hand, the stability plasticity dilemma in learning systems (Simpson, 1992; Goldberg, 1989; Grossberg, 1980) is also related to the catastrophic
forgetting problem. The dilemma aims to address a number of issues, e.g. how a learning system can remain plastic enough to learn new information, and how to retain previously learned information when new information is provided (Carpenter and Grossberg, 1987; Carpenter and Grossberg, 1988). Solving the stability plasticity dilemma is a crucial issue in ANN learning, especially when the number of data samples increases with time and the ANN has to learn these samples in an autonomous and incremental manner. As a result, Simpson $(1992,1993)$ proposed two hybrids ANN models, i.e., the Fuzzy min-max (FMM) networks, in an attempt to combat this stability-plasticity dilemma. The first ANN is for pattern classification (Simpson, 1992); while the second is for pattern clustering (Simpson, 1993). In this thesis, the focus is on the supervised FMM (hereafter known as just FMM) network for pattern classification, owing to a number of reasons, as follows.

The FMM network is a supervised CI model which integrates both ANN and fuzzy set theory together in a unified framework. It uses hyperbox fuzzy sets to create and store knowledge (as hidden nodes) in its network structure. Each hyperbox is defined by its minimum (min) and maximum (max) points in an $n$ dimensional pattern space. The fuzzy part in FMM is created by combining the hyperbox min-max points with the fuzzy membership function. The fuzzy membership function determines the degree by which an input pattern belongs to one particular class or another. There are a number of useful properties for FMM to handle pattern classification problems (Simpson, 1992):
a) Online Learning: the ability to learn new classes and refine existing classes quickly and without losing old information. This property is important to tackle the stability plasticity dilemma.
b) Nonlinear Separability: the ability to build decision regions that separate classes of any shape and size.
c) Overlapping Classes: the ability to form a decision boundary to minimize the misclassification rate by eliminating the overlapping regions from different classes.
d) Training Time: the ability to learn and revise the decision boundaries of different classes within a short training time and using only one pass learning.
e) Soft and Hard Decisions: the ability to provide both soft and hard classification decisions. The hard decision indicates whether a pattern is in a specific class or otherwise (either 0 or 1 ), while the soft decision describes the degree to which an input pattern fits within a particular class.

All the above salient properties make FMM a unique pattern classifier. However, there are rooms to enhance the FMM learning algorithm. In particular, its expansion, overlapping test, and contraction processes need further improvements. Another shortcoming of FMM is its network complexity and noise tolerance capability. Learning with large data sets increases the FMM network complexity, while learning with noisy data samples results in spurious knowledge stored in its network structure. All these limitations affect the performance of FMM. Therefore, this thesis addresses techniques and strategies to solve these limitations and improve the robustness of FMM for tackling pattern classification problems.

In addition to a robust and efficient learning algorithm, improved performance in CI-based systems for pattern classification can also be achieved by using multiple classifier decisions, instead of using single classifier. In this case, each classifier has to make independent classification errors. By using multiple classifiers with a suitable decision combination scheme, the chance of avoiding classification errors and getting the correct decision for each input pattern can be increased; hence improving the overall classification performance. One of the ways is to use ensemble methods to combine the decisions from a set of classifiers (Dietterich, 2000). In this aspect, the Multi Agent System (MAS) is a viable ensemble method, which has been widely used in different fields, e.g. decision support (Fazlollahi and Vahidov, 2000), industrial steel processing (Gao et al., 2003), robot navigation (Ambastha et al., 2005), power systems (Baxevanos and Labridis, 2007), medical service (Lopez et al., 2008), mobile agent technology (Chen et al., 2009), urban traffic signal control (Balaji and Srinivasan, 2010), military network (Hancock and Lamont, 2011), and energy management (Mets et al., 2012). While the MAS framework can be used for building an ensemble of pattern classifiers, designing an effective MAS model is not as simple as it seems. One of the potential problems is the trust measurement of MAS agents (Yu et al., 2013), i.e. how can one agent trust another agent, and how to measure the trustworthiness of an agent. Therefore, this thesis investigates how to solve these problems in an MAS model, in order to improve the robustness of MAS functioning as a useful ensemble method for tackling pattern classification problems.

