EXTENSION OF NUMERICAL MANIFOLD METHOD FOR PLASTICITY AND FRACTURE MODELLING OF ROCK MASS

by

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LIST OF ABREVIATIONS

BEM	Boundary Element Method
BPM	Bonded Particle Method
DDA	Discontinues Deformation Analysis
DEM	Discrete Element Method (Distinct Element Method)
DEN	Discrete Fracture Network
EFG	Element Free Galerkin
EPFM	Elastic Plastic Fractured Mechanics
FCM	Finite Cover Method
FDM	Finite Difference Method
FEM	Finite Element Method
FF	Free Formulation
GFEM	Generalized Finite Element Method
GSI	Geological Strength Index
GUI	Graphical User Interface
LEFM	Linear Elastic Fracture Mechanics
NATM	New Austria Tunnelling Method
NMM	Numerical Manifold Method
PDE	Partial Deferential Equation
PFC	Partial Flow Code
RMR	Rock Mass Rating
RQD	Rock Quality Designation
RSRWT	Pahang–Selangor Raw Water Transfer
SIF	Stress Intensity Factor
TBM	Tunnel Boring Machine
XFEM	Extended Finite Element Method

LIST OF SYMBOLS

C	Damping matrix
D	Displacement vector
$\dot{D}$	Velocity vector
$\ddot{D}$	Acceleration vector
E	Young's modulus
F	Force
G	Griffith crack growth energy
J	*J*-integral
K	Stiffness matrix
M	Global mass matrix
I_1	First principal invariant
J_2	Second principal invariant
J_3	Third principal invariant
K_I	Mode i of stress intensity factor
K_{II}	Mode ii of stress intensity factor
d_n	Normal penetration distance
d_s	Shear penetration distance
$\bar{t}$	Traction on the boundary
$\dot{u}$	Velocity
$\ddot{u}$	Acceleration
$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$	Shear strain
σ_1	Major principal stress

σ_3	Minor principal stress
β	Lode's angle
κ	Hardening parameter
Π_e	Strain potential energy
Π_f	Potential energy of friction force
Π_i	Potential energy of inertia
Π_p	Potential energy of point load
Π_s	Potential energy of contact springs
Π_σ	Potential energy of initial stresses
Π_ω	Potential energy of body load
b	Body force per unit volume
c	Cohesion
d	Penetration distance
g	Gravity
k	Spring stiffness
m	Mass
p	Penalty value
r	Radius in polar coordination system
t	Time
u	Displacement
γ	Specific weight
ε	Normal strain
η	Velocity weighting parameters of Newmark method
θ	Angle in polar coordination system

λ	Lagrange multiplier
μ	Friction
ν	Poisson's ratio
ξ	Acceleration weighting parameters of Newmark method
ρ	Density
τ	Shear stress
φ	Friction angle

LANJUTAN KAEDAH MANIFOLD BERANGKA BAGI PERMODELAN KEPLASTIKAN DAN RETAKAN DI DALAM JISIM BATUAN

ABSTRAK

Kajian terdahulu mengenai permodelan berangka bagi masalah kejuruteraan geoteknik menunjukkan kepentingan permodelan yang selamat dan mampan bagi rekaan projek geoteknik. Secara amnya, kestabilan struktur kejuruteraan yang dibina di dalam batu secara langsungnya dipengaruhi oleh kestabilan jisim batu yang mengandungi struktur tersebut. Di samping itu, reka bentuk kejuruteraan yang ekonomi biasanya mencadangkan penggunaan batu yang sedia ada untuk menyokong struktur bawah tanah itu bagi mengurangkan pengendalian tambahan sistem sokongan. Oleh itu, penilaian kestabilan batuan yang tepat dan jitu diperlukan, yang berkemungkinan besar boleh dicapai dengan permodelan kestabilan jisim batuan. Walau bagaimanapun, banyak peranggaran yang dipertimbangkan dalam teknik-teknik permodelan tradisional jisimbatuan tidak dapat meniru kedua-dua permasalahan ketakselanjaran dan selanjaran. . Oleh itu,, model baru telah dibangunkan dan diperbaiki untuk menyediakan teknik-teknik permodelan tepat untuk mekanik batuan. Kaedah Manifold Berangka (NMM) yang dibangunkan oleh Shi (1997) adalah salah satu model cantuman selanjaran-ketakselanjaran yang sesuai untuk model masalah yang kompleks dalam mekanik batuan. Walau bagaimanapun, kaedah NMM memerlukan kajian lanjutan untuk menyelesaikan masalah keplastikan dan keretakan di dalam memodelkan kestabilan jisim batuan dan mendapatkan keputusan simulasi yang dipercayai.

