NEW SINGLE VARIABLES CONTROL CHARTS BASED ON THE DOUBLE EWMA STATISTICS

TEH SIN YIN

UNIVERSITI SAINS MALAYSIA

2012

NEW SINGLE VARIABLES CONTROL CHARTS BASED ON THE DOUBLE EWMA STATISTICS

by

TEH SIN YIN

Thesis submitted in fulfillment of the requirements
for the degree of
Doctor of Philosophy

February 2012

In loving memory of my mother

Law Siew Gaik

ACKNOWLEDGEMENTS

I would like to take this golden opportunity to acknowledge and express my sincere appreciation and gratitude to several people for continuously providing me with ideas, advice, guidance, support and assistance throughout my postgraduate studies until the completion of this thesis. Their commitments have given me the added strength to complete this daunting task. I would like to express my utmost appreciation to the numerous sources of support behind this effort.

First and foremost, my deepest appreciation goes to my Ph.D. supervisor, Professor Michael Khoo Boon Chong for his invaluable guidance, relentless contributions of ideas, suggestions, useful information, constant advice, persistent encouragement and generous contribution of his valuable time in guiding me from the beginning of my research till the end. He has continuously inspired me by showing enthusiasm in my research. His knowledge, professional expertise and rich experience in statistical quality control have greatly benefited me in my research. He is also my source of inspiration and determination that surge me forward in my task.

My gratitude is also extended to the School of Management (SoM), Universiti Sains Malaysia (USM), and the Malaysian Ministry of Higher Education for awarding me the Academic Staff Training Scheme (ASTS) to financially support me in my postgraduate study. I am so grateful to SoM and the School of Mathematical Sciences (PPSM), USM, for giving me a great deal of opportunities of exposure (by attending workshops and conferences) to sharpen my skills and knowledge in the area of statistical quality control. I am indebted to the Dean of PPSM, USM, Professor Ahmad Izani Md. Ismail, the Dean of SoM, YBhg. Dato Professor Ishak Ismail, lecturers and staff of the two departments for their kindness and hospitality.

Special thanks and appreciation goes to Professor Michael Khoo and Mr. Foo Kok Keong, for their kind assistance in providing inputs on my writing of this thesis. I am grateful to their assistance in improving the quality of this thesis, particularly in terms of the English language. I would like to extend my thanks to both of my former supervisors, Professor Abdul Rahman bin Othman and Professor Low Heng Chin, for giving me unwavering encouragement and moral support.

I would also like to take this opportunity to express my sincere thanks to my friends who have constantly given me encouraging support and motivation throughout my study. They were always willing to support me spiritually and physically whenever help was needed. My profound acknowledgements go to my lab mates who have allowed me to use their computers to run simulations. I wish to record my special appreciation to Mr. Low Chun Kit, Mr. Wong Voon Hee and Mr. Ang Lee Sin for providing valuable computing assistance during my difficult times.

Last but not least, my affectionate appreciation goes to my loving family members and dearest relatives for their love, understanding, patience, sacrifices, constant prayers, confidence and endless encouragements at all times during my study. I would like to convey my heartfelt gratitude to my beloved father, brother and two sisters, for being the pillars of my strength and inspiration. Their continued moral support is a boost to my morale that has continuously driven and motivated me to always do my best.

In closing, my grateful recognition goes to all my friends who have rendered me their friendship, lecturers who have given me unwavering encouragements and tremendous support. Once again, my cordial appreciation goes to those who have directly or indirectly contributed in bringing this research into a completion. Words are inadequate to express my gratitude and I love you all with all my heart.

And may the peace and God’s blessings be upon all of you

TABLE OF CONTENTS

CHAPTER 3 – DEWMA–MAX CHART

CHAPTER 4 – MAX–DEWMA CHART

CHAPTER 5 – SS–DEWMA CHART

CHAPTER 6 – COMPARISONS OF THE PERFORMANCES OF THE PROPOSED CHARTS

CHAPTER 7 – CONCLUSIONS

APPENDIX A

APPENDIX B

APPENDIX C

LIST OF TABLES

LIST OF FIGURES

LIST OF NOTATIONS

The following abbreviations and notations are used in the thesis:

ARL	Average run length
ARL_0	In-control ARL
ARL_1	Out-of-control ARL
cdf	Cumulative distribution function
CL	Center line
CUSUM	Cumulative sum
CV-DEWMA	Coefficient of variation DEWMA
CV-EWMA	Coefficient of variation EWMA
DCUSUM	Double CUSUM
DEWMA	Double EWMA
DEWMA-Max	DEWMA maximum
DGWMA	Double generally weighted moving average
DOE	Design of experiments
EWMA	Exponentially weighted moving average
EWMA-Max	EWMA maximum
EWMA-SC	EWMA semicircle
LCL	Lower control limit
Max	Maximum
Max-DEWMA	Maximum DEWMA
Max-EWMA	Maximum EWMA
PCB	Printed circuit board
pdf	Probability density function
QC	Quality control

SAS	Statistical Analysis Software
SDRL	Standard deviation of the run length
$SDRL_1$	Out-of-control SDRL
SPC	Statistical process control
SQC	Statistical quality control
SS-DEWMA	Sum of squares of DEWMA
SS-EWMA	Sum of squares of EWMA
TQM	Total quality management
UCL	Upper control limit
UCL_{DM}	Upper control limit of the DEWMA-Max chart
UCL_{EM}	Upper control limit of the EWMA-Max chart
UCL_{MD}	Upper control limit of the Max-DEWMA chart
UCL_{ME}	Upper control limit of the Max-EWMA chart
UCL_{SD}	Upper control limit of the SS-DEWMA chart
UCL_{SE}	Upper control limit of the SS-EWMA chart
a	Magnitude of a shift in the mean
a_{opt}	Magnitude of a shift in the mean, where a quick detection is needed
b	Magnitude of a shift in the standard deviation
b_{opt}	Magnitude of a shift in the standard deviation, where a quick detection is needed
c	Count of non conformities
c_4	The $\bar{X} - S$ charts' constant