### 1.4 Research Aim and Objectives

The main aim of this research is to investigate the efficacy of the FMM network as a useful and usable pattern classification system. In addition to algorithmic investigations of FMM, the MAS framework is capitalized as a robust ensemble method to devise a multiple classifier model. The rationale is to formulate a useful trust measurement for exploiting the strength of different FMM-based classifiers using the MAS framework. The research objectives are as follows:

1. to propose an Enhanced FMM (EFMM) model by modifying the learning algorithm of FMM through tackling issues related to the expansion, overlapping test, and contraction processes among different classes;
2. to reduce the EFMM network complexity and enhance its noise tolerance ability in handling large and/or noisy data sets;
3. to devise a novel trust measurement scheme that is able to differentiate between good and poor predictions from different classifiers based on performance indicators;
4. to evaluate the usefulness of individual and ensemble EFMM-based models in undertaking pattern classification problems using both benchmark and realworld data sets, quantify their performances using statistical indicators, as well as analyze and compare their effectiveness with different classifiers.

A step-by-step approach is taken in this research to achieve the above objectives. The research methodology is explained in the next section.

### 1.5 Overview of Methodology

Figure 1.1 depicts an overview of this research. The scope of this research is focused on pattern classification, which is an important part of the overall pattern recognition system as explained in section 1.1. As such, the FMM neural network as single classifier and the MAS framework that supports multiple decisions are discussed, studied, and analysed in this research. The FMM network is selected as the backbone for developing the pattern classifier used in this research, owing to a number of salient features, especially the ability to combat the catastrophic forgetting problem or the stability-plasticity dilemma as explained in section 1.2. In order to enhance the classification performance, two directions are focused: (i) enhancing the learning algorithm of FMM; and (ii) devising a robust MAS-based ensemble framework to combine the predictions from multiple EFMM-based networks.

In this research, in-depth investigations are conducted in two stages, i.e., the learning stage of individual EFMM networks, and the decision combination stage of multiple EFMM networks using the MAS framework. In the learning stage, the challenge is to overcome the expansion, overlapping test, and contraction processes among different classes in FMM and EFMM. In addition, the network complexity and noise tolerance issues resulting from large, noisy data sets are taken into consideration. In the decision combination stage, the challenge is to formulate a useful trust measurement scheme for an ensemble of EFMM-based networks in an MAS framework. This problem is viewed from the angle of "how to measure trust for the predictions from different agents with different (good and poor) performances, and how to arrive at the best final decision". As a result, a number of enhancements to the learning algorithm of FMM are proposed. The EFMM networks
are evaluated thoroughly and systematically, then, the MAS framework is utilized, and a novel trust measurement scheme is proposed. All these investigations aim to improve the classification performance of EFMM-based models, making them a robust and useful pattern classifier for tackling real-world problems. Numerous simulations using benchmark and real data sets are conducted along the course of this research, with the results analyzed, discussed, and compared with those from other related classifiers reported in the literature.


Figure 1.1: Research relationships

A summary of the research methodology is shown in Figure 1.2. The following activities are conducted in order to achieve the overall research objectives.

- Step 1: Examining the FMM and EFMM learning algorithms. Three aspects of improvements are identified in order to overcome some of the existing limitations, i.e. the expansion, overlapping test, and contraction processes.
- Step 2: Benchmarking the EFMM network with publicly available data sets. The results are analyzed and compared with the original FMM network and those from other methods reported in the literature. This step is necessary to evaluate the effectiveness of the enhanced processes in EFMM.
- Step 3: Reducing the network complexity and increasing the noise tolerance capability of the EFMM network. Useful techniques to select the winning hyperbox and to prune hyperboxes with low confidence factors in EFMM are proposed.
- Step 4: Benchmarking the second enhanced FMM network (EFMM2) with publicly available noisy and noise-free data sets. The results are analyzed and compared with the previous FMM and EFMM networks, and with those from other methods reported in the literature. This step is necessary to evaluate the effectiveness of the EFMMs performance in terms of network complexity and noise tolerance capability, in addition to classification accuracy.
- Step 5: Formulating a trust measurement scheme for a Multi-Agent Classifier System (MACS), with the EFMM-based networks (i.e., EFMM and EFMM2) as its constituent agents. This MACS model is designed to tackle issues related to multiple predictions from an ensemble of EFMM-based networks, each with a different performance indicator. This step aims to devise an MACS framework for combining different EFMM-based networks developed in previous steps.
- Step 6: Demonstrating the efficacy of the MACS model with publicly available noisy, noise-free data sets, as well as, real-world data sets. The results are analyzed and compared with individual EFMM-based networks and those from other methods reported in the literature. This step is necessary to ascertain the efficacy of the MACS model in real-world environments.