Dalam kajian ini, kaedah NMM telah dilanjutkan untuk menganalisis perubahan bentuk tanah dan anjakan bersama-sama dengan penilaian kegagalan plastik dan analisis pertumbuhan retak dalam jisim batuan. NMM telah dilanjutkan bagi memodelkan mekanisma keretakan dan keplastikan di dalam batuan, bersama-sama dengan mekanis sentuhanma . Penyelesaian matematik dan pengiraan algoritma telah dibangun dan diperluaskan bagi pengaturcaraan model. Teknik-teknik permodelan retak telah dimasukkan ke dalam NMM bagi memodelkan retak permulaan dan pertumbuhan di dalam batu yang utuh. Mohr - Coulomb kriteria bagi elastik-plastik telah digunakan di dalam NMM bagi model keplastikan tanah dan batu-batu lembut. Teknik-teknik baru permodelan elastik-plastik bilinear telah dicadangkan bagi mengsimulasikan fenomena penutupan retak di dalam batu rapuh. Akhirnya, algoritma NMM sentuhan telah dibangunkan bagi meniru keplastikan, sentuhan dan masalah retak di dalam batuan. Model lanjutan NMM ditentukur, disahkan dan dinilai dengan masalah penanda aras dari keputusan eksperimen. Keputusan mengesahkan model lanjutan dapat menganalisis masalah mekanik batuan untuk reka bentuk kejuruteraan geoteknik.

EXTENSION OF NUMERICAL MANIFOLD METHOD FOR PLASTICITY AND FRACTURE MODELLING OF ROCK MASS

ABSTRACT

Previous studies on numerical modelling of geotechnical engineering problems indicate the importance of modelling in safe and sustainable designing of geotechnical projects. Generally, the stability of built engineering structures in rock is directly influenced by stability of the rock mass that contains the structure. In addition, the economical engineering design usually suggests the usage of existing rock to support the underground structure hence minimizing the additional handling of support systems. Therefore, an accurate and precise assessment of rock stability is required, that can be hugely achieved by modelling the rock mass stability. However, many approximations considered in the traditional modelling techniques of the rock mass were not able to simulate both discontinuity and continuum problems. Hence, new models have been developed and improved for providing accurate modelling techniques for rock mechanics. The Numerical Manifold Method (NMM) developed by Shi (1997) is one of the hybrid continuum–discontinuum models which is suitable for modelling complex problems in rock mechanics. However, the NMM method requires extension for solving the plasticity and cracking problems to model rock mass stability and determines reliable simulation results.

In this study the NMM method was extended to analyse ground deformation and displacement together and evaluation of plastic failure and crack growth analysis in rock mass. NMM was extended to model rock crack mechanism and plasticity, alongside with contact mechanics. The mathematical solutions and computational

algorithms were developed and expanded for programming the model. The crack modelling techniques was adopted with NMM for modelling crack initiation and growth in the intact rock. The Mohr–Coulomb elasto–plastic criterion was applied to the NMM for modelling the plasticity of geomaterials such as soils and soft rocks. A new bilinear elasto–plastic modelling techniques was suggested for simulation crack closure phenomena in brittle rocks. Ultimately, the NMM contact algorithms were developed to simulate plasticity, contact and crack problems in rock. The extended NMM model calibrated, verified and assessed by benchmark problems with experimental result. The results confirmed the extended model able to analyse rock mechanic problems for geotechnical engineering design.

CHAPTER 1
INTRODUCTION

1.1 Background

To protect the environment and, in particular, to avoid noise and pollution in residential areas, railways and highways have more frequently been constructed underground or cut through mountains. With so many geotechnical structures being built, it is important to have a comprehensive understanding of induced displacements and stresses due to rock mechanics and their impact on nearby structures.

Rock masses consist of intact rock and discontinuous components like joints, faults, and bedding planes is a natural non-elastic, inhomogeneous, anisotropic, and largely discontinuous material, which makes modelling the rock structure difficult (Jing, 2003; Ning et al., 2012). Generally, rock can act both as an elastic and inelastic material; however a discontinuous mass results in a much more inelastic material. Also, soft rocks and clays from soil mechanics behave mostly in a non-elastic manner (Hoek, 2000). Therefore, the use of a variety of modelling techniques has been reported in the literature for modelling the different problems in rock mechanics.

Based on the different problems facing a rock engineering project, modelling techniques can be categorized into three groups. Numerical modelling that assumes the rock medium as a single homogeneous material, such as the Finite Element Method (FEM), is a continuum model. The second group is focused on discontinuities in the rocks and models rock problems as a discontinuum medium with a rigid block. This group includes the well known Discrete Element Method (DEM) and Discontinues Deformation Analysis (DDA) methods. These two groups of modelling techniques are limited to an assumed approximation. Therefore, a third group of modelling techniques has been developed recently for handling both discontinuity and deformation of the rock medium (Jing, 2003). The hybrid FEM-DEM and Numerical Manifold Method (NMM) are the examples of this third group.