C^+	One-sided upper CUSUM statistics
C^-	One-sided lower CUSUM statistics
d_2	The $\bar{X} - R$ charts' constant
$E(X)$	Mean of a random variable X
H	Decision interval for the tabular CUSUM chart
$H_{n_i-1}(\cdot)$	Chi-square cumulative distribution function with $n_i - 1$ degrees of freedom
K	A multiplier controlling the width of the EWMA chart's limits
K_{DM}	A multiplier controlling the width of the DEWMA-Max chart's limit
K_{EM}	A multiplier controlling the width of the EWMA-Max chart's limit
K_{MD}	A multiplier controlling the width of the Max-DEWMA chart's limit
K_{ME}	A multiplier controlling the width of the Max-EWMA chart's limit
K_{SD}	A multiplier controlling the width of the SS-DEWMA chart's limit
K_{SE}	A multiplier controlling the width of the SS-EWMA chart's limit
m	Number of in-control samples used in the estimation of parameters in a Phrase-I process
n	Constant sample size
n_i	Size of the i^{th} sample, for $i = 1, 2, \ldots$

$\bar{n}$	Average sample size
np	Number of nonconforming
p	Fraction nonconforming
R	Sample Range
$\bar{R}$	Average sample range
$\bar{S}$	Average sample standard deviation
S^2	Sample variance
u	Count of non conformities per unit of inspection
$V(X)$	Variance of a random variable X
$X \sim N\left(\mu, \sigma^2\right)$	A random variable X, having a normal distribution with mean μ and variance σ^2
X_{ij}	The j^{th} observation in the i^{th} sample, for i = 1, 2, … and j = 1, 2, …, n_i
$\bar{X}$	Sample mean of random variable X
$\bar{\bar{X}}$	Sample grand average of a random variable X
μ_0	Target value of the process mean
μ	Process mean
σ_0^2	Target value of the process variance
σ^2	Process variance
α	False alarm rate or Type-I error size
λ	EWMA chart's smoothing constant
$\Phi(\cdot)$	Standard normal cumulative distribution function
$\Phi^{-1}(\cdot)$	Inverse standard normal cumulative distribution function

$\phi(\cdot)$	Standard normal probability density function
$[y]$	The largest integer smaller than or equal to y
$m+$	An increase in only the mean
$m-$	A decrease in only the mean
$v+$	An increase in only the variance
$v-$	A decrease in only the variance
$++$	Increases in both the mean and variance
$+-$	An increase in the mean but a decrease in the variance
$-+$	A decrease in the mean but an increase in the variance
$--$	Decreases in both the mean and variance
χ_r^2	Chi-square distribution with r degrees of freedom
$G[\beta_1, \beta_2]$	Gamma distribution with scale parameter β_1 and shape parameter β_2.

LIST OF PUBLICATIONS

Book Chapter

1. **Teh, S. Y.** and Michael Khoo, B. C. (2011). DATA STEP and PROC GPLOT in constructing a Max-DEWMA control chart. In M. T. Ismail and A. Mustafa (Eds.), *Research in mathematics and economics* (pp. 67-80). Universiti Sains Malaysia: Brand Interactive Malaysia. [ISBN: 978-967-394-007-3]

Journals

1. Michael Khoo, B. C., **Teh, S. Y.** and Wu, Z. (2010). Monitoring process mean and variability with one double EWMA chart. *Communications in Statistics-Theory and Methods*, 39, 3678-3694. [Print ISSN: 0361-0926; Online ISSN: 1532-415X] [5 years Impact factor (2010): 0.429]

2. **Teh, S. Y.**, Michael Khoo, B. C. and Low, C. K. (2010). Comparing the performance of the optimal SS-DEWMA and Max-DEWMA control chart. *Journal of Statistical Modeling and Analytics*, 1(2), 1-9. [ISSN: 2180-3102]

3. **Teh, S. Y.**, Michael Khoo, B. C. and Wu, Z. (2011). A sum of squares double EWMA chart. *Computers and Industrial Engineering*, 61(4), 1173-1188. [ISSN: 0360-8352] [5 years impact factor (2010): 1.823]

4. **Teh, S. Y.** and Michael Khoo, B. C. (2011). A study on the effects of non-normality on the performances of Max-DEWMA versus SS-DEWMA charts. *Journal of Mathematics and Statistics*. In press. [ISSN: 1549-3644]

5. **Teh, S. Y.** and Michael Khoo, B. C. (2009). Joint monitoring of process mean and variability with DEWMA-Max Chart. *Sains Malaysiana*. Under review. [ISSN: 0126-6039] [Impact factor (2010): 0.152]

6. **Teh, S. Y.** and Michael Khoo, B. C. (2010). Comparing the effect of non-normality on the performances of Max-DEWMA and Max-EWMA control charts. *Pertanika Journal of Science and Technology*. Manuscript submitted for publication. [ISSN: 0128-7680]

Proceedings

1. **Teh, S. Y.** and Michael Khoo, B. C. (2009). *Joint monitoring of process mean and variability with DEWMA-Max Chart*. The proceeding of the International Conference on Quality, Productivity and Performance Measurement 2009, Palm Garden Hotel, Putrajaya, 1-6. [ISSN: 978-967-5048-76-0]

2. **Teh, S. Y.** and Michael Khoo, B. C. (2009). *Comparing the performances of Max-DEWMA and Max-EWMA charts for skewed populations*. Proceeding of the Simposium Kebangsaan Sains Matematik Ke-17, Mahkota Hotel, Malacca, 714-721. [ISBN: 978-967-960-256-2]

3. **Teh, S. Y.**, Michael Khoo, B. C. and Low, C. K. (2010). *Comparing the performance of the optimal SS-DEWMA and Max-DEWMA control chart*. The proceedings of the Regional Conference on Statistical Science 2010, Renaissance Kota Bahru Hotel, Kota Bahru, Kelantan, 74-82. [ISBN: 978-967-363-157-5]

4. **Teh, S. Y.**, Michael Khoo, B. C. and Low, C. K. (2010). *Performance of the optimal Max-DEWMA chart versus the optimal Max-GWMA chart*. The proceeding of the Applied Mathematics International Conference 2010 & The Sixth East Asia SIAM Conference**,** PNB Darby Park Executive Suites, Kuala Lumpur, 85-90.