Figure 1.2: Research methodology

### 1.6 Thesis Outline

This thesis is organized in accordance with the objectives mentioned above. A review of methods and models for improving classification performances based on CI is given in Chapter 2. The review covers the FMM neural network, ensemble methods for pattern classification, as well as multi agent systems as a useful ensemble method.

The learning dynamics of FMM is explained in Chapter 3. A detailed description of the FMM learning algorithm along with a numerical example is presented. Limitations of FMM learning algorithm are highlighted and analyzed. Based on the analysis, novel modifications are proposed to enhance the FMM learning algorithm; hence resulting in the EFMM network. A number of simulations are conducted using benchmark data sets, and the results are compared with those from other methods published in the literature.

Complexity and noise are important issues in pattern classification. As such, a review of existing methods to tackle these issues is presented in Chapter 4. The problems related to network complexity in EFMM are analyzed and discussed. Novel modifications for EFMM are further proposed; hence resulting in EFMM2. Again, a number of simulation studies are conducted using benchmark data sets, and the results are compared with those obtained from other methods.

In Chapter 5, the notion of agent and the MAS model is introduced. A review of trust and its importance in MAS is presented. A novel trust measurement scheme
is proposed, which is known as Certified Belief in Strength (CBS). CBS is incorporated into the MACS model, which comprise EFMM and EFMM2 networks as its agents. The resulting system, MACS-CBS, is evaluated using a number of benchmark data sets, and the results are compared with those from other methods.

To demonstrate the applicability of the MACS-CBS model devised in Chapter 5, and the individual EFMM2 networks proposed in Chapter 4, two real-world case studies are considered in Chapter 6. These data sets are obtained from the medical and power systems domains, as an attempt to ascertain the efficacy of MACS-CBS in real-world environments.

Finally, conclusions are drawn in Chapter 7. Contributions of this research as well as a number of areas to be pursued as further work are presented in Chapter 7 too.

## CHAPTER 2

## LITERATURE REVIEW

### 2.1 Introduction

As explained in Chapter 1, the main focus of this research is to investigate the efficiency of the FMM-based network as a useful and usable pattern classification system. Besides that, the MAS model is capitalized as a robust ensemble framework to devise a multiple classifier system. As such, this chapter presents a review on the FMM network and its variants, ensemble methods, as well as MAS models, whereby all these are utilized in this research. A summary of the review is presented at the end of this chapter.

### 2.2 Background of the Fuzzy Min-Max Neural Network

An artificial neural network (ANN) is a computational model that consists of an interconnected group of artificial neurons organized in a network structure, which emulates the biological neural system (Li and Ma, 2010; Graupe, 1997). Nowadays, ANNs are widely used in many fields, which include healthcare (Lin et al., 2013; Das and Kundu, 2013), business (Salles et al., 2011), marketing (Abhishek et al., 2012; Azcarraga et al., 2008), financial economics (Li and Ma, 2010), security (Alvarez, 2009; Teoh and Tan, 2010), power (Wei, 2010; Wu and Rastgoufard, 2004), robot programming (Stoica et al., 2010), fault detection (Seera et al., 2012; Seera and Lim, 2013), and airline (Turkmen and Korkmaz, 2010). Among different domains, pattern classification is one of the active areas of ANN applications
(Zhang, 2000). As an example, ANN models have been successfully applied to a variety of real-world classification tasks in industry, business, and science (Zhang, 2000); medical prognosis and diagnosis (Economou et al., 1994), harmonic currents (Yap et al., 2011), as well as industrial fault detection and diagnosis (Quteishat et al., 2009; Seera et al., 2012; Seera and Lim, 2013).