The choice of continuous or discrete method depends on many factors that are mostly dependant on the scale of the problem and the fracture system. The continuum approach can be used if small amount of discontinuity is present. The discrete approach is particularly suitable for moderately fractured rocks where the number of fractured elements is large, or when the problem involves large displacements of individual blocks. No method provides an absolute advantage over the others. However, some disadvantages of the two continuous models can be overcome by combining discrete and continuum methods (hybrid models) (Jing and Hudson, 2002).

1.2 Problem Statement

Analysis and design of geotechnical projects involving rock mass is dependent on the combined strength of the intact rock and the various discontinuities in the rock mass. Failures in the rock mass are categorized in three types. The first category is related to failure of intact rock, jointed rock or heavily jointed rock which is induced from overstressing. Then, rock blocks may be formed in this stage due to the coalescence of the newly generated fractures in the intact rock with the pre-existing internal discontinuities. Secondly, failure may occur due to instability along the discontinuities. Finally, the block failure happens caused by loosening or falling of the rock blocks. For example, in rock slope failures, movement of the collapsed rock blocks governs the disaster to a great extent; in the other case during the blasting process, the spread distance of the rock fragments may needs to be controlled within the expected range.

The importance of underground structure safety for rock mechanical engineering has become the subject of special attention in accurate modelling of rock structures. The nonlinear, inhomogeneous and anisotropic behaviour of rocks and especially rock mass make rock mechanical problems very complex, and increase the amount of special cases that must be addressed by numerical modelling of any geotechnical project. Well known numerical computation methods usually apply large approximation assumptions to the system of modelling. However, the approximation selection usually depends on the importance of the project and computational resources to provide enough accuracy for an engineering design. Then, engineering design usually involves a large safety factor to ensure the stability of the structures.

Different modelling techniques have been developed based on the design requirements and different conditions of rock engineering structures. Discontinuity is an important aspect in rock mechanical design, and therefore, most discontinuum modelling techniques were developed to meet the need for accurate rock mechanical design. However, because of limitations in computational resources, many approximations are considered in the modelling techniques.

In recent years, improved computational processors have become available for better computational resources; and many studies provided better and more accurate solutions in mechanical engineering. The necessity of developing a general computational model has motivated many studies, and well known numerical models, such as FEM, have been extended and integrated for modelling the crack and plasticity individually. However, existing modelling techniques can not model plasticity, fracture analysis and discontinuity all together. Although, presented modelling techniques usually require modification for improving the computational time and accuracy. One of the suitable numerical models for modification and extension is NMM, which potentially can be extended for crack, plasticity and discontinuity analysis of rock mechanical problems. However, the NMM still requires extension and modification for modelling complex problems. The NMM also requires adaptation for modelling crack and plasticity in combination with the contact simulation.

1.3 Objective

This study was conducted to provide the fundamental tools, for developing the professional and universal rock mechanic simulation model. Hence, a comprehensive modelling technique was developed for analysing the displacement and failure of the geotechnical construction in rock mass using the NMM. Main objectives of this study are as follows:

- To construct mathematical solutions and computational algorithms, as a requirement for analysing crack tip stress, crack propagation, plasticity and contact problems of discontinuities of geomaterials using developed mathematical models.

- To develop an improved NMM model for simulating the stress strain behaviour of intact rock based on elasto–plastic theories.

- To extend the NMM model for simulating the crack growth, failure and breakage potential of the rock under static loading.

1.4 Scope of Research

A constitutive modelling method was developed in this study for simulating the behaviour of rock mass. The research was focused on developing a general model which can handle main problems in modelling of the fractured rocks. In this study, three main modelling difficulties of rock masses, crack initiation, plasticity and contact were accumulated in a single computation program. The presented model is an extension to NMM model which was developed by Shi (1988). All mathematical and numerical solutions, algorithms and techniques are provided as fundamental tools for modelling the complex rock modelling. The model was calibrated, tested and verified for some well known resolved problems. A simple tunnel section was also modelled for validation and verification of the model.

1.5 Structure of Thesis

This thesis is divided into six chapters. Chapter 1 briefly introduces the research, including objective and scope of works for study. A review of the previous research and modelling techniques is presented in Chapter 2 to provide the background of this research. Chapter 3 is provided all the developed and required algorithms and techniques for modelling of the rock mass problems. Chapter 4 describes the experimental testing methods and characteristics of rocks used for verification of the model. First section of chapter 5 presents the assessment and calibration of the model. In addition the verification of the model and sample modelled problems are described in this chapter. The conclusion and recommendation for this research are presented in Chapter 6.

CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

Design of geotechnical engineering projects is directly related to the geological material such as soils and rocks that are substrates for the project. Most geotechnical design involves either surface or underground structures. Various geotechnical medium depends on the origin, location and environmental condition of the site.