5. **Teh, S. Y.** and Michael Khoo, B. C. (2010). *Comparing the effects of skewed distributions on the performances of Max-DEWMA & SS-DEWMA control charts*. The proceeding of the 2nd International Conference on Mathematical Sciences, ICMS2 2010, PWTC, Kuala Lumpur, 523-530. [ISBN: 978-967-5878-03-9]

6. **Teh, S. Y.** and Michael Khoo, B. C. (2010). *A study on the effects of skewed distributions on the performances of Max-EWMA and Max-GWMA charts*. Abstracts book of the International Conference on Applied Statistics and Financial Mathematics, ASFM2010, The Hong Kong Polytechnic University, Hong Kong, 16.

7. **Teh, S. Y.**, Michael Khoo, B. C. and Low, C. K. (2011). *An evaluation of the steady state mode Max-DEWMA control chart*. Proceeding of the 5th Quality Conference in the Middle East, Atlantis, The Palm Dubai, United Arab Emirates, 397-405. [ISBN: 978-9948-03-638-8]

CARTA-CARTA KAWALAN TUNGGAL YANG BARU BAGI DATA SELANJAR BERDASARKAN STATISTIK EWMA GANDA DUA

ABSTRAK

Dalam situasi pemantauan kawalan proses berstatistik (SPC), min proses dan varians proses berkecenderungan beranjak secara serentak. Secara tradisional, dua carta kawalan yang berasingan, setiap satu untuk min dan varians digunakan secara serentak untuk memantau min proses dan varians proses. Walau bagaimanapun, dalam banyak situasi pemantauan proses yang sebenar, kawalan serentak min proses dan varians proses diperlukan. Hal ini telah mendorong kami untuk membangunkan carta-carta DEWMA tunggal (dikenali sebagai purata bergerak berpemberat eksponen ganda dua) yang mampu memantau anjakan serentak dalam kedua-dua min dan varians proses, apabila taburan pendasar proses adalah normal. Statistik DEWMA adalah berdasarkan pendekatan melaksanakan pelicinan eksponen sebanyak dua kali pada statistik asal proses pendasar. Objektif kajian ini adalah untuk mencadangkan tiga carta DEWMA tunggal, iaitu carta-carta DEWMA-Max (dikenali sebagai DEWMA maksimum), Max-DEWMA (dikenali sebagai maksimum DEWMA) dan SS-DEWMA (dikenali sebagai hasiltambah kuasa dua DEWMA). Selain membandingkan prestasi ketiga-tiga carta ini, setiap carta juga dibandingkan dengan carta EWMA tunggal yang setara dengannya. Pada keseluruhannya, keputusan simulasi menunjukkan bahawa prestasi carta-carta DEWMA-Max, Max-DEWMA dan SS-DEWMA mengatasi carta-carta EWMA tunggal setara, masing-masing, daripada segi prestasi purata panjang larian (ARL) dan sisihan piawai panjang larian (SDRL), serta keupayaan diagnostik dalam pengecaman sumber dan arah anjakan dengan tepat. Antara carta-carta yang dicadangkan, carta SS-DEWMA dan carta Max-DEWMA didapati mempunyai kelajuan terpantas dalam pengesanan anjakan kecil dan sederhana dalam min dan/atau varians proses.

Kata-kata kunci: carta DEWMA-Max; carta Max-DEWMA; carta SS-DEWMA; purata panjang larian (ARL); sisihan piawai panjang larian (SDRL); min proses; varians proses; carta kawalan tunggal

NEW SINGLE VARIABLES CONTROL CHARTS BASED ON THE DOUBLE EWMA STATISTICS

ABSTRACT

In Statistical Process Control (SPC) monitoring situations, there is a tendency for both the process mean and process variability to shift simultaneously. Traditionally, two separate control charts, each for the mean and variance are used concurrently to monitor the process mean and process variance. However, in many real life process monitoring situations, a simultaneous control of the process mean and process variance is necessary. This has motivated us to develop single DEWMA (called Double Exponentially Weighted Moving Average) charts which are capable of monitoring simultaneous shifts in both the process mean and process variance, when the underlying distribution of the process is normal. The DEWMA statistics are based on the approach of performing exponential smoothing twice on the original statistics of the underlying process. The objective of this study is to propose three single DEWMA charts, namely the DEWMA-Max (called the DEWMA maximum), Max-DEWMA (called the maximum DEWMA) and SS-DEWMA (called the sum of squares of DEWMA) charts. Besides comparing the performances of the three charts, each of these charts is also compared with its corresponding single EWMA chart counterpart. Overall, the simulation results show that the DEWMA-Max, Max-DEWMA and SS-DEWMA charts outperform their corresponding single EWMA chart counterparts, in terms of the average run length (ARL) and standard deviation of the run length (SDRL) performances, as well as the diagnostic abilities in identifying the source and direction of a shift accurately. Among the proposed charts, the SS-DEWMA chart and Max-DEWMA chart are found to have the quickest speed in detecting small and moderate shifts in the process mean and/or variance.

Key words: DEWMA-Max chart; Max-DEWMA chart; SS-DEWMA chart; average run length (ARL); standard deviation of the run length (SDRL); process mean; process variance; single control chart

CHAPTER 1
INTRODUCTION

1.1 Overview

Since the last decade, many organizations are increasingly concerned with improvements on quality, in order to survive in an increasingly competitive global market. In other words, quality improvement is becoming a major concern to many corporations. The field of statistical quality control (SQC) can be broadly defined as those statistical and engineering methods that are used in measuring, monitoring, controlling and improving quality (Gupta & Walker, 2007).

Dating back to the 1920s, Dr. Walter A. Shewhart of the Bell Telephone Laboratories was one of the pioneers who formulated a statistically-based approach to quality control or improvement. In 1924, he wrote a memorandum showing a modern control chart, which was the most powerful tool in statistical process control (SPC), used for monitoring the quality characteristics of a process over time. Dr. W. Edwards Deming (philosophy of 14 points) and Dr. Joseph M. Juran (philosophy of Quality Trilogy) were then influential in spreading SQC methods for quality management, quality planning, process control and process or product quality improvement (DeVor et al., 2007). Since then, SPC was proven to be an effective means to improve the quality and productivity of processes. From here onwards, a process is referred to as a set of causes and conditions that repeatedly come together to transform inputs into outputs. The inputs refer to raw materials, machineries, human resources and information, while the outputs refer to products and services (Thaga, 2003).