Out of many different types of ANNs, the Fuzzy Min Max (FMM) neural network and its variants have been the focus of many researchers for tackling pattern classification problems (Simpson, 1992; Simpson, 1993; Gabrys and Bargiela, 2000; Nandedkar and Biswas, 2007a; Nandedkar and Biswas, 2007b; Quteishat and Lim, 2008; Zhang et al., 2011; Bargiela et al., 2004; Kim and Yang, 2005). The design of these FMM-based classifiers originates from two FMM models introduced by Simpson (Simpson, 1992; Simpson, 1993). FMM was first presented as a supervised classification neural network (Simpson, 1992), and later as an unsupervised clustering neural network (Simpson, 1993). Both FMM networks combine ANN and fuzzy set theory into a common framework for tackling pattern classification and clustering problems. The FMM structure is built from hyperboxes. A hyperbox is defined by its minimum and maximum points which are encoded from the input patterns. The FMM learning algorithm consists of three steps: expansion, overlapping test, and contraction (Simpson, 1992). The learning procedure in FMM starts by selecting an input pattern and then finding the closest hyperbox that matches the input pattern, as shown in Figure 2.1. The closest hyperbox in the FMM model is found by using a fuzzy membership function, which is defined with respect to hyperbox min-max points (Simpson, 1992).


Figure 2.1: The FMM learning process

The membership function represents the degree to which an input pattern fits in the hyperbox, and the membership value ranges between 0 and 1 . If the input pattern does not belong to any hyperboxes, even with the expansion process in FMM, a new hyperbox is created to include the input pattern. The overlapping test is carried out to check whether there are any overlaps among hyperboxes from different classes caused by the expansion process, while the contraction process takes place to
eliminate the overlapping area. In other words, FMM entails a dynamic network structure with an online learning capability whereby the number of hyperboxes can be increased when necessary; therefore, avoiding the problem of re-training as faced by many neural network models with an off-line learning capability (Nakashima et al., 2010; Odeh and Khalil, 2011). Further details of FMM can be found in Chapter 3.

A number of FMM variants are available in the literature. An extension of the FMM classification network known as the General Fuzzy Min-Max (GFMM) neural network was proposed by Gabrys and Bargiela (2000). GFMM is established using the expansion and contraction principles, and is able to handle both labelled and unlabelled data simultaneously. The structure of GFMM is similar to that of FMM with three network layer. The advantages of GFMM include the abilities to process input data such as confidence limits, incorporate new information, avoid retraining the network, and combine both supervised and unsupervised learning strategies within a single structure (Gabrys and Bargiela, 2000).

A stochastic FMM network for reinforcement learning was proposed by Likas (2001), which was an extension of the reinforcement FMM model (Likas and Blekas, 1996). It uses the concept of random hyperboxes for reinforcement learning problems. Unlike FMM, the proposed extension (Likas, 2001) uses a stochastic automaton instead of the action label (or class label), where the probability vector of the stochastic automaton determines the corresponding action through random selection. Each hyperbox is associated with a stochastic learning automaton. The location, boundaries of each hyperbox, as well as probability vector of each stochastic automaton are adjusted by the stochastic FMM network.

Inspired by the FMM network, the adaptive resolution Min-Max model was proposed as a new neuro-fuzzy classifier (Rizzi et al., 2002). It employs two algorithms, i.e. the Adaptive Resolution Classifier (ARC) and the Pruned Adaptive Resolution Classifier (PARC). In the proposed model, the hyperbox expansion process is not limited by a fixed maximum size, in order to overcome some undesired properties of the original FMM algorithm. As such, the proposed model possesses a less complex network structure, as compared with original FMM (Rizzi et al., 2002).

