

**USING A CONCRETE-REPRESENTATIONAL-  
ABSTRACT (CRA) APPROACH IN LEARNING  
FRACTIONS AMONG GRADE FIVE IRAQI PUPILS**

by

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ABSTRACT (CRA) APPROACH IN LEARNING  
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## **TABLE OF CONTENTS**

Acknowledgement	ii
Table of Contents	iii
List of Tables	ix
List of Figures	xi
Abstrak	xii
Abstract	xiv

### **CHAPTER 1: INTRODUCTION**

1.0 Introduction	1
1.1 Background of the Study	1
1.2 Problem Statement	7
1.3 Objectives of the Study	14
1.4 Research Questions	15
1.5 Null Hypotheses	16
1.6 Significance of the Study	17
1.7 Limitations of the Study	18
1.8 Operational Definition of Key Terms	19
1.9 Thesis Organization	24
1.10 Summary	25

## **CHAPTER 2: LITERATURE REVIEW**

2.0	Introduction	26
2.1	Theoretical Framework	26
2.1.1	Jerome Bruner's Theory	26
2.1.2	Zoltan Dienes's Theory	27
2.1.3	Richard Skemp's Theory	28
2.1.4	Related Theories of CRA Module	29
2.1.5	Related Theories for Proficiency of Mathematics	31
2.2	Previous Studies	38
2.2.1	Previous Studies on CRA approach	38
2.2.2	Previous Studies on Proficiency of Mathematics	45
2.2.3	Previous Studies on Motivation in Learning Mathematics	50
2.2.4	Previous Studies on Fractions	57
2.2.5	Previous Studies on Pupils' Perceptions Towards Mathematics	63
2.3	Conceptual Framework of the study	69

## **CHAPTER 3: METHODOLOGY**

3.0	Introduction	74
3.1	Research Design	74
3.2	Research Variables	76
3.3	Population and Sample	76
3.4	The Concrete-Representation-Abstract approach	77

3.5	Instruments	77
3.5.1	Proficiency in Fractions Test	77
3.5.2	Motivation in Learning Fractions Questionnaire	83
3.5.3	Perceptions of Learning Fractions Using CRA approach Questionnaire	85
3.6	The Training for the teacher	87
3.7	Pilot Study	88
3.8	Data Collection Procedure	89
3.9	Data Analysis	91
3.10	Ethical Issues	93
3.11	Development of CRA approach	94

## **CHAPTER 4: RESEARCH FINDINGS**

4.0	Introduction	100
4.1	Descriptive Data Analyses	100
4.2	Research Questions	104
4.2.1	Research Question 1	104
4.2.2	Research Question 2	108
4.2.3	Research Question 3	111
4.2.4	Research Question 4	114
4.2.5	Research Question 5	118
4.2.6	Research Question 6	120
4.2.7	Research Question 7	122
4.2.8	Research Question 8	124

## **CHAPTER 5: DISCUSSION AND CONCLUSION**

5.0	Introduction	127
5.1	Summary of Findings and Discussion	127
5.1.1	Pupils' Proficiency in Fractions	127
5.1.2	Motivation in Learning Fractions through the CRA approach	131
5.1.3	Pupils' Perceptions of Learning Fractions Using the CRA approach	134
5.1.4	The Reasons for Grade Five Pupils' Perceptions of Learning Fractions Using the CRA approach	136
5.1.5	The Strength, Weaknesses and suggestion to improve the CRA approach from the teacher's perspective	138
5.1.6	The Strength, Weaknesses and suggestion to improve the CRA approach from the pupil's perspective	139
5.2	Implications of the study	140
5.3	Recommendations for future studies	142
5.4	Conclusion	142

<b>REFERENCES</b>	144
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## **APPENDICES**

Appendix A	CRA Approach	160
Appendix B	Validation of CRA Approach	256
Appendix C	Proficiency in Fractions Test (Pretest )	263

Appendix D	Proficiency in Fractions Test (Posttest )	275
Appendix E	Proficiency in Fractions Test (Retention test )	287
Appendix F	Questionnaire of Motivation in Learning Fractions (Post-survey)	299
Appendix G	Validation of Proficiency in Fractions Test	305
Appendix H	Validation of Translation of Proficiency in Fractions Test	315
Appendix I	Questionnaire of Perceptions of Learning Fractions Using CRA Approach	317
Appendix J	Validation of Perceptions of Learning Fractions Using CRA Approach Questionnaire	321
Appendix K	Validation of Translation of the Perceptions of Learning Fractions Using CRA Approach Questionnaire	325
Appendix L	Reliability Analysis for Proficiency in Fractions Test	331
Appendix M	Reliability Analysis for Perceptions of Learning Fractions Using CRA Approach Questionnaire	332
Appendix N	Semi-Structured Interview Protocol	333
Appendix O	Validation of Motivation in Learning Fractions Questionnaire	335
Appendix P	Questionnaire of Motivation in Learning Fractions (Pre- survey)	341
Appendix Q	Reliability Analysis for Motivation in Learning Fractions Questionnaire	347
Appendix R	Questionnaire of Motivation in Learning Fractions (Retention -survey)	348
Appendix S	Dependent -Sample T-Test for Proficiency Test (Pretest-Posttest)	354
Appendix T	Dependent -Sample T-Test for Proficiency Test (Posttest-retention test)	355
Appendix U	Validation of Translation of Motivation in Learning Fractions Questionnaire.	356
Appendix V	Dependent -Sample T-Test for Motivation in Learning Fractions (Pre-Post) survey	362



Appendix W	Dependent -Sample T-Test for Motivation in Learning Fractions (Post- retention) survey	363
Appendix X	Percentage and Median of the grade five pupils' perceptions of learning fractions	364
Appendix Y	Approval Letter from the School of Educational Studies, University Sains Malaysia (USM)	367