In any production process, regardless of how well-designed or carefully maintained it is, a certain amount of variability will always exist. Shewhart, Deming and Juran all clearly pointed out that the variability present in a process falls into two categories, i.e. common causes and assignable causes of variation (Montgomery, 2009). The common (chance) causes of variation or "background noise" are the cumulative effect of many small, essentially unavoidable causes inherent in the process. When the background noise in a process is relatively small, we usually consider it an acceptable level of process performance. In the framework of SQC, this common cause of variation is frequently called a "stable system of chance causes". A process that is operating with only the presence of common causes is said to be in statistical control. The assignable (special) causes of variation are sources of variability that are not part of the chance causes. Assignable causes which are occasionally present in a process (very few and perhaps only one or none) usually arises from sources, such as improperly adjusted machines, machine wear, machine downtime, operator errors, introduction of new workers, defective raw materials, materials contamination, a change in the inspection method or standard and other factors that can be controlled. A process that is operating in the presence of assignable causes is said to be out-of-control (Montgomery, 2009).

The primary objective of SPC is to quickly identify the presence of assignable cause(s) or process shift(s) so that an investigation of the process and corrective actions can be taken to bring the process into statistical control before many nonconforming units are being manufactured (DeVor et al., 2007). Control charts have an excellent history of more than 80 years. Most processes do not operate in a state of statistical control. Therefore, routine and attentive use of control charts enable the detection of assignable causes present in a process. If these causes can be eliminated, variability will be reduced and the process will be improved.

Control charts are classified into two categories. Control charts with quality characteristics that can be measured and expressed as numbers on some continuous scale of measurement, like weight, length, width, diameter, thickness, volume, density and temperature, are called variables control charts. In such cases, it is convenient to describe the quality characteristics with a measure of central tendency (mean) or a measure of variability (variance). On the contrary, attributes control charts judge a unit of product as either conforming or nonconforming on the basis of whether or not the product possesses certain attributes.

Before the introduction of control charts, practitioners inspect every single unit of the end product, in order to produce high-quality products. Thus, in those days, quality control was in fact quality inspection and not quality improvement. Improving quality and productivity using conventional methods, such as upgrading of technology and modifying of the existing system are usually not practical, besides consuming more resources, time, manpower and cost. Therefore, improving the quality of processes and products by means of control charts is more practical and effective (Woodall, 1997).

Shewhart (1931) published a complete exposition of the theory, practical applications and economics of control charts, where he pointed out that control charts are useful (i) to set goals or standards for a process, for practitioners to control against predefined standards; (ii) as a device to achieve goals; and (iii) to judge whether the goals have been met. However, conventional control charts have several limitations. Firstly, if the underlying distribution of the quality characteristic is nonnormal and the sample points are not independent and identically distributed over time, the Shewhart chart (Shewhart, 1931) in particular, may not perform well. Secondly, the Shewhart chart which focuses only on the current sample point and pays no attention to the historical information about the process when a new point is plotted, is not effective in detecting small shifts in the process; while the cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts are ineffective in detecting large shifts. Thirdly, two control charts, each for the mean and variance are required to be plotted concurrently to monitor the process mean and variability, hence, making process monitoring cumbersome and time consuming.

1.2 Problem Statements

Traditionally, the mean and variance type charts are plotted concurrently to monitor the process mean and variance, respectively. However, in many real life process monitoring situations, a simultaneous control of the mean and variance is necessary. For example, Gan et al. (2004) have shown a case in the integrated circuit manufacturing, where the solder paste is printed onto the printed circuit board (PCB) before the mounting of circuit components. Here, the thickness of the solder paste influences the soldering ability of circuit components on the PCB. If the process goes out-of-control, the thickness of the paste is off-target and meanwhile the process variability is large because the solder paste thickness is not uniformly distributed over the PCB. Therefore, the process mean and variance are simultaneously affected by the same assignable cause in this manufacturing setting. The need for an effective chart to simultaneously monitor the mean and variance has motivated us to develop three single variables DEWMA (Double EWMA) charts. These charts are referred to as the DEWMA-Max (called the DEWMA maximum), Max-DEWMA (called the maximum DEWMA) and SS-DEWMA (called the sum of squares of DEWMA) charts, in the thesis. The proposed charts are capable of simultaneously monitoring shifts in both the process mean and variance, which are more effective than the existing single EWMA charts, in terms of out-of-control detection speed and diagnostic abilities. The term ‘variables’ refers to quality characteristics that can be measured and expressed as a number on some continuous scale of measurement (Montgomery, 2009).

1.3 Objectives of the Thesis

The objectives of this thesis are:

(i) to propose the DEWMA-Max chart. The technique of performing exponential smoothing twice to construct a DEWMA chart is applied to the maximum of the absolute values of the statistics controlling the process mean and variance (whichever is larger), so that a new chart is proposed.

(ii) to propose the Max-DEWMA chart. The statistics of this chart are based on the maximum of the absolute values of two DEWMA statistics, one for controlling the process mean while the other the process variance.

(iii) to propose the SS-DEWMA control chart. This chart uses the sum of squares statistics and it simultaneously monitors the process mean and variance in a single chart.

1.4 Organization of the Thesis

Chapter 1 gives an overview of control charts and highlights the problem statements of the study. It also mentions the objectives of the study. Chapter 2 introduces the principles of SQC and discusses the developments in quality control. A literature review is given to explain existing works on single variables charts.

Three single variables DEWMA charts are proposed and described, each in Chapters 3, 4 and 5. Here, the performances of the proposed optimal single DEWMA charts are compared with that of their competing optimal single EWMA counterparts, in terms of the out-of-control detection speed and diagnostic abilities. Examples on how these charts are put to work in a real situation are also shown in these chapters.

In Chapter 6, the performances of the three proposed single DEWMA charts are compared, in terms of their average run length (ARL), standard deviation of the run length (SDRL) and diagnostic abilities performances. Finally, the main contributions of the thesis and some suggestions for further research are summarized in Chapter 7.

The derivation of the statistics for the proposed single DEWMA charts and the numerous computer programs written in the Statistical Analysis Software (SAS) and FORTRAN codes are included in Appendices A to C. The programs are used to compute the ARLs and SDRLs, as well as, to study the diagnostic abilities of the charts.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

Quality has always been an essential part of almost all products and services. Statistical quality control (SQC) is a set of interrelated tools used to monitor and improve the performance of a process producing a product or service. This chapter presents a review on the literature related to this study. The discussion covers both the philosophical and analytical sides. Section 2.2 is directed towards the philosophical side of quality control (QC). In this section, some important milestones or historical perspectives in the evolutionary process of SQC with some of the pioneers and their basic principles will be briefly reviewed.