In Bargiela et al. (2004), an inclusion/exclusion fuzzy hyperbox classifier was proposed. As its name implies, this model creates two types of hyperboxes, i.e., the inclusion and exclusion hyperboxes. The purpose of the inclusion hyperboxes is to contain the input patterns that belong to the same class. Other overlapped patterns are contained by the exclusion hyperboxes. The use of the exclusion hyperboxes helps reduce the training process from three steps (expansion, overlap test, and contraction) to two (expansion and overlap test). This is achieved through a contentious area of the pattern space to approximate the complex topology of data samples, which helps to solve the overlapping problem in FMM (Bargiela et al., 2004).

A Weighted FMM network (WFMM) was proposed by Kim and Yang (2005). In this model, both the hyperbox contraction and overlap test steps do not restrict hyperbox expansion. A new membership function and a learning method (hyperbox creation, expansion, contraction, and weight update) are defined in WFMM (Kim and Yang, 2005). On the other hand, a new model called FMM with compensatory neuron (i.e., FMCN) was proposed by Nandedkar and Biswas (2007a). Based on the compensatory neuron (CN) architecture, FMCN is used as a supervised
classification model that supports online learning. The CNs are activated when the test sample falls in the overlapped area between two different classes, with the hyperbox contraction process eliminated.

Later, another General Reflex Fuzzy Min-Max Neural Network (GRFMN) was proposed by Nandedkar and Biswas (2007b). GRFMM combines both FMM clustering and classification algorithms as well as the concept of human reflex mechanism to solve the problem of class overlaps (Nandedkar and Biswas, 2007b; Ries et al., 2006; Yuan et al., 2006) into one framework. GRFMN offers on-line training, and exhibits a better ability to identify the underlying data structure as compared with GFMM; hence enhancing classification accuracy (Nandedkar and Biswas, 2007b).

A Modified FMM (MFMM) network that aimed to improve the FMM classification performance was proposed by Quteishat and Lim (2008). MFMM is able to tackle the problem related to a small number of large hyperboxes formed in the network as well as to facilitate rule extraction. In MFMM, the data set is divided into three sub-sets: a training set for learning; a prediction set for pruning and rule extraction; and a test set for performance evaluation. MFMM improves the classification performance with a two-stage process, i.e. (i) network pruning and rule extraction; (ii) prediction by using the Euclidean distance and membership function. Later, MFMM was further enhanced with a Genetic Algorithm (GA)-based rule extractor to form another model called FMM-GA (Quteishat et al., 2010). In the particular study, an application of FMM-GA to medical diagnosis problems has been demonstrated.

A Data-Core-Based Fuzzy Min-Max Neural Network (DCFMN) for pattern classification was recently proposed by Zhang et al. (2011). DCFMN uses new membership functions for two types of neurons, i.e., Classifying Neurons (CNs) and Overlapping Neurons (OLNs). These membership functions take into consideration three factors: noise, the geometric center of the hyperbox, and the data core. The FMM contraction processes are removed in DCFMN. When a new pattern falls in the overlapped area of different classes, DCFMN uses the OLN membership to determine the target class of the new pattern (Zhang et al., 2011).

Table 2.1 shows the characteristics of FMM-based networks discussed in the section. Even though many investigations have been conducted to improve the original FMM neural network, some limitations in the FMM learning algorithm remain unsolved (Hyperbox expansion rule, overlapping test rule, contraction rule, network complexity and noise tolerance capability), which affect its performance. It can be observed that all FMM variants suffer from at least two limitations, as shown in Table 2.2. As such, there are rooms for improving the FMM learning algorithm and performance, and making it a more robust classifier.

In this thesis, two strategies are adopted to enhance FMM. The first is to improve its learning algorithm by solving issues related to overlapping hyperboxes, as detailed in Chapters 3 and 4. Instead of single classifier, the second strategy examines the use of a multiple classifier framework to form an ensemble of classifiers so that the overall performance can be improved, as presented in Chapter 5. The next section reviews some of the ensemble methods that are related to this research.