## LIST OF TABLES

	<b>Page</b>
Table 1.1 Data from pupils' interview	12
Table 2.1 Summary of Previous Studies Using CRA Approach	39
Table 2.2 Summary of Previous Studies about Proficiency in Mathematics	46
Table 2.3 Summary of Previous Studies about Motivation in Learning Mathematics	52
Table 2.4 Summary of Previous Studies about Fractions	59
Table 2.5 Summary of Previous Studies about Pupils' Perceptions in Learning Mathematics	65
Table 3.1 Research Design used in this study	75
Table 3.2 The Proficiency in Fractions Test Items	79
Table 3.3 The Validation Process for Proficiency Instrument	82
Table 3.4 The Validation Process for Motivation Questionnaire	84
Table 3.5 The Validation Process for Perception Questionnaire	86
Table 3.6 Data Analysis for each research question in this study	92
Table 4.1 Number of pupils in the experimental group	101
Table 4.2 Mean and Standard Deviation for the Pretest, Posttest and Retention Scores of Proficiency in Fractions test of the Experimental Group	101
Table 4.3 Mean and Standard Deviation for the Pre-survey, Post-survey and Retention- survey Scores for Motivation of the Experimental Group	103
Table 4.4 Normality Distribution for pretest, posttest for Proficiency	105

	of the Experimental group	
Table 4.5	Results of the dependent- samples t- tests	106
Table 4.6	The Effect Size of Proficiency in Fractions	107
Table 4.7	Normality Distribution for posttest, retention test for Proficiency of the Experimental Group	109
Table 4.8	Results of the dependent- samples t- test	110
Table 4.9	The Effect Size of Retention Proficiency in Fractions	110
Table 4.10	Normality Distribution for Motivation pre-survey and post- survey of the Experimental Group	112
Table 4.11	Results of the dependent- samples t- test	113
Table 4.12	The Effect Size of Motivation in Learning Fractions	113
Table 4.13	Normality Distribution for motivation post-survey and retention -survey of the Experimental Group	115
Table 4.14	Results of the dependent- samples t- test	116
Table 4.15	The Effect Size of Retention of Motivation in Learning Fractions	116
Table 4.16	Summary of Hypotheses Testing	117
Table 4.17	Percentage and Median of the grade five pupils' perceptions of Learning fractions	118
Table 4.18	Percentage for pupils' answers to Q1	124
Table 4.19	Percentage for pupils' answers to Q2	125

## LIST OF FIGURES

	<b>Page</b>
Figure1.1 Intertwined strands of Mathematical Proficiency	4
Figure 1.2 Mathematics Achievement for the sixth grade in primary schools in Iraq	9
Figure 2.1 Conceptual Understanding mind map	33
Figure 2.2 Procedural Fluency mind map	34
Figure 2.3 Conceptual Framework of the study	73

**PENGGUNAAN PENDEKATAN KONKRIT, PERWAKILAN DAN  
ABSTRAK (KPA) DALAM PEMBELAJARAN PECAHAN DI KALANGAN  
MURID IRAQ GRED LIMA**

**ABSTRAK**

Dalam sistem pendidikan Iraq, pecahan diperkenalkan dalam kalangan murid sekolah rendah bermula daripada Tahun Empat dan pengetahuan tersebut dikembangkan sehingga ke peringkat sekolah menengah. Walau bagaimanapun, adalah agak mengejutkan apabila murid sekolah di Iraq selalu menghadapi masalah dalam mempelajari pecahan. Oleh itu, objektif kajian ini adalah untuk mengkaji kesan pendekatan KPA terhadap kecekapan dan pengekalan kecekapan murid Iraq gred lima dalam pecahan. Kajian ini juga mengkaji kesan pendekatan KPA terhadap motivasi dan pengekalan motivasi murid dalam pembelajaran pecahan. Selain itu, ia juga mengkaji persepsi murid serta maklumbalas guru dan murid terhadap pembelajaran pecahan menggunakan pendekatan KPA. Kajian ini menggunakan reka bentuk kajian satu kumpulan praujian, pasca ujian dan ujian peyekalan yang melibatkan satu kelas seramai 34 orang murid gred lima dari sekolah Iraq di Kuala Lumpur, Malaysia dan pendekatan KPA digunakan untuk memperkenalkan konsep pecahan. Dua minggu selepas intervensi, murid-murid tersebut menduduki ujian pengekalan. Kaedah pengumpulan data kuantitatif seperti ujian pra, ujian pos, ujian pengekalan, dan tinjauan pra, tinjauan pos dan tinjauan pengekalan serta soal selidi persepsi digunakan untuk mengumpul data. Di samping itu, kaedah pengumpulan data kualitatif seperti temubual digunakan untuk mengumpul data tentang maklumbalas guru dan murid terhadap pembelajaran pecahan menggunakan pendekatan KPA. Ujian-t sampel bersandar digunakan untuk menganalisa data bagi

setiap soal kajian kuantitatif. Keputusan kajian menunjukkan bahwa terdapat kesan pendekatan KPA yang signifikan terhadap kecekapan tetapi tidak signifikan terhadap pengekatan kecekapan murid dalam pecahan. Keputusan kajian juga menunjukkan bahawa terdapat kesan pendekatan KPA yang signifikan terhadap motivasi tetapi tidak signifikan terhadap pengekatan motivasi murid dalam pembelajaran pecahan. Di samping itu, dapatan daripada soal selidik persepsi murid dalam pembelajaran pecahan menggunakan pendekatan KPA menunjukkan bahawa murid dalam kumpulan eksperimen melaporkan persepsi yang positif terhadap pembelajaran pecahan menggunakan pendekatan KPA. Secara keseluruhannya, dapatan daripada temubual menunjukkan bahawa maklumbalas guru dan murid terhadap pembelajaran pecahan menggunakan pendekatan KPA adalah positif dan pendekatan tersebut adalah berkesan dalam meningkatkan kecekapan murid dalam pecahan serta motivasi dalam pembelajaran pecahan.

