

MODELING AND CONTROL OF LANE KEEPING SYSTEM FOR AUTONOMOUS VEHICLE

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**MODELING AND CONTROL OF LANE KEEPING SYSTEM
FOR AUTONOMOUS VEHICLE**

by

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LIST OF ABBREVIATIONS

| | |
|-------|--|
| ASM | Active Set methods |
| ASTM | American Society for Testing and Materials |
| AHS | Automated Highway System |
| CG | Center of Gravity |
| CCD | Charge Coupled Devices |
| CMOS | Complementary Metal Oxide Semiconductor |
| DARPA | Defense Advanced Research Projects Agency |
| Dist | Disturbance |
| GM | General Motor |
| KF | Kalman Filter |
| LDWS | Lane Departure Warning System |
| LKAS | Lane Keeping Assista System |
| lim | limit |
| MPC | Model Predictive Controller |
| min | Minimum |
| max | Maximum |
| MIMO | Multi Input Multi Output |
| MV | Manipulated Variable |
| PI | Proportinal Integral |
| PO | Percentage of Overshoot |
| PID | Proportinal Integral and Derivative |
| QP | Quadratic Programming |
| Ref | Reference |
| SSV | Structured Singular Value |
| SISO | Single Input Single Output |
| SITO | Single Input Two Output Motor |
| SAE | Society of Automotive Engineers |
| sup | Supremum |
| TISO | Two Input Single Output |

LIST OF SYMBOLS

| | |
|----------------|--|
| A | System matrix |
| a_y | Inertial acceleration of the vehicle |
| A_k | System matrix at time k |
| A_o | Nominal system matrix |
| A_i | Uncertain system matrix |
| A_c | Real matrix |
| B_o | Nominal input matrix |
| B_i | Uncertain input matrix, |
| B | Input matrix |
| b | Real vector |
| B_k | Input matrix at time k |
| C | Output matrix |
| C_o | Nominal output matrix |
| C_i | Uncertain output matrix |
| \mathcal{C} | Observability matrix |
| c^T | Transpose of vector c |
| C_α | Cornering stiffness |
| C_{α_f} | Cornering stiffness of the front tire |
| C_{α_r} | Cornering stiffness of the rear tire |
| C_m | Output matrix for measured variable |
| D_o | Nominal feedthrough matrix |
| D_i | Uncertain feedthrough matrix |
| D_m | Feedthrough matrix for measured variable |
| d_{o1} | Disturbance to y_1 |
| d_{o2} | Disturbance to y_2 |
| d_1 | Disturbance to input |
| D | Feedthrough matrix |
| $d(s)$ | Disturbance of plant in s -domain |

| | |
|-------------|--|
| D_s | Denominator of the transfer function of a system |
| e | Exponential |
| \dot{e}_y | Derivative of lateral error variable of the vehicle |
| \dot{e}_p | Derivative of angular error of the vehicle |
| e_1 | Error for <i>output</i> ₁ |
| e_2 | Error for <i>output</i> ₂ |
| e_{ymin} | Lower limit of lateral error variable constraint |
| e_{ymax} | Upper limit of lateral error variable constraint |
| e_p | Angular deviation of the vehicle with respect to lane centerline |
| e_y | Lateral deviation of the vehicle with respect to lane centerline |
| $e(t)$ | Error variable vector at time t |
| f | Vector function |
| F_x | Longitudinal force acting on the tire |
| F_x | Lateral force on the tires |
| F_y | Lateral force acting on the tire |
| F_{x_r} | Longitudinal force acting on the rear tire |
| F_{y_r} | Lateral force acting on the rear tire |
| F_{x_f} | Longitudinal force acting on the front tire |
| F_{y_f} | Lateral force acting on the front tire |
| f_c | Continuous time domain vector function |
| f_d | Discrete time domain vector function |
| $F_t(s)$ | Transfer function of a system |
| g | Vector function |
| G_p | Plant transfer function |
| G_c | Controller transfer function |
| G_d | Output to disturbance transfer function |
| G_y | output to control input transfer function |
| G_{y_1} | <i>Output</i> ₁ to control input transfer function |

