

**BEHAVIOUR OF UNDER-REINFORCED  
SHALLOW FIBROUS CONCRETE BEAMS  
SUBJECTED TO PURE TORSION**

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**by**

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# **KELAKUAN RASUK GENTIAN KONKRIT CETEK BERTETULANG KURANGAN DI BAWAH KILASAN TULEN**

## **ABSTRAK**

Rintangan kilasan untuk rasuk gentian konkrit cetek kurangan-tetulang di bawah pengaruh zon tegangan unggul, ketebalan penutup konkrit, ikatan kekuatan dan indek tetulang telah dikaji. Sehingga ini, sumbangan zon tegangan unggul dan penutup konkrit untuk merintangi kilasan berdasarkan tiub berdinding nipis, analogi kekuda ruang telah diabaikan. Dalam kajian ini, tigapuluh (30) rasuk gentian konkrit cetek kurangan-tetulang telah disediakan dan diuji di bawah kilasan tulin. Didapati rintangan kilasan pada beban puncak telah bertambah baik disebabkan kemasukan tetulang tambahan pada luas keratan zon tegangan unggul, penambahbaikan ikatan kekuatan tetulang membujur dan matrik gentian konkrit dan pengurangan dalam indek tetulang. Dalam pada itu, rintangan kilasan pada beban puncak dan keretakan ditambah baik hasil daripada penebalan penutup konkrit. Walau pun keterikan tetulang membujur memberi kesan terhadap bilangan keretakan pada rasuk gentian konkrit pada kegagalan, didapati indek tetulang dan ikatan kekuatan dipengaruhi sudut kecondongan keretakan pada kegagalan. Analisis dimensi dan model kekuda ruang yang sedia wujud telah diubahsuai untuk mencadangkan satu pendekatan baru bagi membuktikan kesan kemasukan tetulang tambahan pada zon tegangan unggul, ketebalan penutup konkrit dan ikatan kekuatan tetulang bagi rasuk konkrit gentian terhadap kapasiti kilasan. Kesimpulannya, kajian ini telah membuktikan sumbangan parameter-parameter yang disebut di atas dan rumusan-rumusan yang dicadangkan untuk meramal rintangan kilasan pada retak dan beban puncak adalah munasabah dengan dapatan.

# **BEHAVIOUR OF UNDER-REINFORCED SHALLOW FIBROUS CONCRETE BEAMS SUBJECTED TO PURE TORSION**

## **ABSTRACT**

Torsional resistance of under-reinforced shallow fibrous concrete beams with the influence of the idealized core zone, thickness of concrete cover, bond strength and reinforcement indexes were investigated. Up-to-date, the contribution of the idealized core zone area and thickness of concrete cover to resist torsion based on thin-walled tube, space truss analogy have been ignored. In this investigation, thirty samples (30) of under-reinforced shallow fibrous concrete beams were prepared and tested under pure torsion. As a result, the torsional resistance of peak load was improved due to additional reinforcements in the idealized core zone area of the section, enhancement of bond strength between longitudinal reinforcement and fibrous concrete matrix, and reduction in the reinforcement indexes. Meanwhile, the torsional resistance at the crack and peak loads were improved due to thickening of concrete cover. Although the strain in longitudinal reinforcement was effected on crack number in fibrous concrete beams at failure, the reinforcement indexes and their bond strength in fibrous concrete were found to influence on the inclination angle of the crack at failure. The dimensional analysis and space truss model based on the established models were modified to propose a new approach for proving the effect of additional reinforcements in the idealized core zone, thickness of concrete cover and bond strength of embedded reinforcement in fibrous concrete on the torsional capacity of the beam. In conclusion, this study has proven the contribution of all the above mentioned parameters and the proposed equations for predicting torsional resistance at crack and peak loads are reasonably agreed with the results.



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