**USING A CONCRETE-REPRESENTATIONAL-ABSTRACT (CRA)  
APPROACH IN LEARNING FRACTIONS AMONG GRADE FIVE IRAQI  
PUPILS**

**ABSTRACT**

In the Iraqi education system, fractions is a topic introduced to primary school pupils since Grade Four until secondary school. However, pupils in Iraqi schools often face difficulties in learning fractions. Thus, the two main objectives of this study were to examine the effects of the CRA approach on Iraqi Grade Five pupils' proficiency and retention of proficiency in learning fractions; and also in their motivation and retention of motivation in learning fractions. In addition, it also investigated the pupils' perception, feedback of teacher and pupils about learning fractions using the CRA approach. This study adopted a one-group pretest-posttest-retention test research design where a class of 34 Grade Five pupils from an Iraqi school in Kuala Lumpur, Malaysia participated and the CRA approach was used to introduce the concept of fractions. Two weeks after the intervention, the pupils sat for the retention test. Quantitative methods of data collection such as pretest, posttest, and retention test, pre-survey, post-survey, and retention survey and perception questionnaire were used to collect the data. Meanwhile, qualitative method of data collection such as interviews were used to collect the data on the feedback of teacher and pupils about learning fractions using the CRA approach. The dependent-samples t-test was used to analyse the data for each of the quantitative research questions. The results of the study indicated that there was a significant difference of the CRA approach on pupils' proficiency and no significant difference on retention of proficiency in learning fractions. The results of the study also showed that there was a significant difference of the CRA approach on pupils' motivation and no significant

difference on retention of motivation in learning fractions. Meanwhile, the result from the questionnaire of pupils' perceptions in learning fractions using the CRA approach revealed that the pupils reported positive perceptions in learning fractions using the CRA approach. Overall, the findings of the interviews showed that the feedback of teacher and pupils about learning fractions using the CRA approach was positive and that the approach was effective in improving the pupils' proficiency in fractions as well as their motivation in learning fractions.



# CHAPTER 1

## INTRODUCTION

### 1.0 Introduction

This chapter starts with the background of the current study which presents a summary of previous research conducted within the relevant scope. It focuses on the general explanation of education weaknesses in Iraq. The next section interprets the weaknesses of proficiency in fractions. The main problem statements of study and the literature gaps are later addressed. Moreover, research objectives, research questions, together with the hypotheses of the study are clearly presented. The significance of the study and operational definitions are presented at the end of the chapter.

### 1.1 Background of the study

Mathematics knowledge during schooling predicts academic and occupational success years later (Siegler & Forgues, 2015). Among topics of Mathematics, knowledge of fractions is important to pupils' future success. Indeed, it is necessary to learn fractions because before pupils proceed to decimal numbers (including measurement) with understanding, they need a deep understanding of the concept of fractions (Baturu & Cooper, 2008). A child would feel confident in his understanding of fractions because it is a corner stone for other mathematics skills. It is the fact that fraction plays a crucial role in helping pupils to understand the nature of numbers (Lisa, 2009).

It is necessary to teach and learn fractions because fractions are widely used in our daily life. For example, fractions can be used in cooking, building, sewing and in the stock market (Math Open Reference, 2009). Fractions are defined as divisions

and were invented before decimal numbers. They refer to a way of showing portions less than one. More specifically, the conceptual understanding of fractions is a prerequisite to deal with other areas of Mathematics. The conceptual understanding develops when pupils are able to connect between concepts and procedures and can explain why some facts are consequences of others (National Research Council, 2001; Wong & Evans, 2007). Thus, misconception in fractions learning may cause problems with other domains in Mathematics such as Algebra, Measurement, Ratio and Proportion concepts (Behr, Lesh, Post, & Silver, 1983).

Fraction concepts are understood as “part of a whole,” pupils can practise the steps involved in reading and writing fractions. A variety of physical materials can be used to show the meaning of a fraction as “part of a whole.” For example, fraction cubes, counters, fraction bars, or geometric shapes can indicate a fraction (e.g., 3 red cubes (part) out of the 5 cubes (whole, the total number of cubes). Representations and numeric symbols of the fraction can develop the skills of reading and writing fractions. The abstract stage is developed by writing a numeric symbol of the number of squares or parts of the whole in a correct fraction form. This step involves the order in which digits should be read or written.

In educational system in Iraq, pupils begin to go to school in Grade 1 at the age of 6. The primary school consists of six grades. It is compulsory for pupils to learn main subjects (i.e., Islamic Education, Arabic language, Science and Mathematics), but not compulsory to learn extra subjects (i.e., Art, Sports, Music). At primary level, pupils take oral exams for Mathematics in all grades till fourth grade. The maximum test score is 10 points for the first, second, third and fourth grades. However, in grade five and six, the maximum test score is 100 points for all subjects (i.e., Islamic Education, Arabic language, Science, Mathematics, English language,

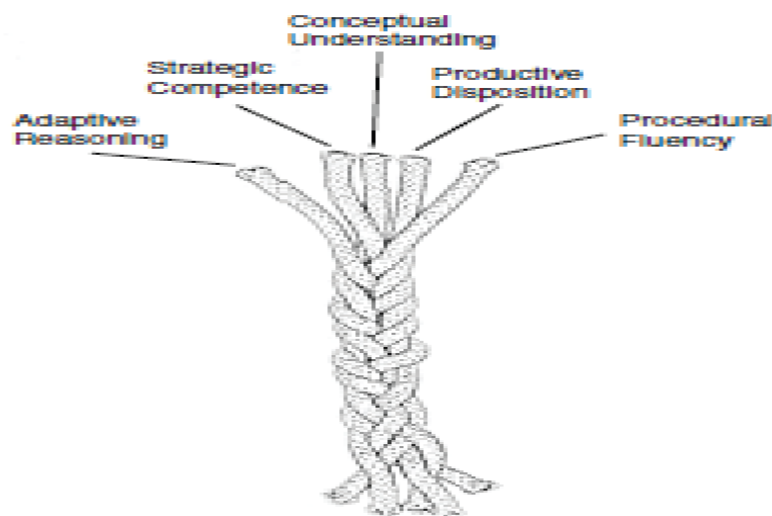
History and Geography) and extra subjects such as Art, Sports, Anthem and Music (UNESCO, 2003).

It is noticed that primary pupils in Iraq are exposed to learning fractions (i.e., numbers that express a part of a whole) for two years. They begin to learn fractions in the fifth grade. At this level, fractions are introduced to pupils in two units. Pupils are required to name and write fractions with denominators, express equivalent fractions to proper fractions, adding two proper fractions with denominators, and subtract proper fractions with denominators, multiply fractions and divide fractions (Mathematics Year 5, 2013).