Control charts are the basic tools of SPC to identify the presence of a special cause of variation in a process and an analysis can suggest causes of any out-of-control occurrences in the process. Theoretical basis of control charts, like the common steps in constructing a control chart, the theory of hypothesis testing employed in control charts and the benefits of control charts are discussed in Section 2.3, as they form the foundation of this study.

Both the philosophical and analytical sides of the developments in quality control charts will be covered in Section 2.4. The development of variables control charts, in chronological order, from the Shewhart, CUSUM, EWMA to DEWMA charts, as well as the differences between these charts will be explained.

The existing works on single variables control charts, for both mean and variance are highlighted in Section 2.5. Two different approaches used in constructing single variables charts will be discussed. The discussion will emphasize on the EWMA-type single variables control charts because the EWMA control chart is superior to the Shewhart chart as explained in Section 2.4. The performance of EWMA chart is approximately equivalent to that of the CUSUM chart, and it is less sensitive to the normality assumption.

In Section 2.6, the ARL and the SDRL which are used to measure the performance of a chart will be defined.

2.2 Statistical Quality Control (SQC)

From here onwards, SQC is referred to as the collection, analysis and interpretation of data for application in QC activities aimed at monitoring and improving the performance of a process (Besterfield, 2009). SQC is different from SPC, as SPC is one of the statistical tools that make up SQC (Gupta & Walker, 2007).

In ancient times, people were already concerned about the quality of products and it was known that elementary techniques for QC must have existed (Wierda, 1994). Before industrialization, that is in the early production era before the 1920's, operators inspected the quality of their own works through their eyes and were responsible for the quality of products that they produced. They inspected every single product, in order to ensure that the products were all identical and were able to meet the market's quality demand. This is in fact 100% quality inspection because the products which were unacceptable were discarded but the assignable causes leading to the defects were neither identified nor eliminated (Besterfield, 2009).

During the industrial revolution from the 18th to the 19th century, high volumes of products were produced and the use of a 100% visual inspection to avoid defective products from being produced was impractical. In addition, a major setback of a 100% visual inspection is that no variability patterns could be gauged (Besterfield, 2009).

The work of Shewhart in the 1920s led to an approach of an examination of the pattern of the underlying process variation, followed by the removal of the sources of variation. This approach was more effective than a 100% visual inspection of the end products. In 1928, H. F. Dodge and H. G. Romig developed and refined the statistically based acceptance sampling inspection of the process output prior to shipping to decide on the extent to which process output conforms to specifications. This method was also considered in QC as an alternative to 100% inspection. All of the earliest recorded works in QC as mentioned above were conducted at the Bell Telephone Laboratories. Walter A. Shewhart and his colleagues recognized that variation in a process is a statistical incident and developed statistical methods for QC (Chandra, 2001).

In 1924, Walter A. Shewhart presented to his chief at the Bell Telephone Laboratories his first statistical control chart, showing the monthly number of percent defective items in some devices. This was often considered as the formal beginning of SQC. In December 1925, Shewhart published a paper entitled "The Application of Statistics as an Aid in Maintaining Quality of a Manufactured Product" in the *Journal of the American Statistical Association*. In this paper, he officially introduced the control chart to the world. Later in 1931, he published his famous book, i.e. *Economic Control of Quality of a Manufactured Product*, outlining the statistical methods for use in production and his proposed control charting methods (Shewhart, 1931). His proposed charts include the $\bar{X}$, R and S charts for variables data and the p, np, c and u charts for attributes data. Shewhart's is the beginning of SQC, where the main objective is to improve the quality of a product through process monitoring and not correcting defects at the end product. However, during Shewhart's time, the importance of SQC was still not widely recognized and applied by industries (Montgomery, 2009).

The high demand for war equipments during World War II, followed by the high demand for products after World War II, has boosted the productivity of the American manufacturing industries. The consequence was employment of many semiskilled and even unskilled operators who emphasized on the quantity but not the quality of products. As such, many products did not meet customers' expectations and were returned for rework. This brought about the realization of the importance of SQC and the necessity of quality improvement techniques, like control charts, acceptance sampling and design of experiments (DOE) in manufacturing industries (DeVor et al., 2007).

As a result, widespread training courses were established extensively by the American manufacturing industries, where the applications of SQC in manufacturing and service industries were taught during the training. Besides training, publications and conferences for the promotion of SQC techniques were also done by several organizations, such as the American Society for Quality (Control), which was formed in 1946. At the same time, Britain also witnessed similar developments of SQC (DeVor et al., 2007).

SQC gained popularity in the Japanese industries during the 1950s through W. Edwards Deming's (who learned SQC from Shewhart) training programs which emphasized on total quality management (TQM). Since the birth of QC, most industrial organizations have a QC manager, leading the QC department to monitor the quality of products. The concepts of TQM is not only applied in the production floor but in all departments involved in the production process. These include the management, planning, purchasing, sales and accounting departments. Deming emphasized the concept of "do it right the first time", in order to reduce rework costs. In 1954, Joseph M. Juran made his first trip to Japan, where he emphasized on the management's responsibility to achieve quality. Through this, the Japanese set the quality standards for the rest of the world to follow (Montgomery, 2009).

By the late 1970s and early 1980s, the American industrial leaders studied from the Japanese, where a quality renaissance occurred in products and services. The Americans embraced the Deming's philosophy for quality improvement, and the Taguchi's methods and techniques of statistical DOE. The industrial revolution in the 1980s emphasized on SPC and aimed at preventing the manufacturing of defective products through improved process monitoring and diagnostic from the very beginning of the process, especially in the automotive industry (DeVor et al., 2007).

QC in today's context refers to the case, where both quality and productivity go hand in hand, in the correct direction, by means of process control, in identifying the root cause of a process failure and in taking actions to remove the assignable cause when one exists. The next section discusses the theoretical basis of a control chart.