| | |
|------------------|--|
| G_{y_2} | $Output_2$ to control input transfer function |
| $G(s)$ | Overall transfer function of plant |
| $G_{d_1}(s)$ | $Output_1$ to disturbance input transfer function |
| $G_{d_2}(s)$ | $Output_2$ to disturbance input transfer function |
| G_r | Transfer function of regulator |
| H | Hessian matrix |
| H_p | Prediction horizon |
| H_c | Control horizon |
| h | Vector function |
| H_k | Hessian matrix at time k |
| i | Lower limit of constraint |
| I_{zz} | Moment of inertia of the vehicle |
| \tilde{I}_{zz} | Normalized moment of inertia of the vehicle |
| I | Identity matrix |
| J | Cost function |
| j | Laplace variable |
| K_d | Controller |
| K | Compensation matrix for pole placement |
| K_k | Kalman gain at time k |
| K_p | Proportional coefficient |
| K_c | Integral coefficient |
| K_d | Derivative coefficient |
| L_i | Uncertain look ahead distance |
| $-\bar{L}_i$ | Lower limit of interval of parameter L_i |
| \bar{L}_i | Upper limit of interval of parameter L_i |
| l_x | Lower limit of constraint |
| l_f | Distance between the vehicle CG from the front axle of the vehicle |
| l_r | Distance between the vehicle CG from rear axle of the vehicle |

| | |
|---------------|---|
| L | Look ahead distance |
| m_1 | Dimension of output matrix |
| m | Mass of vehicle |
| \tilde{m} | Normalized mass of vehicle |
| M | Nominal closed loop system |
| M_z | Rotating moment of the vehicle around z axis |
| \mathcal{M} | Matrix containing all uncertainties |
| n_{y_1} | Noise input to y_1 |
| n_{y_2} | Noise input to y_2 |
| N_z | Normal force acting on tire |
| n_1 | Dimension of matrix |
| n | Number of dimensional space |
| N_s | Nominator of the transfer function of a system |
| \mathcal{O} | Controllability matrix |
| P | Point |
| P_d | Interconnection matrix |
| p_1 | Dimension of input matrix |
| P_k | Predicted estimated covariance matrix at time k |
| P_o | Predicted estimated covariance matrix at time 0 |
| $P_{k k-1}$ | Predicted estimated covariance matrix |
| p_n | Poles of a system |
| Q | Upper limit of constraint |
| Q_k | Process noise covariance matrix at time k |
| R | Road radius |
| R_1 | Lower limit of constraint |
| R_2 | Upper limit of constraint |
| \dot{r} | Derivative of yaw rate of the vehicle |
| r_1 | Reference signal $output_1$ |
| r_2 | Reference signal $output_2$ |

| | |
|--------------|--|
| r | Yaw rate |
| r_{des} | Desired yaw rate |
| \mathbb{R} | Set of all real numbers |
| R_k | Measured noise covariance at time k |
| S_k | Innovation covariance matrix at time k |
| s_1 | Path length coordinate |
| S | Frequency domain |
| S_I | Input sensitivity |
| S_f | Scale factor |
| T_s | Settling time |
| t | Time |
| T_i | Time unit for integral control |
| t_0 | Initial time |
| T_d | Time unit for derivation control |
| T_s | Discretization time interval |
| t_k | Time step |
| t_w | Distance in between steering wheels of ground vehicle |
| u | Control input |
| u_x | Upper limit of constraint |
| u_{min} | Longitudinal force acting on the rear tire |
| u_{max} | Lateral force acting on the rear tire |
| $u(t_k + 1)$ | Discrete time domain input vector function at time $t_k + 1$ |
| $u(t_k)$ | Discrete time domain input vector function at time t_k |
| U_k | Control input at time k |
| $u(s)$ | Control input of plant in s -domain |
| \mathbf{u} | Input vector |
| v | Feedback signal vector |
| V_x | Longitudinal velocity of vehicle |
| V_y | Lateral velocity of vehicle |