In any geotechnical project that involves rock, various problems may occur because of discontinuity within the rock medium. The numerical modelling solution for each case depends on the condition of the rock and its discontinuity behaviour. Four scale of problem can be defined based of number of discontinuity in the rock as shown in Figure 2.1. In first case, the problems are simply related to the intact rock; for example, when a rock burst occurs during drilling or excavating in good quality rock. In second case, a rock with one or a small number of discontinuities can cause a local rock failure in the roof or wall of the excavation. In the third case, problems related to the rock mass with high discontinuity can be simulated by modelling a group of discrete blocks. In the final and fourth case, problems involving a high level of discontinuity, the discontinuities are considered as rock mass with uniform properties for modelling. Each case can be chosen for simulation based on the scale of the problem domain (Brady and Brown, 2007).

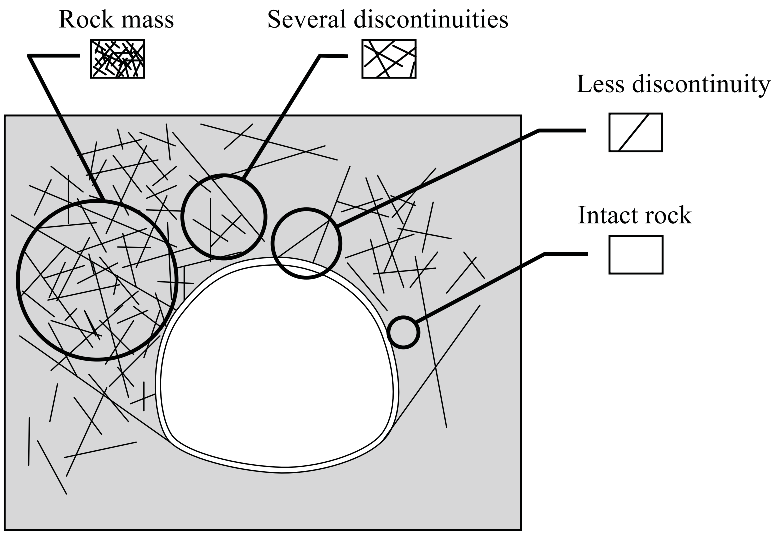

Figure 2.1: Concept of the transition from intact rock to a heavily jointed rock mass (after Brady and Brown (2007))

Different problem condition described above is required different kinds of modelling and simulating techniques, depending on rock characteristics, loading condition and problems regarding the engineering design. Rock mechanics modelling has developed for the design of rock engineering structures with widely different purposes. Different modelling techniques was developed based on (i) previous design experiences, (ii) simplified models and analytical methods such as rock mass classification (RMR, Q, GSI), (iii) modelling which attempts to capture most applicable mechanisms for example basic numerical methods (FEM, BEM, DEM, hybrid); or (iv) widespread modelling which are extended numerical methods and fully-coupled models (Jing, 2003).

2.2 Numerical Techniques in Rock Mechanics

Rock mass deformation is a consequence of deformation of the intact rock and displacement of rock blocks along joints. Therefore, the most realistic modelling should consider both phenomena. However, lack of resources and computation difficulties make rock mass modelling complicated and most researches prefer to simplify the modelling techniques. Rock mass modelling is usually categorized into three groups:

- Continuum based modelling
- Discontinuum based modelling
- Hybrid continuum- discontinuum models

Some well known numerical methods used in geotechnical engineering are summarized in Table 2.1.

Table 2.1: Common numerical method in rock mechanics

Methods	Abbreviation	Continuum	Discontinuum
Finite Difference Method	FDM	**X**	
Boundary Element Method	BEM	**X**	
Finite Element Method	FEM	**X**	
Distinct Element Method	DEM		**X**
Discrete Fracture Network	DEN		**X**
Discontinuous Deformation Analysis	DDA		**X**
Hybrid FEM/BEM	FEM/BEM	**X**	**X**
Hybrid FEM/DEM	FEM/DEM	**X**	**X**
Numerical Manifold Method	NMM	**X**	**X**

The FDM is a straight approximation of the governing partial deferential equations (PDE). Equations of FDM are formed by introducing the system of algebraic equations and placing the original PDEs for each grid points in terms of unknowns. Then, the solution of the system equation can be obtained by assumption of the necessary initial and boundary conditions (Jing, 2003).

In FEM the problem domain divisions are assumed as sub-domains named elements with standard shapes (triangle, quadrilateral, tetrahedral) and fixed number of nodes at the vertices and on the edges. Subsequently, usually polynomial trial function is used to approximate the behaviour of PDEs in each element for generation of the local algebraic equations. The relations between the elements are then assembled the global system of algebraic equations based on to the geometrical relations between the nodes and elements. The solution of the global equations is obtained by imposing the properly defined initial and boundary conditions (Jing, 2003).