2.3 Theoretical Basis of a Control Chart

A control chart is a graphical tool for monitoring the stability of a process. A typical univariate control chart displays a quality characteristic which has been measured from a sequence of samples on a graph versus the sample number or time. Although there are many types of control charts to monitor different processes and various ways to construct the charts, the common steps in setting up a control chart, in practice, can be summarized as follows (Xie et al., 2002):

1) Obtain a sequence of sample points for the process being monitored.

2) Then calculate the process mean and use it as the center line (CL) of the chart representing the target value, μ_0.
3) Calculate the process standard deviation, σ_0.
4) Assuming a normal underlying distribution, the upper control limit (UCL) and the lower control limit (LCL) are established at ± 3 standard deviations from the CL.
5) Plot the sample points on the chart and connect the consecutive points to show how the sequence of points has evolved over time.
6) Assess if any sample point falls beyond the control limits. If at least a sample point falls beyond the limits, an investigation and suitable corrective actions are required to find and eliminate the assignable cause(s) so that the process returns into an in-control state (within the control limits).
7) Revise the CL, UCL and LCL, if necessary. Then, construct the revised chart.
8) Continue plotting whenever a new sample point is obtained.

Practices since 1930 in all types of industries show that the 3 standard deviations width of the control limits from the CL provide an economical balance between the costs resulting from the Type-I and Type-II errors (Umble & Umble, 2000). Unless there are strong practical reasons for doing otherwise, the $\pm 3\sigma$ limits should be applied. For a normal underlying distribution, a total of 99.73% of the population points will fall within the $\pm 3\sigma$ control limits. This means that almost all of the population points fall within the $\pm 3\sigma$ limits if the process is free from any assignable causes. In other words, the false alarm rate or Type-I error size, α is as low as 1 in every 370 (0.27%) random samples (Besterfield, 2009; Gupta & Walker, 2007). This can be determined by using the in-control ARL (ARL_0) formula in Equation (2.1).

$$\mathrm{ARL}_0 = \frac{1}{\alpha}, \tag{2.1}$$

where $\alpha = 0.0027$.

The statistical theory employed in control charts is the theory of hypotheses testing. Applying a control chart can be considered as doing repeated tests of the statistical hypothesis that the process is in a state of statistical control. When a sample point is plotted on a control chart, the hypothesis of statistical control is to be tested based on the information obtained from the sample. A point falling within the control limits is equivalent to accepting the hypothesis of statistical control and a point falling beyond the control limits is equivalent to rejecting the said hypothesis. As in hypothesis testing, there are also two types of errors for a control chart. The probability of a Type-I error represents the probability that a control chart will give an out-of-control signal when in fact the process is actually in-control. The probability of a Type-II error represents the probability that a control chart fails to signal an out-of-control when the process is actually out-of-control. Table 2.1 illustrates the differences between the Type-I error and the Type-II error. An optimal design of a control chart is to achieve the smallest probability of a Type-II error when a desired probability of a Type-I error is specified (Besterfield, 2009; Montgomery, 2009).

Table 2.1 Type-I error and Type-II error

True state of nature	Hypotheses (plotted point shows)	
	Assignable cause is present	Common cause is present
Out-of-control	OK	Type-II error
In-control	Type-I error	OK

There are many benefits that can be obtained from control charts if they are properly designed (Gupta & Walker, 2007):

1. Control charts can help practitioners, i.e. production operators, in ongoing process control.
2. Control charts can help a process to run consistently and predictably.
3. Control charts are proven techniques for improving productivity, quality and capacity, and hence lowering the manufacturing cost.
4. Control charts are effective in defect prevention.
5. Control charts can distinguish between assignable causes and common causes of variations and help practitioners to take corrective actions.

2.4 Developments in Quality Control Charts

Shewhart (1931) formulated a statistically-based approach to QC and introduced the $\bar{X}$, R and S control charts in 1924. A control chart is one of the basic and most powerful tools in statistical process control (SPC), used in the monitoring of a quality characteristic of a process over time.

Assume that the underlying process consists of sample points, $X_{i1}, X_{i2}, \ldots, X_{in_i}$, in the i^{th} sample of size n_i from a normal distribution. The normal distribution is described by its parameters, i.e. the mean, μ and the standard deviation, σ. The $\bar{X}$ chart, which is based on the underlying distribution of the sample mean, is used to monitor the process mean. The R chart utilizes the sample ranges and the S chart utilizes the sample standard deviations to monitor the process variance.

Montgomery (2009) pointed out that it is necessary to monitor the process mean and variance because in real situations, both the mean and variance are more likely to shift simultaneously. The $\bar{X} - R$ charts are used because they are easily comprehensible by practitioners. However, the sample range ignores all information between the two most extreme values and hence the sample range method becomes inefficient as a measure of variability for large sample sizes. The $\bar{X} - S$ charts are sometimes used in place of the $\bar{X} - R$ charts because the sample standard deviation makes use of all the information available and can provide a better estimate of the process variance compared to the sample range.

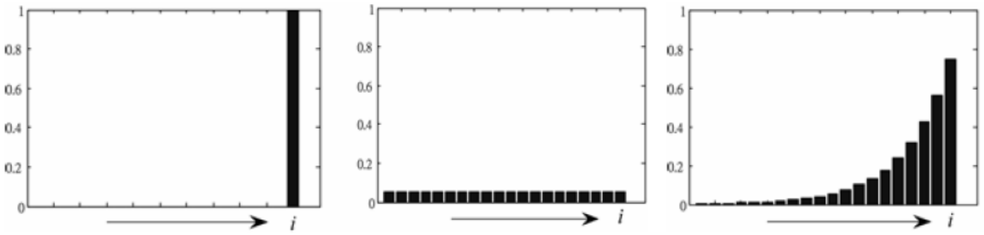
The main drawback of the classical Shewhart control chart (Shewhart, 1931) is that it focuses only on the current sample point and pays no attention to the historical information about the process when a new point is plotted. Therefore, it is inefficient in detecting small shifts in the process. To overcome this drawback, the cumulative sum (CUSUM) and the exponentially weighted moving average (EWMA) charts which take into account historical sample points were proposed for detecting small/moderate shifts in the process.