The Ministry of Education (MOE) of Iraq proposed a strategy for education in September, 2014 extending until 2020. It aims to spread education and improve its quality among the priorities. This strategy will focus on the plan in the next phase, re-examine the curriculum and teaching methods applied in the classrooms. It also aims to keep pace with the global development and plan for training teaching staff including the introduction of information technology in all areas of education. In particular, the strategy of education will help teachers to improve their skills and their scientific levels. It will also help reflect the results of teaching and learning. It involves improving teachers' knowledge, enhancing teachers' love of teaching (Iraq National Commission for Education, Culture and Science, 2014).

It is true that successful mathematical learning requires not only knowledge of skills but also knowledge of procedures. Kilpatrick, Swafford and Findell (2001) proposed five "intertwining strands" of mathematical proficiency, namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Proficiency has some types of affective component. They used a drawing of a braid or rope to portray the interlocking, interpenetrating,

integrated nature of the five strands. However, they ended up with the metaphor of braiding because five strands were found to fit and develop together. It is confirmed that proficiency in Mathematics involves more than just a skill or understanding. Therefore, learners need to develop all five components simultaneously ( Kilpatrick et al., 2001).



*Figure 1.1.* Intertwined strands of mathematical proficiency

(Source: Kilpatrick et al., 2001, p. 117)

" Pupils with a conceptual understanding know more than isolated facts and methods. They understand why a fraction idea is important and the kinds of contexts in which it is useful. They have organised their knowledge into a coherent whole, which enables them to learn new ideas by connecting those ideas to what they have already known" (Kilpatrick et al., 2001, p.118). Furthermore, accuracy and efficiency are often associated with pencil and paper computation, pupils need to develop their ability to perform mental computations flexibly and fluently. Such fluency needs to be based on number sense and the understanding of important

concepts of fraction (Susie Groves, 2012). For strategic competence, it is the ability to formulate mathematical problems, represent them and solve them. This strand is similar to what has been called problem-solving and problem formulation in the literature of mathematics education and cognitive science, and fraction problem solving (Kilpatrick et al., 2001). In adaptive reasoning, pupils are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached when they prove that something is true or false, and when they compare and contrast related ideas and explain their choices (Susie Groves, 2012). Developing a productive disposition requires frequent opportunities to make sense of fractions to recognise the benefits of perseverance, and to experience the rewards of sense making in fractions (Kilpatrick et al., 2001).

Jerome Bruner (1964) theorised that learning consists of three progressive stages. He stated that going through each of the learning stage is essential to deeply understand the concept, and it helps the learner to improve reasoning skills. The three stages of Mathematics learning are Concrete, Representation and Abstract stage. Concrete-Representational-Abstract Approach provides supports for understanding underlying mathematical concepts before rules are learnt. This sequence moves from a concrete model of chips or blocks for multiplication to an abstract representation ( $\frac{2}{4} = \frac{1}{2}$ ), presented as follows:

- Concrete: It refers to the “doing” stage using concrete objects to model problems.
- Representational: It is the “seeing” stage using the representation of objects to model problems.
- Abstract: It means the “symbolic” stage using abstract symbols to model problems (The Access Center, 2009).

Little research had been done either on how precisely visualisation could help pupils to learn, or how to produce ones that are effective pedagogical tools (Philips, 2010). There were studies carried out to investigate the relationship between CRA Approach in mathematics learning and sex differences or problem solving but there was less or even no study which focuses on the relationship between CRA Approach and pupils' knowledge retention. Even though some had proved to be successful, there was still the need of minimizing the difficulty of the understanding of this addition process (Garcia, 2007). Guzman (2000) stated that human perception was very strongly visual. Thus CRA Approach was chosen as a Approach to implement in this study in order to improve pupils' proficiency in fractions.

Motivation can be defined as the force that drives individuals to behave in a particular way. A variety of external influences, including cultural differences, family involvement, and classroom environment, can all impact students' motivational beliefs. Individual beliefs and attitudes can also determine a person's motivation. The pupils in mathematics classroom often feel unmotivated for a variety of reasons. One of these reasons could be the mundane, repetitive nature of the current teacher-centered instructional style which is prevalent in primary school mathematics classrooms. Meanwhile, pupils go through the education system, teachers and administrators need to be concerned with pupils that appear to be unmotivated and have below grade level abilities. According to Banda, Matuszny, and Therrien (2009), "Increasing pupils' motivation is a complex and ongoing process, especially for pupils who have often experienced extensive failure with mathematics in the past" (p.146). School professionals are constantly investigating methods to improve pupils' motivation and help pupils to be successful in school.

On the hand, retention could be defined as a measure of how well a pupil remembered the learnt material over time. It could be considered as the extent to which one could successfully access and use the information from long-term memory (Jukic & Dahl, 2012; Sousa, 2000). Knowledge retention is important because the improvement in skill and knowledge is dependent upon the learner's retention of the previously learned skills and knowledge. Rohrer and Taylor (2006) stated that the benefit of learning will lose when the materials presented during the lesson were forgotten. They emphasised that such forgetting was common for knowledge acquired in school and might even lose within days or weeks of learning. Therefore, it was important to define learning strategies that could promote long-lasting retention. In this study, pupils' retention on the arithmetic skill that is proficiency in fractions was examined to check whether the CRA approach could better extend and retain pupils' mathematics knowledge for meaningfully long period of time when compared to the conventional teaching method. Meanwhile, pupils' retention was examined to check their motivation in learning fractions.

## **1.2 Problem Statement**

Mathematics has always been one of the core subjects in the school syllabus around the world. It is not only applied in the learning process but also commonly used in daily life. Fractions is one of the topics introduced in the primary education. According to Charalambous and Pantazi (2005), fractions is the most complex concept among children in primary education. Other scholars also agreed that students have problems in learning fractions and the problems persist into their adulthood (Bruce & Ross, 2009; Lee, 2008; Naiser, et.al., 2004). This is one of the reasons why students are unable to fully understand fractions taught in class.