The BEM is used the discretization of the domain boundary, which cause reducing the dimensions of problem. Therefore, BEM is greatly simplified the input requirements in compare with FEM. A boundary integral equation is then solved individually based on the information on the boundary. The BEM is the most efficient technique for fracture propagation analysis with better accuracy than FEM at the same level of discretization (Jing, 2003).

The DEM is represented the fractured medium as assemblages of blocks formed by connected fractures. Subsequently, the equations of motion of blocks are solved with DEM using treatment of contacts between the blocks. The original DEM assumed the blocks to be rigid and the deformable block afterwards is simulated with FEM discretizations. DEM can handle large displacements caused by rigid body motion of each block, block rotation, fracture opening and complete detachments which is impossible in the FEM or BEM (Jing, 2003).

2.2.1 Continuous and Discontinuous Modelling of Rock Mass

Some engineering problems can be modelled using a finite number of well-defined components. The behaviour of the components is also well known, or can be individually treated mathematically. Inter-relations between components can be defined the global behaviour of the system. In larger problems, the definition of such independent components may require an infinite sub-division of the problem domain which is known as the continuous problem and have infinite degrees of freedom. Then, the continuous problem is divided into a finite number of subdomains (elements) with approximated mathematical explanation to be used in numerical modelling methods, and whose behaviour is with finite degrees of freedom. Therefore, the computational methods is approximate the infinite degree of freedom by a discrete system with finite degrees of freedom (Jing, 2003).

The elements of a discrete system are usually treated as continuous and their characteristics can be obtained from laboratory tests if the components are macroscopically homogeneous like an elastic beam structures, or may be mathematically derived from homogenization processes for heterogeneous or/and fractured components, such as the fractured rock masses which are considered in this study. Therefore, the concepts of continuum and discontinuum are not absolute and are depending mainly on the size of the problem. This is particularly true for rock mechanics problems. Furthermore, the closed solution is not available for the fractured rock mass and numerical methods must be used for solving practical problems. Due to the differences in the fundamental material assumptions, different numerical methods have been developed for discontinues and continuous problems (Jing, 2003).

2.3 Numerical Methods for Rock Mechanics Modelling

Assumption of infinitesimal element with infinite degrees of freedom can be applied in many problems, which is known as the continuous. The differential equations of the continuous system can be solving by subdividing the domain to the finite number of sub-domains called elements. Therefore, the whole system can be solved numerically by solving the simpler numerical expression with finite degrees of freedom. Then, the continuity condition and elements interface should be satisfy using the principal differential equations. Therefore, the continuum based numerical model was developed using this idea by approximating the continuous system with infinite degrees of freedom by discrete system with finite degrees of freedom (Jing, 2003). Continuum-based models are the most commercial of the available mechanical engineering and material modelling software, and generally use the well known FEM and boundary element method (BEM).

The rock mass can be considered as a system of blocks cut by planes of discontinuity. The geometrical distribution and physical properties of discontinuities always effects on the jointed rock system (Hoek and Bray, 1981). The conventional continuous-based methods can not model the system of block correctly by the assumption of displacement continuity across elements, even using the likewise interface element alone without the displacement continuity assumption. In the large deformations and displacements condition the continuum methods can not satisfy the conditions (Shentu, 2011). Continuum methods for modelling joints and large displacements involve inappropriate approximations, and often researchers choose discontinuum modelling techniques for projects that involve rock mass. Therefore, discontinuum methods are developed to solve the problems involving the discontinuity such as rock mass. The discontinuum methods solve the displacement function across the element boundaries. The Distinct Element Method or Discrete Element Method (DEM) is the most commonly used in commercial geotechnical software. Another discontinuum method currently available, discontinuous deformation analysis (DDA) was developed by Shi (1988).

However, in most cases the individual discrete blocks are also able to deformed or fracture, which both continuous to discontinuous problems should be involved in the problems analysis. The combined continuum-based and discontinuum-based numerical methods that are known as Hybrid methods are able to represent these problems, such as the numerical manifold method (NMM) and the combined finite-discrete element method (Munjiza, 2004). The review and basics of some of well-known numerical methods are presented in the following sections.

2.3.1 Finite Difference Methods (FDM)

The finite difference method is the oldest numerical method used for the solution of group of differential equations, given initial values or boundary values. FDM is directly replaced an algebraic expression for each set of governing equations, The algebraic expressions is defined in terms of the field variables (e.g., stress or displacement) at discrete points in space which are undefined within elements. One of the most commercially available software which has been used in both soil and rock mechanics is FLAC (Mitra, 2006). Some of the studies using FDM method was carried out by (Iannacchione and Vallejo, 1991; Zipf Jr, 1999; Burke, 2003; Esterhuizen and Iannacchione, 2005; Esterhuizen and Karacan, 2005).