The CUSUM chart developed by Page (1954) incorporates past information into each individually plotted sample point to increase the chart's sensitivity for detecting small shifts in the process. This chart plots the cumulative sums of deviations of the sample values from a target value against time. There are two types of CUSUM charts, the tabular CUSUM and the V-mask CUSUM. The tabular CUSUM employs two sample statistics (C^+ and C^-), where the one based on C^+ is the one-sided upper CUSUM that accumulates positive deviations above the target while the other is the one-sided lower CUSUM that accumulates negative deviations below the target. Being similar to the Shewhart charts, the center line of the tabular CUSUM chart represents the target value, μ_0. If either C^+ or C^- exceeds the predetermined decision interval, *H*, the process is considered to be out-of-control.

Instead of the classical control limits, the V-mask CUSUM chart requires the use of a mobile V-shaped mask which can be superimposed on the CUSUM plot to decide whether a shift occurs (Barnard, 1959). The mask is in the shape of a ">" placed over the chart with its vertex placed at a fixed distance from the last sample plotted. If all the samples previously plotted lie within the two arms of the V-mask, the process is assumed to be in-control, otherwise, the process is said to be out-of-control (Oakland, 2008). However, the CUSUM chart's statistics assign the same weight to all the sample points.

Roberts (1959) introduced the EWMA control chart for monitoring the process mean. Then, Hunter (1986) suggested writing the current EWMA as the previous EWMA plus a fraction of the difference between the current observation and the previous EWMA. Crowder (1987) evaluated the properties of EWMA's by formulating and solving a system of integral equations. The EWMA property introduces a weighting factor, λ, which weights the current sample point more heavily than the historical sample points, i.e. a shift in the process can be aggregated in the charting statistics so that it can be detected quickly. In other words, each sample point is assigned a weight, and the weight increases exponentially from the previous sample point to the present one. Thus, the EWMA chart is more sensitive than the Shewhart and CUSUM charts to reflect crucial information on the recent process. The EWMA chart is insensitive to the normality assumption, whereas the CUSUM chart is sensitive to the normality assumption (Hawkins & Olwell, 1998).

The design parameters of the EWMA chart are the multiple of the standard deviation used in the control limits (K) and the smoothing constant (λ). The smoothing constant determines the rate of decay of the weights and hence the amount of information obtained from the historical data. A combination of (K, λ) is chosen to obtain a desired ARL_0 value. The EWMA chart reduces to the Shewhart chart when $\lambda = 1$. The differences between the Shewhart, CUSUM and EWMA charts, in terms of how past sample points are weighted, are shown in Figures 1.1 (a) – 1.1 (c) (Cheng et al., 2007).



(a) Shewhart (b) CUSUM (c) EWMA

Figure 1.1 The weighted functions for the Shewhart, CUSUM and EWMA charts

The DEWMA (called the double EWMA) control chart is an extension of the usual EWMA mean chart by performing exponential smoothing twice on the original statistics of the underlying process. EWMA is a statistic that always gives strictly decreasing weights to historical sample points. However, this is not always applied to DEWMA statistics and weights may be non-monotones occasionally. According to the process characteristics, the previous sample points may have a greater importance than the current sample points. Therefore, the DEWMA can be a more flexible technique than the EWMA in applying the weights (Hong et al., 2011).

The DEWMA chart was originally proposed by Shamma et al. (1991), and Shamma and Shamma (1992), which was later studied by Zhang and Chen (2005). Zhang and Chen (2005) recommended the use of the DEWMA chart for detecting small shifts in the mean of a zero state process. A comparison of the EWMA and DEWMA charts, in terms of the zero state ARL performance indicates that the latter is superior to the former. Zhang (2002) showed that the DEWMA chart can improve upon the EWMA chart's performance for variables data. He also proposed the DEWMA *p* chart for attribute data with time-varying control limits, which dominates the EWMA *p* chart. Zhang et al. (2003) applied the DEWMA chart to the monitoring of Poisson data. Lately, Hong et al. (2011) proposed the DEWMA chart for the coefficient of variation (called the CV-DEWMA chart) that combines the DEWMA technique with the CV chart developed by Chang et al. (2007). The results revealed that the CV-DEWMA chart performs better than the CV-EWMA chart, proposed by Hong et al. (2008) in detecting small shifts in the variance when the sample size *n* is greater than 5.

2.5 Single Variables Control Charts

Most of the Shewhart, CUSUM and EWMA charts discussed in the literature monitor the process mean and variance separately. Using two charts plotted separately to monitor the process mean and variance is inconvenient, besides consuming more resources, time, manpower and cost. According to Gan (2000), by referring to either the mean or the variance chart alone without making reference to the other one might mislead QC engineers into making a wrong decision. In real life, the process variance tends to increase with the process mean. An ideal situation is a decrease in the variance when the mean is in control, but the situation is undesirable if a decrease in the variance is accompanied by a decrease in the mean. For this case, the mean chart becomes insensitive to the change in the process mean because the variance of the sample mean has decreased. Any detection of the mean with a decrease in the variance could lead to a false conclusion that the process mean has improved. The example from Gan et al. (2004) discussed in Section 1.2 of this thesis shows that the process mean and variance are simultaneously affected by the same assignable cause in the said manufacturing setting and they need to be looked at jointly, in order to make meaningful inferences.

For practical concern, this has led to the development of one control chart to simultaneously monitor the process mean and variance. The following discussion focuses more on the EWMA-type single variables control charts because the EWMA control chart is superior to the Shewhart and CUSUM charts, as discussed in the previous section.

White and Schroeder (1987) introduced the use of one control chart to simultaneously monitor the process mean and variance. This simultaneous chart was constructed using resistant measures and a modified box plot display to monitor the process mean and variance. Since then, many efforts have been made to design a single control chart which can simultaneously monitor both the process mean and variance. Iglewicz and Hoaglin (1987) extended and refined the technique discussed by White and Schroeder (1987). The former authors claimed that the information contained in a simultaneous chart with two statistics can be confusing due to its complexity and that the chart is ineffective for small sample sizes. To overcome this setback, Chan et al. (1990) provided an alternative to the box-plot style of simultaneous charts that is effective for both small and large sample sizes.