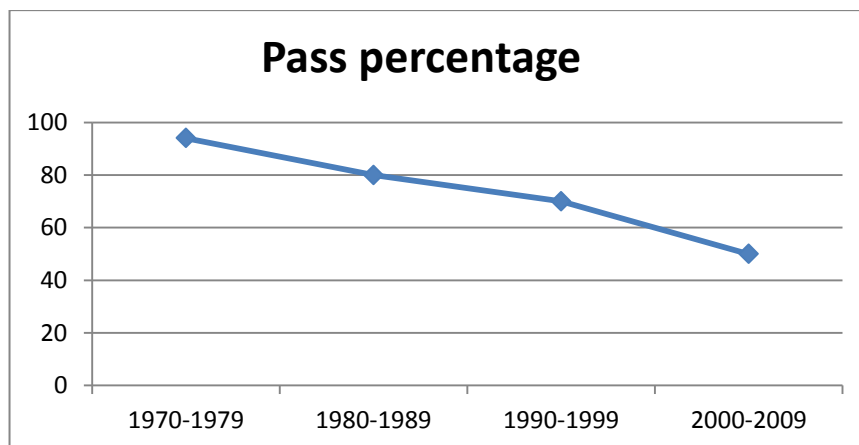
However, students could not visualise the problem or understand the solution and the equivalent fractions of division of fractions.

The problem with equivalent fractions is that students sometimes do not know how to find the equivalent fractions and sometimes they could not find the connection between the equivalence and size of the two fractions given. Charalambous and Pantazi (2005) also highlighted that the obstacles that students encounter are due to the instructional Approaches in teaching fractions. Meanwhile, the National Council of Teachers of Mathematics (NCTM, 2012) clearly points out that students in secondary school need to acquire a deep understanding of fractions and to be able to use them proficiently in problem solving such as algebra. Hence, it is apparent that students need to understand fractions during their primary school. Since then, many efforts have been made to explore the alternative ways of teaching fractions by creating curricula and didactic material which incorporate new tools, pedagogical approaches. Therefore, educators need to find ways of teaching fractions that would not only make students understand fractions but at the same time can catch their attention to the lessons that is being taught.

Education in Iraq is administered by the Iraqi Ministry of Education and for many years, Iraq suffered several wars, which makes the education level in Iraq decreases and the quality of education could not achieve the required level (Figure 2) (UNESCO, 2010).

In addition, Iraq did not participate in international comparative studies such as the Trends in International Mathematics and Science Study (TIMSS) and Programme for International Students Assessment (PISA).





*Figure 1.2.* Mathematics achievement for the sixth grade in primary schools in Iraq

(UNESCO, 2010)

In the final Mathematics exam of Iraqi primary schools in the academic year 2011-2012, the pass percentage of grade six pupils was 79 %; whereas in the academic year 2012-2013, the pass percentage decreased to 77%, and in the academic year 2013-2014, the pass percentage significantly decreased to 70%. In the final Mathematics exam of Iraqi primary schools in Malaysia in the academic year 2011-2012, the pass percentage of grade six pupils was 85%. However, in the academic year 2012-2013, the pass percentage slightly decreased to 83% and in the academic year 2013-2014 the pass percentage continued to be decreasing to 80%. (MOE in Iraq, 2014).

In the Iraqi primary school Mathematics curriculum, fractions are taught in grades five and six. For two years, pupils' learn how to read, write fractions and basic operations of fractions during two units (MOE Iraq, 2000). For the fractions question to measure the pupils' achievement in the final exam in Iraq, the pass percentage was 75% for the academic year grade six 2011-2012. In the academic year 2012-2013, the pass percentage slightly reduced to 72% and in the academic

year 2013-2014, it remained the same, with 72% (Education for all global monitoring report EGMR, 2013-2014). The pass percentage of the Iraqi primary schools in Malaysia in the academic year grade six 2011-2012, accounted for 80%. However, in the academic year 2012-2013, the pass percentage was slightly increased to 81%, and in academic year 2013-2014, the pass percentage continued to increase to 82% (MOE Iraq , 2014).

The learning methods focus on rules and the procedural computation, whereas the conceptual understanding was limited. Undoubtedly, children get difficulties in learning fractions because it seems to be a complex and abstract skill. Previous research on difficulties in learning fractions suggested that errors involve misapplications of prior knowledge (NCTM, 2012).

One of the studies which support the effectiveness of Concrete-Representational-Abstract Approach is Witzel, Mercer and Miller's study (2003). They conducted a study of the sixth-and seventh grade students who have difficulty in learning Algebra. Students who learn how to solve Algebra transformation equations through CRA get higher scores on the post instruction and follow-up tests than the control peers receiving traditional instruction. Furthermore, students who use the CRA sequence of instruction performed fewer procedural errors when solving algebraic variables (Witzel, Mercer, & Miller, 2003).

The CRA Approach is a well-researched intervention and it is documented as an effective practice for a wide range of pupils, including low-achieving pupils (Flores, 2010; Mercer & Miller, 1992), pupils with LD (Cass et al., 2003; Maccini & Ruhl, 2000), pupils with emotional and behavioral disorders (EBD) (Mercer & Miller, 1992 ; Riccommini et al., 2008) and pupils with intellectual disabilities (ID) ( Morin & Miller, 1998). All the studies showed significant gains for pupils who received the

CRA intervention. The gradual sequence has been documented as effective from grades as early as second (Harris, Miller, & Mercer, 1995), to late elementary (Ketterlin-Geller, Chard, & Fien, 2008; Riccomini et al., 2008), to junior high level students (Butler et al., 2003; Cass et al., 2003; Witzel, 2005), and throughout high school math classes (Maccini & Ruhl, 2000).