2.3.2 Finite Element Method (FEM)

The concept of discretization in solving PDE provided the basis for developing the FEM, which is the most commonly, used numerical method. The FEM is the most popular method for rock mechanics because the flexibility the method provides for overcoming most geotechnical problems. The domain discretization is the initial step in FEM, and is followed by defining the approximation for each discretization part (elements). Finally, the resulting linear equations can be solved using any available numerical linear algebra methods (Jing, 2003). The approximation functions are identified using linear or higher order equations and specified for each unknown values for each element. The weight function definition is based on Galekin's method for each element:

$$u_i = \sum_{j=1}^{m} N_{ij} u_i^j \tag{2-1}$$

where N_{ij} is the FEM shape function and m is the order of elements. Subsequently, according to the elastic problem (Hook's law), the partial differential equation of elasticity problem can be written as follows:

$$\sum_{i=1}^{n} [f]_i = \sum_{i=1}^{n} [K_{ij}][u_j] \tag{2-2}$$

where $[K_{ij}]$ is the coefficient matrix which is known as the FEM stiffness matrix, $[u_j]$ is unknown nodal displacement vector, and $[f_i]$ is the vector of external forces. The stiffness matrix is defined by the relation between the elasticity matrix $[E_i]$, the shape function, and the element geometry matrix $[B_i]$, given by:

$$[K_{ij}] = \int_{\Omega} ([B_i][N_i])^T [E_i] [B_j] \, d\Omega \tag{2-3}$$

The original finite element theories have been modified for many rock mechanical problems. Some models were developed to improve FEM for overcoming the difficulties encountered with the original FEM. Such modifications resulted in extended finite element method (XFEM) (Moës et al., 1999) and generalized finite element method (GFEM) (Strouboulis et al., 2000; Strouboulis et al., 2001), which uses the theories of partition of unity proposed by (Melenk and Babuška, 1996).

2.3.3 Distinct Element Method (DEM)

Cundall (1971) developed the discontinuum method referred to as the DEM, which uses an explicit time-marching scheme to solve the equation of motion for blocks. The first DEM used an assembly of rigid blocks and interaction between the blocks to model jointed rocks. The DEM is commonly used in large-scale block problems with large displacements and rotations. Fundamentally, the DEM solves Newton's second law of motion to determine the displacements and velocities through a finite difference algorithm (Bobet et al., 2009). DEM uses an explicit time-marching scheme for solving the dynamic equations of motion of the rigid block system. For deformable block systems, can be applied to DEM using explicit solution with finite volume discretization of the block interiors for the treatment of block deformability, which is not required to solving large-scale matrix equations. The other alternative is using an implicit solution with finite element discretization of the block interiors which leads to a matrix equation of block system deformability.

The method has found many applications in rock mechanics, soil mechanics, structural analysis, granular materials, material processing, fluid mechanics, multi-body systems, robot simulation, computer animation, etc. and became one of most rapidly developing method in computational mechanics. DEM uses the rigid or deformable blocks as the problem domain and then contact among them identified and continuously updated during analysis process (Jing, 2003). This fundamental conception leads to three central issues:

1. Identification of blocks of the system and the fractures geometry in the problem domain
2. Producing the formulation of blocks motion and solve the equations
3. Detection the contacts of blocks and updating the deformation and motion of the discrete system.

The DEM consider the contact pattern between the blocks of the system which are continuously changing with the deformation process, but are fixed for the latter difference which make difference between DEM and continuum based methods. The DEM method is suitable for simulating the mechanical processes in rock mechanics applications.

The DEM is commonly used in commercial numerical programs. The most well-known explicit DEM methods is the Distinct Element Method created by Cundall (1980) and the computer codes UDEC (ITASCA, 2012) was developed based on DEM. The bonded particle method (BPM) developed by Potyondy and

Cundall (2004) is based on DEM and includes an assumption that discontinuous medium is comprised of circular disks in two dimensions. The method uses cantered particle approximation to solve the global equation. Simulation using BPM assumes the deformation is the result of particle contact or relative displacement between the particles (Bobet et al., 2009). Partial Flow Code (PFC) (Cundall and Strack, 1999) is a BPM–based commercial computer program, which was previously used by researchers to evaluate rock mechanical problems.

Other parallel developments were made based on the distinct element approach that named discrete element methods such as in (Taylor, 1983; Williams and Mustoe, 1987; Williams, 1988; Williams and Pentland, 1992; Williams and O'Connor, 1995). Researchers was studied the application of discrete element methods for solving rock engineering problems (Wang and Garga, 1993; Hu, 1997; Li and Wang, 1998; Li and Vance, 1999). The Discontinuous Deformation Analysis (DDA) is an implicit DEM was originated by Shi (1988), further review of DDA is presented in next section.