Table 2.1: A summary of the reviewed FMM-based networks

| Model | Characteristics |
| :---: | :---: |
| FMM (Simpson, 1992) | Suitable for supervised learning; combine ANN and fuzzy set theory into a common framework for tackling pattern classification problems; supports on-line adaptation which enables it to avoid the retraining process; |
| FMM (Simpson, 1993) | Suitable for unsupervised learning; combine ANN and fuzzy set theory into a common framework for tackling pattern clustering problems; supports on-line adaptation. |
| General Fuzzy Min-Max (GFMM) (Gabrys and Bargiela, 2000) | Handles both labeled data and unlabeled data; suitable for supervised and unsupervised learning. |
| Stochastic FMM (Likas, 2001) | Uses a stochastic automaton, instead of the action label or class label in original FMM; suitable for reinforcement learning. |
| Adaptive resolution Min-Max model (Rizzi et al., 2002) | Comprises of the Adaptive Resolution Classifier (ARC) and Pruned Adaptive Resolution Classifier (PARC); the hyperbox expansion process is not limited by a fixed maximum threshold as in the original FMM algorithm; possesses a less complex network structure than that of the original FMM. |
| Inclusion/Exclusion fuzzy hyperbox classifier (Bargiela et al., 2004) | Uses the inclusion hyperboxes to contain input patterns from the same class; uses the exclusion hyperboxes to contain overlapped patterns. |
| Weighted FMM network (WFMM) (Kim and Yang, 2005) | Does not restrict hyperbox expansion through the hyperbox contraction and overlap test steps; defines a new membership function; defines a new learning method for hyperbox creation, expansion, contraction, and weight update. |
| FMM with compensatory neuron (FMCN) (Nandedkar and Biswas, 2007a) | Uses the compensatory neuron ( CN ) architecture; uses new membership functions for two types of neurons, Classifying Neurons (CNs) and Overlapping Neurons (OLNs). |
| General Reflex Fuzzy MinMax Neural Network (GRFMN) (Nandedkar and Biswas, 2007b) | Combines both FMM clustering and classification algorithms; <br> deploys the concept of human reflex mechanism to solve the problem of class overlaps. |
| Modified FMM (MFMM) (Quteishat and Lim, 2008) | Performs network pruning and rule extraction; uses the Euclidean distance and membership function for prediction. |
| Modified FMM with Genetic <br> Algorithm (MFMM-GA) <br> (Quteishat et al., 2010) | Performs network pruning and rule extraction; uses the Euclidean distance and membership function for prediction; incorporates a Genetic Algorithm (GA) - based module for rule extraction. |
| Data-Core-Based Fuzzy MinMax Neural Network (DCFMN) (Zhang et al., 2011) | Uses membership functions for Classifying Neurons (CNs) and Overlapping Neurons (OLNs); <br> Removing the contraction process. |

Table 2.2: Limitations of the FMM-based networks

| Limitations <br> Model Of | Expansion rule | Overlapping test rule | Contraction rule | Complexity | Noise tolerance |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Simpson (1992) | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ |
| Simpson (1993) | $\checkmark$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ |
| Gabrys and Bargiela (2000) | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ |
| Likas (2001) | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | X | $\sqrt{ }$ |
| Rizzi et al. (2002) | $\checkmark$ | $\sqrt{ }$ | $\checkmark$ | X | X |
| Bargiela et al. (2004) | $\sqrt{ }$ | $\sqrt{ }$ | X | $\sqrt{ }$ | $\sqrt{ }$ |
| Kim and Yang (2005) | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | X |
| Nandedkar and Biswas (2007a) | $\sqrt{ }$ | $\sqrt{ }$ | X | $\sqrt{ }$ | $\sqrt{ }$ |
| Nandedkar and Biswas (2007b) | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ |
| Quteishat and Lim (2008) | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | X | X |
| Quteishat et al. (2010) | $\sqrt{ }$ | $\sqrt{ }$ | $\sqrt{ }$ | X | X |
| Zhang et al. (2011) | $\sqrt{ }$ | $\sqrt{ }$ | X | X | X |



### 2.3 Ensemble Methods

As explained in the previous section, FMM is a useful neural network that is able to produce good and accurate results in undertaking pattern classification problems. In FMM (as well as other online ANN models), learning can be unstable whereby any small change in the sequence of the training samples, or parameters of the network could affect the network performance (Navone, 2001). Therefore, it is useful to enhance the learning algorithm and improve its performance. To minimize the classification errors, one useful way is to deploy a group of classifiers for