Domangue and Patch (1991) proposed simultaneous omnibus EWMA charts for detecting changes in both the location and dispersion. The EWMA statistic is based on the exponentiation of the absolute value of the standardized sample mean. The setback of this chart is that it is incapable of identifying the source and direction of a shift. To overcome this, Gan (1995) proposed a combined scheme consisting of a two-sided EWMA mean chart and a two-sided EWMA variance chart. To ensure that charts controlling the mean and variance are interpreted jointly, Gan (1997) constructed a joint monitoring of both the mean and variance using simultaneous EWMA charts, i.e. plotting the EWMA of $\log(S^2)$ against the EWMA of $\bar{X}$. The position of an out-of-control point for this chart is able to provide insights to both the magnitude and direction of a process shift. Later, Gan (2000) developed a joint EWMA chart to monitor the process mean and variance using a rectangular or an elliptical chart.

The single EWMA charts discussed in the preceding paragraph plots two statistics on the same chart, for a joint monitoring of both the process mean and variance. However, there are two different approaches that are used in constructing single variables charts. The first is to plot two statistics, one representing the mean and the other the variance, both having a standard scale on the same chart as discussed above. The second approach uses one plotting variable to represent both the mean and variance. The first approach is not simple because it requires plotting two different types of quantities on the same chart. The following paragraph discusses single charts that use the second approach.

Xie (1999) proposed several single EWMA control charts that use only one plotting characteristic. Firstly, he extended the Max chart, proposed by Chen and Cheng (1998) to the EWMA-Max chart. The EWMA technique is applied to the Max statistic to construct the EWMA-Max chart. It is capable of detecting small changes in the process mean and/or variance. Xie (1999) also suggested the Max-EWMA and EWMA-SC charts, which were later published by Chen et al. (2001) and Chen et al. (2004), respectively. The Max-EWMA chart plots the maximum of the two EWMA statistics containing the mean and variance, while the EWMA-SC chart applies the EWMA techniques to the statistics employed in the semicircle chart. In addition, Xie (1999) proposed the SS-EWMA chart, based on the sum of squares of the maximum standard EWMA values. Costa and Rahim (2004) proposed an EWMA chart based on a non-central chi-square statistic for a joint monitoring of the process mean and variance and found that their chart has a similar ARL performance to that of the Max-EWMA chart. Costa and Rahim (2006) proposed a single EWMA chart for a simultaneous monitoring of the process mean and variance as an extension to the chart studied by Chen et al. (2004). Cheng and Thaga (2006) provided a comprehensive overview of single variables charts. They concluded that single charts are more appealing than the simultaneous charts because single charts are easy to construct, and the source and direction of a shift is easily identified and interpreted.

The works of Xie (1999) on single EWMA charts and Zhang and Chen (2005) on DEWMA chart have motivated us to propose three single DEWMA control charts, where each uses only one plotting variable to represent both the mean and variance. This study extends the EWMA-Max chart to the DEWMA-Max chart; the Max-EWMA chart to the Max-DEWMA chart; and the SS-EWMA chart to the SS-DEWMA chart. To the best of our knowledge, no attempt has been made to develop single DEWMA charts for a simultaneous monitoring of the mean and variance, prior to the work in this thesis.

2.6 Average Run Length (ARL) and Standard Deviation of the Run Length (SDRL)

The performance of control charts for monitoring a process is commonly measured by the ARL which is defined as the average (expected) number of sample points that must be plotted on the chart before the first out-of-control signal is detected (Xie et al., 2002). In other words, ARL is a measure of the speed of a chart in detecting the occurrence of assignable causes (Zhang & Chen, 2005).

When the process is in-control, the ARL_0 should be sufficiently large to avoid too frequent false alarms produced by the chart. When the process is out-of-control, it is desirable to have a small out-of-control ARL (ARL_1), so that an out-of-control condition can be detected quickly. The out-of-control condition is represented by a significant change or shift in the mean and/or variance (Montgomery, 2009).

Using the ARL as a sole measure of performance of a chart is insufficient. Instead, supplementing the ARL with other characteristics of the run-length distribution is important (Chakraborti, 2007; Radson & Boyd, 2005). For example, in addition to the ARL, the standard deviation of the run length (SDRL) can also be computed to get an idea about the variation of the run length distribution. SDRL measures the spread of the run length distribution. A small SDRL value is desirable, while a large one is undesirable. A chart with a smaller ARL_1 and a small out-of-control SDRL ($SDRL_1$) compared to its counterparts having the same ARL_0 value is said to be more effective in detecting a process change.

Thaga (2003) commented that using ARL alone to measure a chart's performance is inadequate as only a fraction of the behavior of the control chart is shown by the size of the ARL. Thus, it would be better to study the chart's behavior by investigating the properties of its run length distribution, i.e. via the use of SDRL. Moreover, Di Bucchianico et al. (2005) also commented that when the run length distributions are highly skewed, it is less meaningful to judge the performance of a control chart by considering its ARL only. Instead, the SDRL should also be taken into account.

The simulation, integral equation and Markov chain approaches have been used in the literature to compute the ARL and SDRL values of control charts (Champ & Ridgon, 1991; Chen et al., 2004; Costa & Rahim, 2006; Zhang & Chen, 2005). The simulation method will be used in this study to obtain the ARL and SDRL values.

CHAPTER 3
DEWMA-MAX CHART

3.1 Introduction

As discussed in Chapter 2, there is an abundance of new developments in control chart techniques. The EWMA-type charts are more sensitive in detecting small shifts in the process mean and/or variance and thus they serve as alternatives to the Shewhart-type charts. Firstly, Section 3.2 elucidates the EWMA-Max chart proposed by Xie (1999), which effectively combines two EWMA charts into one chart with a single quality characteristic to monitor both the process mean and variance. The EWMA-Max chart is an extension of the Max chart (Chen & Cheng, 1998). It indicates the source and direction of a shift when an out-of-control signal is detected. The EWMA technique applied to the Max statistic has increased the sensitivity of the Max chart, as the Max chart is not sensitive to small changes in the process.

Secondly, Section 3.3 introduces the proposed DEWMA-Max chart as a superior alternative to the EWMA-Max chart. It is assumed that an assignable cause of variation may shift the process mean and/or variance. The proposed chart is based on the assumption that the underlying distribution is normally distributed. The technical details are provided in Section 3.4. The charting procedure and the optimal design of the DEWMA-Max chart are presented in Section 3.5 and Section 3.6, respectively.

A simulation study is conducted in Section 3.7 to compare the performances of the optimal DEWMA-Max and the optimal EWMA-Max charts. In addition, an example of application is presented in Section 3.8 to show how the DEWMA-Max