2.3.4 Discontinues Deformation Analysis (DDA)

Shi developed DDA based on his block theory (Shi and Goodman, 1985; Shi, 1988). Both DDA and DEM analysis treats joined rock mass as a discrete block system. However, there is fundamental difference between two methods, in the DEM each block is analyzed separately to calculate the solution while the DDA is calculating the block problems by minimizing the total potential energy of whole blocks (Bobet et al., 2009). DDA has two advantages over the explicit DEM which make it more practical in engineering, (i) DDA can handle relatively larger time steps and (ii) DDA uses closed-form integrations for the stiffness matrices of elements. In addition, the typical FEM code can transform into a DDA code and keeping all the advantageous features of the FEM (Jing, 2003).

The unknown displacement is solved in DDA by using motion theory and simulating block interaction by applying the force and stiffness to the system, the interaction between blocks is calculated based on the mechanical spring and penalty function. The system of equations is then solved in the same way as the FEM by minimizing the potential energy. Shi (1992) was assumed the total displacement and deformation of the system as the accumulation of small displacements and deformations of time steps. Additional development for stress-deformation analysis was done by (Chang, 1994). Also, some studies has improved the contact algorithm of DDA (Yeung et al., 2007; Bao and Zhao, 2010; Wang et al., 2013). Many studies were used, extended and verified the DDA using the comparison of predicted results of NMM with the analytical solutions (Doolin and Sitar, 2002; Wu et al., 2005; MacLaughlin and Doolin, 2006; Tsesarsky and Hatzor, 2006).

2.3.5 Hybrid Models

Hybrid models are frequently used in rock engineering for flow and stress–deformation problems of fractured rocks. The most acknowledgeable types of hybrid models are the hybrid BEM/FEM, DEM/BEM and hybrid DEM/FEM models. The BEM is most commonly used for simulating far-field rocks as an equivalent elastic continuum, and the FEM and DEM for the non-linear or fractured near-field where explicit representation of fractures or non-linear mechanical behaviour, such as plasticity, is needed. This harmonizes the geometry of the required problem resolution with the numerical techniques available, thus providing an effective representation of the effects of the far-field to the near-field rocks. (Jing, 2003).

2.3.5(a) Hybrid FEM/BEM

Zienkiewicz et al. (1977) were proposed the hybrid FEM/BEM, afterward Brady and Wassyng (1981) and Beer (1983) were used it as a general stress analysis technique. The hybrid FEM/BEM has been used mainly for simulating the mechanical behaviour of underground excavations in rock mechanics (Varadarajan et al., 1985; Von Estorff and Firuziaan, 2000; Rizos and Wang, 2002).

The coupling algorithms are also developed by Beer and Watson (1992). The technique builds an artificially symmetrized stiffness matrix for treatment of the BEM region using the least-square techniques. Therefore, the matrix can be easily included in the symmetric FEM stiffness matrix, which is easier to handle than the non-symmetric BEM stiffness matrix. This symmetrization process initiates additional errors into the final system equations. The coupling may performed in the opposite direction by treating the FEM region as a BEM element, and apply the corresponding FEM stiffness matrix into the final BEM stiffness matrix. Then, this makes an asymmetric stiffness matrix of the final equation, which needs additional computational process (Jing, 2003).

2.3.5(b) Hybrid DEM/BEM

The hybrid DEM/BEM model was created by Lorig and Brady (1982) and applied to the explicit Distinct Element Method which is used in code of UDEC by de Lemos (1985). The super block of BEM was defined the cover the DEM region interface, the DEM can be treated by standard DEM contact concepts. The model should satisfy the conditions of: (i) the kinematics continuity along the interfaces of the two regions during the time-marching process, (ii) the elastic properties of the two regions near the interface are similar, (iii) blocks of DEM region assumed to be deformable (Jing, 2003).

Hybrid DEM/BEM was applied for modelling of hybrid discrete-continuum models for coupled hydro-mechanical analysis of fractured rocks (Wei and Hudson, 1998). They used Discrete Fracture Network (DFN) for covering the near-field of a fractured rock mass simulation by using independent DFN and DEM codes. Also, the far-field flow and stress-deformation in a continuum is simulated by BEM codes. The equations of flow and motion are coupled through an internal linking algorithm with the time-marching process after independently analysis of each DFN, DEM and BEM codes (Jing, 2003).

2.3.5(c) Numerical Manifold Method (NMM)

The NMM was developed by Shi (1997) based on DDA contact theories and partition of unity. The NMM can identify as an extension for DDA which can describe the deformability of the materials based on most common numerical modelling technique (FEM). The NMM is used the main concept of FEM in solving the element based problems. However, Shi (1997) was resolved some of limitation of FEM by introducing the mathematical and physical cover as separate layer which are linked together based on NMM frameworks. The NMM also known as Finite Cover Method (FCM) is extended and well adopted for solving special problems such as week and strong discontinuities (Kurumatani and Terada, 2009). A few studies also used the plasticity theories in combination with the NMM for modelling the nonlinear problems (Terada et al., 2003).

