ANALYTICAL AND NUMERICAL STUDY ON CARBUNCLE PHENOMENON

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ANALYTICAL AND NUMERICAL STUDY ON CARBUNCLE PHENOMENON

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LIST OF ABBREVIATIONS

ARS	Approximate Riemann Solvers
AUSM	Advection Upstream Splitting Method
CFD	Computational Fluid Dynamics
CFL	Courant-Friedrichs-Lewy
EC	Entropy-Consistent
EOS	Equation of State
FDS	Flux Difference Splitting
FVS	Flux Vector Splitting
HLL	Harten-Lax-van Leer
KHI	Kevin-Helmholtz Instability
RH	Rankine-Hugoniot
RHS	Right Hand Side
RMI	Richtmeyer-Meshkov Instability
RTI	Rayleigh-Taylor Instability
vNR	von Neumann and Richtmyer

LIST OF SYMBOLS

δ	intermediate perturbation
γ	specific heat ratio
ω	eigenvalues
ρ	density or mass
ς	dissipation coefficient
a	speed of sound
j_e	the ratio of pertubed total energy to mass
j_h	the ratio of perturbed total enthalpy to mass
j_u	the ratio of perturbed x-momentum to mass
j_v	the ratio of perturbed y-momentum to mass
j_r	the ratio of perturbed mass to mass
j_w^2	total of j_u^2 and j_v^2
k	latitudinal wave number
l	longitudinal wave number
E	total energy
H	total enthalpy
M	Mach number
S	entropy
p	pressure
R	right eigenvectors
U	conservative variables matrix

KAJIAN ANALITIKAL DAN BERANGKA KE ATAS FENOMENA INAS

ABSTRAK

Kebanyakan kaedah terbaru didalam literatur bagi menyelesaikan ketidakstabilan kejutan bagi persamaan konservasi hiperbolik lebih tertumpu kepada penambahan faktor penyebaran tanpa mendalami tunjang masalah tersebut. Salah satu contoh ketidakstabilan kejutan adalah fenomena inas yang terbentuk apabila simulasi aliran berkelajuan tinggi ke atas badan tumpul dijalankan dimana gelombang kejutan yang terbentuk adalah tidak menepati ketentuan fizikal. Oleh itu, objektif kajian ini adalah untuk mencari sekurang-kurangnya satu punca masalah dan memulihkan ketidakstabilan melalui punca yang ditemui tersebut. Pencarian punca masalah dijalankan melalui proses penyisihan dengan mengurangkan penglibatan pembolehubah konservatif dalam setiap persamaan yang digunakan bermula dari persamaan Burgers diikuti persamaan isoterma dan persamaan Euler. Kemudian, definisi gangguan digunakan untuk melinearisasikan persamaan yang akan diuji. Analisa menggunakan kaedah normal mod bagi melihat faktor-faktor ketidakstabilan dan salah satu darinya adalah berpunca dari gangguan pada ketumpatan. Ujian pengkomputeran dijalankan bagi mengesahkan penemuan ini dan hasilnya adalah sama dengan jangkaan analisa. Akhir sekali, kaedah penyebaran dikenakan keatas persamaan ketumpatan sahaja dengan meletakkan satu pekali yang boleh diubah. Ujian telah mendapati bahawa julat pekali pada 0.02 – 0.09 memadai untuk menstabilkan kesemua skema serta tidak terlalu menyebar pada lokasi kejutan.

ANALYTICAL AND NUMERICAL STUDY ON CARBUNCLE PHENOMENON

ABSTRACT

Most newly developed schemes in the literatures to solve the shock instability in hyperbolic conservation laws mainly focused on adding ad hoc diffusion factor without properly indulging into the sources of the problem. An example of shock instabilities is the carbuncle phenomenon which occurs when simulating a blunt body subjected to a high speed flow. The shock formed ahead of the body is unphysical. Therefore, the goals of this study are to find at least one possible cause of the problem and to fix the instability from that cause. Extruding a possible source of the problem, herein the elimination process was applied to reduce the number of conservative variables involve, starting from the Burgers' equation followed by isothermal equations to the full Euler equations. Then, a small perturbation definition to the hyperbolic conservation equations was used as a mean to ease the nonlinearity from the equations. After that, the method of normal mode was used to analytically analyze the instability mechanism. The cause was found to be the perturbation from density which seeding into the instability. Numerical tests were then used to check the validity of the analytical result and they gave a good agreement with the analysis. Finally, a tunable dissipative coefficient was inserted only to the density equation and a range value of $0.02 - 0.09$ was found to stabilize all the involved schemes without smearing the shock too much.