In fact, pupils face difficulties in understanding fractions and presenting equivalent fractions (Munirah et al., 2003). This problem could be explained by learning fractions in the classroom that focuses on memorizing activities instead of emphasising the meaning (National Council of Teachers of Mathematics, 1989). From the theoretical background, the researcher attempted to conduct an interview to get deep insights into what problems teachers faced most when they teach fractions through their teaching experiences. The interview started with Mrs. Anaam, who has 25 years of experiences in teaching Mathematics for primary pupils in the fifth and sixth grades. Then, the interview was conducted with Mr. Haider, who has 20 years of experiences in teaching Mathematics in primary schools. Furthermore, Mrs. Anaam reported that most of the pupils did not understand fractions as a part of things. They were confused between fractions and integers. Therefore, they could not imagine the fractional part of the numbers. Pupils thought that because fractions have a numerator and denominator, therefore fractions are bigger than integers. Mr. Haider also reported that pupils considered fractions as symbols instead of concrete things. Therefore, they could not compare between fractions or distinguish which is bigger or smaller. Also, the interview was then conducted with some pupils randomly. The results of the interview are presented in Table 1.1.

*Table 1.1: Data from pupils' interview*

Learning Objectives	Percentage (%) of responses with understanding	Percentage (%) of responses after explanation	Percentage (%) of no response
Equivalent fractions	40%	0 %	60%
Fractional numbers	0%	50%	50%
Converting fractions to fractional numbers	0%	60%	40%
Converting fractional numbers to fractions	0%	80%	20%
Simplifying fractions	0%	100%	0%
Comparing fractions	0%	60%	40%
Order fractions	30%	40%	30%

Table 1.1 shows equivalent fractions, most pupils could not give responses to the questions despite that the questions are explained. However, with help for understanding the questions from the teacher, a big number of pupils could answer sections related to fractional numbers such as converting fractions into the fractional numbers, comparing fractions and order fractions. Furthermore, they could not understand the relation between converting fractional numbers to fractions and converting fractions to fractional numbers. Data from the interview revealed that pupils' proficiency in fractions remains low. In particular, most of them get a low proficiency level in fractions although some of them can answer the questions of fractions tests correctly.

Moreover, it was found that pupils who study more broad-based curriculum tend to do tests on skills and perform the assessments of conceptual understanding and problem solving better than those who experience skill-focused instruction (Schoenfeld, 2014). Witzel and Bradley (2008) also conducted an intervention to teach fractions for primary school pupils systematically. The instruction aimed to help pupils master important fraction concepts from the concrete level to the abstract level. Miller and Mercer (1993) stated that mathematics instruction should aim to help learners to move from needing concrete manipulatives to solving a problem to a point where they are able to think abstractly through the steps to solve a problem.

It is obvious that Mathematics is a challenging subject for pupils who have difficulties in understanding, especially, it is more difficult for pupils to understand when the concepts and instructional methods become more abstract. Therefore, using direct and explicit instruction with the purpose of helping pupils understand difficulties in Mathematics was emphasised by many researchers (e.g., Baker, Gersten & Dae-Sik, 2002; Gersten, Chard, Jayanthi, Baker, Morphy & Flojo, 2009; Zheng, Flynn & Swanson, 2013). In addition, the CRA Approach systematically and explicitly instructed pupils through three levels of learning process in which pupils solve Mathematics problems through the physical manipulation of concrete materials, followed by learning through representations of the concrete manipulations, and ending with solving problems through abstract notation (Witzel, 2005).

In mathematics education there has not been done much work on pupils' motivation to date (Evans & Wedege, 2004; Hannula, 2006), although cognitive predictors were usually found to explain large amounts of variance (up to 50%) in achievement of mathematics, detailed analyses showed that the variance explained

by cognitive factors is reduced to 25% when motivational variables are held constant by statistical means (Schoenfeld, 1985). Few researchers have distinguished between intrinsic and extrinsic motivation in mathematics (Goodchild, 2001; Holden, 2003; Middleton & Spanias, 1999), or between task orientation and ego orientation (Nicholls, Cobb, Wood, Yackel, & Patashnick, 1990; Yates, 2000). Hannula (2006) points out that many of the above approaches fail to describe the quality of the individual's motivation for learning mathematics in sufficient detail. He suggests that the reason for this is that the conventional authors' approaches aim to measure predefined aspects of motivation, not to describe it (Hannula, 2006, p.166).

Research-based studies show that pupils who use concrete materials develop more precise and more comprehensive mental representations, often show more on task behavior, understand mathematical ideas, and better apply these ideas to life situations (Harrison & Harrison, 1986; Suydam & Higgins, 1977). Therefore, this study aims to propose using the CRA Approach to enhance proficiency achievement in fractions and motivation in learning fractions for grade five pupils in the Iraqi school in Kuala Lumpur, Malaysia.

### **1.3 Objectives of the study**

The objectives of this study are as follows:

1. To determine whether there is a significant difference between the pretest mean score and posttest mean score of grade five pupils' proficiency in fractions who learn fractions through the CRA approach.
2. To determine whether there is a significant difference between the posttest mean score and retention test mean score of grade five pupils' proficiency in fractions who learn fractions through the CRA approach.

3. To determine whether there is a significant difference between the pre-survey mean and post-survey mean score of grade five pupils' motivation in learning fractions who learn fractions through the CRA approach.
4. To determine whether there is a significant difference between the post-survey mean and retention-survey mean score of grade five pupils' motivation in learning fractions who learn fractions through the CRA Approach.
5. To elicit grade five pupils' perceptions of learning fractions using the CRA approach.
6. To identify the reasons for pupils' perceptions of learning fractions using the CRA approach.
7. To obtain feedback on the strengths, weaknesses and suggestions to improve the CRA approach from the teacher's perspective.
8. To obtain feedback on the strengths, weaknesses and suggestions to improve the CRA approach from the pupils' perspective.

#### **1.4 Research Questions**

Based on the research objectives above, this study aims to answer the following research questions:

1. Is there a significant difference in proficiency in fractions between the pretest mean score and posttest mean score of grade five pupils' proficiency in fractions who learn fractions through the CRA approach?
2. Is there a significant difference in retention of proficiency in fractions between the posttest mean score and retention test mean score of grade five pupils' proficiency in fractions who learn fractions through the CRA approach?
3. Is there a significant difference in motivation in learning fractions between the pre-survey mean and post-survey mean score of grade five pupils' motivation in learning fractions who learn fractions through the CRA approach?