The main function $F(x, y)$ on the whole physical domain is defined. The physical domain is divided to finite small parts, physical cover ($U_i, i = 1, \ldots, n$), to simplify the function. The cover function $u_i(x, y)$ is defined for each physical cover U_i.

$$u_i(x, y) \qquad (x, y) \in U_i \tag{2-4}$$

The cover function $u_i(x, y)$ can be constructed by constant, linear or higher order polynomial functions. The local displacement functions in NMM approximation are solved over the mathematical cover and are independent of the physical domain. The global displacement function is the same as other normal analytical methods defined over the material volume comprised of the local displacement functions. For each mathematical cover, a cover function $\omega_i(x, y)$ is defined by the following:

$$\begin{aligned} &\omega_i(x, y) \geq 0 \qquad (x, y) \in U_i \\ &\omega_i(x, y) = 0 \qquad (x, y) \notin U_i \end{aligned} \tag{2-5}$$

and

$$\sum_{(x,y) \in U_i} \omega_i(x, y) = 1 \tag{2-6}$$

The estimation of global function $u(x, y)$ in the whole physical domain is defined from the sum of cover functions:

$$u(x,y) = \sum_{i=1}^{n} \omega_i(x,y)\, u_i(x,y) \tag{2-7}$$

The manifold-based equilibrium equations are solved in the same way as the finite element theories using the principal of virtual work (Ma et al., 2010). The weak form of the NMM approximation equation is based on virtual work and can be describe by assuming that the virtual work of external forces is equal to the virtual strain energy of the system. Therefore:

$$\int_{\Omega} \delta\varepsilon^T \sigma dV + \int_{\Omega} \delta u^T \rho \ddot{u}\, dV = \int_{\Omega} \delta u^T b dV + \int_{\Gamma_t} \delta u^T \bar{t} d\Gamma \tag{2-8}$$

where

ε = strain tensor

σ = stress tensor

u = displacement vector

b = body force per unit volume

$\bar{t}$ = traction on the boundary

ρ = is the density

$\ddot{u}$ = the acceleration vector

In the same manner as the FEM, equilibrium is reached by minimizing the total potential energy (Π). The total potential energy is obtained from the summation of the potential energy of different sources for each force, stress and strain.

Table 2.2 lists the different types of potential energy for force or stress and the name of the corresponding matrix.

Table 2.2: Different types of potential energy

Source	Symbol	Matrix
Strain potential energy	Π_e	Stiffness matrix
Potential energy of initial stresses	Π_σ	Initial stress matrix
Potential energy of inertia	Π_i	Mass matrix
Potential energy of point load	Π_p	Point load matrix
Potential energy of body load	Π_ω	Body load matrix
Potential energy of contact springs	Π_s	Contact matrices
Potential energy of friction force	Π_f	-

The equation of motion is used to mathematically describe the dynamics of the system with assumption that the F force vector depends on time as well as D, $\dot{D}$ and $\ddot{D}$ (displacement vector, velocity and acceleration, respectively).

$$M\ddot{D}_{n+1} + C\dot{D}_{n+1} + KD_{n+1} = F_{n+1} \quad (2\text{-}9)$$

In Equation (2-9) the parameter K, C and M are the stiffness, damping and mass matrices. It is also possible to set the initial condition, $D_0 = d_0$ and $\dot{D}_0 = v_0$. Equation (2-10) can describe the static condition can be obtained from Equation (2-9) and can be used for simulating with NMM.

$$KD_{n+1} = F_{n+1} \quad (2\text{-}10)$$

Otherwise, the total potential energy has the following form:

$$\Pi = \frac{1}{2}\begin{bmatrix} D_1^T & D_2^T & \cdots & D_n^T \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ K_{n1} & K_{n2} & \cdots & K_{nn} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_n \end{bmatrix} + \begin{bmatrix} D_1^T & D_2^T & \cdots & D_n^T \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{bmatrix} + C \tag{2-11}$$

The equilibrium of the load and forces acting on cover i (node *i*) in the x and y directions is as follows:

$$\frac{\partial \Pi}{\partial u_i} = 0, \qquad \frac{\partial \Pi}{\partial v_i} = 0 \tag{2-12}$$

The minimization of the total potential energy of Equation (2-11) produces *n* submatrix equations for *n* physical cover by using the following differential equation for the unknowns d_{js} and d_{ir}:

$$\frac{\partial^2 \Pi}{\partial d_{ir} . \partial d_{js}}, \qquad r, s = 1,2 \tag{2-13}$$

Adding whole matrices for each physical cover is produces the same of motion:

$$\begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ K_{n1} & K_{n2} & \cdots & K_{nn} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_n \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{bmatrix} \tag{2-14}$$