| | |
|---------------------|---|
| w | Noise and disturbances |
| w | White gaussian noise |
| w_1 | Noise <i>output</i> ₁ |
| w_2 | Noise <i>output</i> ₂ |
| $w(k)$ | White noise at time k |
| x | Horizontal axis |
| x_0 | State vector at initial time |
| $x(t)$ | State vector at time t |
| $x_c(k+1)$ | Controller state at time $k+1$ |
| $x_c(k)$ | Controller state at time k |
| x_n | State vector of dimension n |
| \hat{x}_k | Estimation of vector x at time k given all measurements available |
| $\hat{x}_{k k-1}$ | Estimation of vector x at time k given first $k-1$ measurements |
| $\hat{x}_{k-1 k-1}$ | Estimation of vector x at time $k-1$ given first $k-1$ measurements |
| \hat{x}_o | State vector x at time 0 |
| x_l | Lane width |
| x^T | Transpose of vector x |
| \hat{x} | State vector |
| X | Global Cordinate along horizon |
| $\dot{x}(t)$ | Derivative of state vector at time t |
| y | Output vector |
| y_1 | <i>Out put</i> ₁ |
| y_2 | <i>Out put</i> ₂ |
| $y(t)$ | Output vector at time t |
| $y_{meas}(k)$ | Measured plant output at time k |
| $y(s)$ | Output of plant in s -domain |
| y_l | Car width |

| | |
|------------------|---|
| \hat{y}_k | Innovated output variable at time k |
| \dot{y} | Lateral velocity in temrs of lateral position of the vehcile |
| y | Verticle axis |
| Y | Global corodinate along verticle |
| z_r | Vector including reference signal, controlled signal and error signal |
| z | Plane |
| z_l | Avaibale space in the lane |
| z | Right angle axis |
| Z | Global corodinate along right angle |
| \hat{z}_k | Measurements at time k |
| z_n | Zeros of a system |
| α_d | Disturbance attenuation coefficient |
| β_f | Side-slip angle of the front tire of the vehicle |
| $\dot{\beta}$ | Derivative of the side-slip angle of the vehicle |
| β_m | Real Number |
| β | Side-slip angle |
| δ | Steering angle |
| Δ | Uncertain transfer matrix |
| Δ_i | Uncertain element |
| δ_{i_L} | Uncertain element for L |
| δ_{i_μ} | Uncertain element for μ |
| Δu_{min} | Lower limit of rate of change of control input constraint |
| Δu_{max} | Upper limit of rate of change of control input constraint |
| δ_i | Inner wheels steering angle |
| δ_o | Outer wheels steering angle |
| δ_1 | Output of PID_1 |
| δ_1 | Output of PID_1 |
| δ_2 | Output of PID_2 |

| | |
|----------------------|---|
| Δ_{u_p} | Optimization variable |
| $\eta(t_k + 1)$ | Discrete time domain output vector function at time $t_k + 1$ |
| $\eta(t_k)$ | Discrete time domain output vector function at time t_k |
| η | Output vector |
| η_d | Uncertainty vector |
| $\eta_{t+i,t}$ | Lower limit of lateral error variable constraint |
| $\eta_{ref_{t+i,t}}$ | Upper limit of lateral error variable constraint |
| \in | Is an element of |
| ε | Vector of cotinuous time domain |
| $\dot{\gamma}$ | Parameterized curve |
| μ_i | Uncertain road-tire friction coefficient |
| $-\bar{\mu}_i$ | Lower limit of interval of parameter μ_i |
| $\bar{\mu}_i$ | Upper limit of interval of parameter μ_i |
| μ_Δ | Analysis parameter for measuring robustness |
| μ | Road tire friction coefficient |
| ω | Laplace variable |
| ω_n | Natural frequency |
| ψ | Orientation of the vehicle |
| Π | Uncertainty model set |
| ρ | Road curvature |
| τ | Time coordinate |
| θ | Orientation angle |
| θ_i | Parameter of a system |
| $-\bar{\theta}_i$ | Lower limit of interval of parameter θ_i |
| $\bar{\theta}_i$ | Upper limit of interval of parameter θ_i |
| θ_{v_f} | Velocity vector angle of front tire |
| θ_{v_r} | Velocity vector angle of rear tire |
| ζ | Damping ratio |