4. Is there a significant difference in retention of motivation in learning fractions between the post-survey mean and retention-survey mean score of grade five pupils' motivation in learning fractions who learn fractions through the CRA approach?
5. What are grade five pupils' perceptions of learning fractions using the CRA approach?
6. What are the reasons for grade five pupils' perceptions of learning fractions using the CRA approach?
7. What are the feedback on the strengths, weaknesses and suggestions to improve the CRA approach from the teacher's perspective?
8. What are the feedback on the strengths, weaknesses and suggestions to improve the CRA approach from the pupils' perspective?

### **1.5 Null Hypotheses**

The null hypotheses of the study are as follows:

1.  $H_{01}$ : There is no significant difference in proficiency in fractions between the pretest mean score and posttest mean score of grade five pupils' proficiency in fractions who learn fractions through the CRA approach.
2.  $H_{02}$ : There is no significant difference in retention of proficiency in fractions between the posttest mean score and retention test mean score of grade five pupils' proficiency in fractions who learn fractions through the CRA approach.
3.  $H_{03}$ : There is no significant difference in motivation in learning fractions between the pre-survey mean and post-survey mean score of grade five pupils' motivation in learning fractions who learn fractions through the CRA approach.
4.  $H_{04}$ : There is no significant difference in retention of motivation in learning fractions between the post-survey mean and retention-survey mean score of grade five pupils' motivation in learning fractions who learn fractions through the CRA approach.



## **1.6 Significance of the study**

It is difficult for Iraqi primary school pupils to understand fractions, especially for those in grade five. The ultimate goal of education is, therefore, to provide pupils with a wide knowledge of fractions. To achieve this goal, it is important for Iraqi teachers to improve their teaching strategies, which helps them explain fractions in a more clearly and simple method of teaching to pupils. This study therefore, aims to develop proficiency in fractions for Iraqi pupils in grade five in Malaysia through the CRA Approach. It is true that fractions contribute 36% of the topics in the fifth grade curriculum (MOE Iraq, 2013).

Therefore, using the CRA approach is expected to help pupils learn fractions in an effective way. It will improve pupils' proficiency in fractions, raise the standard of fractions among pupils and help them to obtain good grades in their final examination. It is also believed to improve the level of understanding of fractions among pupils, promote collaborative learning among teachers, provide guidelines for teachers to carry out lesson plan for improving their teaching strategies.

In addition, it will provide guidelines for teachers to carry out the CRA approach for improving the teaching strategies, provide guidelines for the Iraqi Ministry of Education to carry out development in fractions for Mathematics teachers. Moreover, it will provide guidelines for teacher-training colleges with the application of the CRA Approach in their training. In other words, the methodology used in the present study can be duplicated and improved in other studies to provide insights in other settings. The goal of mathematics instruction in a step-by-step manner, is to allow the learners to move from needing concrete manipulatives to solve a problem to a point where they can think abstractly through the steps to solve a problem (Miller & Mercer, 1993).

## **1.7 Limitations of the study**

This study has some limitations. The first limitation of this study is related to the sampling technique which is not a random sampling, because there is only one Iraqi primary school in Kuala Lumpur, Malaysia. Thus, the sample of this study consists of grade five pupils in the Iraqi school in Kuala Lumpur, Malaysia.

The second limitation of this study was the aspect of the Approach that had been applied in this study. The Approach that had been proposed in this study might benefit the low achievers, the average pupils and the high achieving pupils in improving the skills involving fractions. Moreover, this approach could only be used for operations involving fractions and it worked better especially for equivalent fractions, fractional numbers, converting fractions to fractional numbers, converting fractional numbers to fractions, simplifying fractions, comparing fractions and order fractions.

The third limitation of this study was the lack of prior research related to the title of this study. There are studies related to CRA and arithmetic problem solving, CRA and addition, subtraction, multiplication and division of integers, CRA and subtraction of integers, CRA and understanding and manipulating one variable equations in word problems, but there is no related study that focuses on the effect of CRA Approach on grade five pupils' proficiency and retention of proficiency in fractions and this had caused difficulties especially during the citation of prior research studies for the literature review and to define the research problem in this study .

The fourth limitation of this study was the schools selected for this study. The school that was selected for this study only involved the Iraqi primary school in

Kuala Lumpur, Malaysia because there is only one Iraqi primary school in Malaysia. Furthermore, the bad situations in Iraq make the researcher could not involve the Iraqi schools in this study. Both of these constraints had limited the researcher in this study to select the pupils from one Iraqi school to be used in this study

The fifth limitation of this study was that the findings of this study could only be used to make generalisation on grade five pupils in Iraqi primary schools in Kuala Lumpur. The findings can not generalise to other grades in primary schools except grade five pupils because the method which had been proposed in this study was only suitable in helping those pupils who were just being introduced to fractions. This method was also believed to be the most suitable method in helping grade five pupils in the Iraqi school in Kuala Lumpur, Malaysia to understand the concepts of fractions. Last but not least, the findings of this study could only be used to make generalization to the population in the educational context only and not to other population of grade five pupils' in the Iraqi schools in proficiency in fractions.

## **1.8 Operational Definitions of Key Terms**

### **1.8.1 Fractions**

A fraction is two quantities written one above the other that shows how much of a whole thing we have. The bottom part is called the denominator, and indicates how many parts it is divided into. The top part is called the numerator and tells us how many of these parts we have.

There are many types of fractions as follows:

*Proper fraction:* Numerator is less than the denominator ( $\frac{2}{3}$ )

*Improper fraction:* Numerator is greater than or equal to the denominator ( $\frac{5}{4}$ )

*Mixed number:* Whole number and a fraction ( $2\frac{1}{5}$ )

*Equivalent fractions:* Fractions that represent the same number ( $\frac{2}{4} = \frac{4}{8}$ )

*Reciprocal:* The multiplicative inverse of a number. For a fraction, it is obtained by turning the fraction over ( $\frac{3}{4}$  &  $\frac{4}{3}$  are reciprocal ) (Math Open Reference, 2009)

### **1.8.2 Proficiency in fractions**

Pupils with proficiency in fractions understand basic concepts, are fluent in performing basic operations, exercise a repertoire of strategic knowledge, reason clearly and flexibly, and maintain a positive outlook (Kilpatrick et al., 2001). Proficiency in fractions is necessary for pupils to learn fractions successfully. It has five components or strands:

1. Conceptual understanding- refers to comprehension of concepts, operations, and relationships in fractions.
2. Procedural fluency- refers to skills of carrying out procedures in fractions flexibly, accurately, efficiently and appropriately.
3. Strategic competence- is the ability to formulate, represent, and solve problems in fractions.
4. Adaptive reasoning- refers to the capacity for logical thought, reflection, explanation and justification of solutions to problems in fractions.
5. Productive disposition- is the habitual inclination to see fractions as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy. (Kilpatrick et al., 2001).

### **1.8.3 The Concrete-Representational-Abstract (CRA) Approach**

The CRA is a sequential three-level strategy that promotes overall conceptual understanding, procedural accuracy and fluency by employing several instructional techniques to introduce new concepts. Each level builds on the concepts taught previously (Witzel, Riccomini & Schneider, 2008).

The CRA is an intervention for Mathematics teaching including three-part instructional strategy. Each part has instruction to promote pupils' learning and retention and address conceptual knowledge.

The CRA instructional sequence consists of three stages including concrete, representational and abstract.

**Concrete:** In the concrete stage, the teacher begins to represent by modeling each mathematical concept (fractions) with concrete materials.

**Representational:** In this stage, the teacher transforms the concrete model into a representational (semi concrete) level, which may involve drawing pictures, using circles, squares, or using the colors to more precisions.

**Abstract:** In this stage, the teacher models Mathematics concept at a symbolic level using only numbers, notation and mathematical symbols to represent the parts of fractions. The teacher uses operation symbols ( $=$ ,  $<$ ,  $>$ ) to indicate equivalent, bigger, or smaller (The Access Center, 2009).

### **1.8.4 Retention of proficiency in fractions**

Retention could be defined as a measure of how well a pupil remembered the learnt material over time. It considers the extent to which one could successfully

access and use information from a long-term memory (Jukic & Dahl, 2012; Sousa, 2000). The retention period for this study was two weeks after the post test. After two weeks, pupils were required to sit for the retention test. The score for the retention test for the experimental group was compared to the score of their posttest to examine if teaching method would (CRA) approach help pupils to improve their proficiency in fractions taught by the teachers.

### **1.8.5 Motivation in Learning Fractions**

Pupils' motivation in learning fractions refers to the cognitive and affective psychological processes that influence the learning of fractions (Slavin & Davis, 2006). This internal process activates, guides and maintains a learner's behavior over time (Schunk, 1990), propels and directs pupils to engage in academic activities and determines how much is learnt from such activities, and from other information sources to which learners are exposed (Slavin & Davis, 2006; Tuckman & Sexton, 1992). They strive to understand the fractions, improve their performance in fractions (Woolfolk, 1990). The motivation questionnaire for this study will be administered before and after the intervention for experimental group.

### **1.8.6 Retention of Motivation in Learning Fractions**

Retention could be defined as a measure of how well a pupil remembered the learnt material over time. It considers the extent to which one could successfully access and use information from a long-term memory ( Jukic & Dahl ,2012; Sousa, 2000). The retention motivation questionnaire for this study will be two weeks after the intervention for the experimental group.

### **1.8.7 Conventional Method**

Conventional method is still the predominant instructional pattern in most of Iraqi primary schools, typically begins with an explanation of whatever idea is on the current page of the text followed by showing children how to do the assigned exercises. Meanwhile, the conventional teacher activity is guiding pupils, telling them exactly how to use the materials in a prescribed manner. The focus of the lesson is primarily on getting answers. Pupils rely on the teacher to determine if their answers are correct or not. Pupils emerge from these experiences with a view that mathematics is a series of arbitrary rules, handed down by the teacher, who in turn got them from some very smart source (<http://www.education.com/reference/article/traditional-views-mathematics>).

### **1.8.8 Grade Five in Iraqi Primary Schools in Kuala Lumpur, Malaysia**

The Iraqi primary school in Kuala Lumpur, Malaysia established for the Iraqi children who live in Malaysia. It follows the same school rules and curriculum in Iraq. The Iraqi primary school in Kuala Lumpur, Malaysia had 350 pupils divided into six grades. The grade five had 64 pupils (40 males and 24 females).

Grade five in Iraqi primary schools in Malaysia is the fifth grade in primary school and includes pupils who are 12 years old. These pupils are taught basic subjects (e.g., Mathematics, Arabic, English, Islamic Education and Science) and other subjects (e.g., History, Geography, the National Anthem, Music, Sports and Art Education). The total score of a written exam paper is 100 points. Pupils study for 5 to 6 hours per day including 6 to 7 lessons. Each lesson is taught in 45 minutes. There is a 15- minute break after each two lessons.

The Mathematics curriculum in the Iraqi primary school in Malaysia followed the same curriculum and the method of teaching in the Iraq schools. The curriculum consists of equivalent fractions, fractional numbers, converting fractions to fractional numbers, converting fractional numbers to fractions, simplifying fractions, the order and comparison of fractions. The teachers follows conventional method for teaching mathematics based on the rules of Iraqi Ministry of Education (MOE Iraq, 2013).

The teachers in Iraq, primary school in Malaysia have good experiences in Mathematics teaching in primary schools in Iraq. Most of them graduated from educational institutions or have a diploma in teaching. The teachers came to Malaysia for further education, so their children must follow their family to Malaysia for studying.

### **1.9 Thesis Organisation**

This study is organised into five chapters.

Chapter 1 begins with the background of this study. The research problem statements are then addressed. The research objectives, the research questions, null hypotheses, limitations of the study, the significance of the study and the scope of the study are also presented as well. The chapter ends with operational definitions of key terms.

Chapter 2 presente the relevant literature review. Previous studies related to proficiency in fractions and the CRA Approach are clearly summarised and critically reviewed. The purpose of literature review is to provide readers with the literature related to the study as well as highlight the flaws in previous studies and outline the gaps which this research tends to fulfill.