# BICUBIC B-SPLINE AND THIN PLATE SPLINE ON SURFACE APPROXIMATION

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# BICUBIC B-SPLINE AND THIN PLATE SPLINE ON SURFACE APPROXIMATION

**by**

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| | |
|---|---|
| **CAGD** | Computer Aided Geometric Design |
| **3D** | Three-Dimensional |
| **NURBS** | Non-Uniform Rational B-Spline |
| **MLS** | Moving Least Square |
| **LOP** | Locally Optimal Projector |
| **EAR** | Edge-Aware Resampling |
| **RBF** | Radial Basis Function |
| **kNN** | *k*-Nearest Neighbour |
| **PCA** | Principal Component Analysis |
| **OFF** | Object File Format |
| **HC** | Humphrey's Classes |

**SPLIN-B DWIKUBIK DAN SPLIN PLAT NIPIS UNTUK PENGHAMPIRAN PERMUKAAN**

## ABSTRAK

Dalam hidup hakiki, titik data 2D atau 3D sedia ada biasanya tidak berpola dan dicemari oleh hingar. Hingar ditakrifkan sebagai variasi dalam suatu set titik data. Kaedah penghampiran ialah cara yang sesuai digunakan berbanding kaedah interpolasi dalam menyesuaikan titik data tersebut. Adalah penting bagi suatu kaedah penghampiran untuk mengekalkan bentuk dan sifat-sifat model dalam kehadiran hingar. Penghampiran splin-B dan splin plat nipis dikaji dalam tesis ini. Keberkesanan penggunaan algoritma penghampiran splin-B yang diubahsuai telah dikaji dalam penghampiran permukaan splin-B dwikubik daripada sampel titik data bertaburan yang diambil daripada model set titik. Algoritma tersebut digunakan untuk menentukan titik kawalan splin-B yang tidak diketahui dan diikuti dengan penghasilan penampal permukaan splin-B dwikubik. Keputusan eksperimen menunjukkan bahawa algoritma penghampiran splin-B yang diubahsuai berjaya membina permukaan yang menyerupai bentuk sampel titik set asal tanpa melalui penghampiran permukaan berbilang aras. Pengekalan pinggir tajam dalam permukaan splin-B dwikubik turut dikaji. Algoritma yang berasaskan kaedah pemurataan cangkuk but dicadangkan dan pinggir tajam dapat dikekalkan. Kajian dilanjutkan bagi melihat kesan hingar dalam pengekalan pinggir tajam. Kaedah penghampiran splin plat nipis, yang merupakan salah satu fungsi asas jejarian juga digunakan untuk menyesuaikan titik data dalam kajian ini. Perbandingan kesan hingar antara penyesuaian permukaan splin-B dwikubik dan penyesuaian permukaan splin plat nipis dilakukan dan didapati bahawa permukaan splin-B adalah sensitif akan hingar berbanding splin plat nipis. Dalam kehadiran data hingar, skim penghampiran splin plat nipis digunakan, maka, nilai parameter pelicinan yang sesuai diperlukan untuk mengawal kualiti penyesuaian bagi mengelakkan masalah terlebih suai dan terkurang suai. Bagi mendapatkan parameter pelicinan otpimum, ujian penganggaran cangkuk but *leave-one-out* digunakan bagi model set titik dengan liku dan tanpa liku. Perbandingan juga dilakukan antara penyesuaian splin plat nipis dengan penyesuaian permukaan polinomial kubik dan kuartik dalam konteks pembinaan semula permukaan yang berasaskan ujian penganggaran ralat cangkuk but. Didapati splin plat nipis merupakan penyesuaian permukaan terbaik jika parameter pelicinan yang sesuai dipilih. Parameter pelicinan optimum splin plat nipis boleh digunakan untuk melicinkan permukaan. Kehadiran data hingar ialah suatu perkara biasa dalam pemodelan set titik, maka penyah-hingaran set titik perlu diberi perhatian. Penggunaan algoritma pelicinan yang bergantung kepada fungsi asas jejarian berdasarkan cangkuk but telah dicadangkan. Kaedah yang dicadangkan telah menggabungkan carian jiran terdekat *k* dan kemudian set titik diunjurkan ke permukaan splin plat nipis yang telah dihampirkan. Akhir sekali, kaedah yang dicadangkan telah dibandingkan dengan algoritma pelicinan Laplacian *Humphrey's Clasess* dan algoritma pelicinan permukaan set titik algebra. Keputusan menunjukkan bahawa algoritma pelicinan yang dicadangkan telah menunjukkan keupayaan yang setanding untuk melicinkan model titik set hingar dan memelihara sifat-sifat model set titik.

# BICUBIC B-SPLINE AND THIN PLATE SPLINE ON SURFACE APPROXIMATION


## ABSTRACT

In real life, the available data points which are either 2D or 3D are normally scattered and contaminated with noise. The noise is defined as the variation in a set of data points. To fit these data points, the approximation methods are considered as a suitable mean compared to the interpolation methods. It is important for the approximation methods to preserve the shape and features of the model in the presence of any noise. B-spline and thin plate spline approximation are being studied in this thesis. The effectiveness of the modified B-spline approximation algorithm is investigated in approximating the bicubic B-spline surface from the samples of scattered data points taken from the point set model. The algorithm is used to determine the unknown B-spline control points and followed by the construction of bicubic B-spline surface patch. The experimental results show that the modified B-spline approximation algorithm manages to construct the surfaces that resemble the shape of original samples point set without going through the multilevel surface approximation. The sharp edge preservation in bicubic B-spline surface is also being studied. On top of that, an algorithm which is based on bootstrap averaging method is proposed and able to achieve the sharp edge preservation. Further studies are carried out to investigate the effect of noise with different noise levels in preserving the sharp edge. The approximation scheme of the thin plate spline which is one of the radial basis functions is also used to fit the data points in this study. A comparison is carried out to observe the effect of noise in the bicubic B-spline surface fitting and the thin plate spline surface fitting, which reveals that the B-spline surface is sensitive to the noise compared to the thin plate spline. The approximation scheme for a thin plate spline is used in the presence of noisy data; hence, an appropriate value for the smoothing parameter is needed to control the quality of fitting, that is to prevent the problem of overfitting and underfitting. The bootstrap leave-one-out error estimator is applied to find an optimum smoothing parameter for the point set models with and without the feature. Apart from that, a comparison is also carried out to compare the thin plate spline fitting with the cubic and quartic polynomial surface fitting in the context of surface reconstruction, which is based on the bootstrap test error estimation. It is found that the thin plate spline is the best surface fitting among the three if an appropriate smoothing parameter is selected. The optimum smoothing parameter of thin plate spline can be applied in the process of surface smoothing. The presence of noisy data is considered a common issue in the point set modelling, thus the point set denoising is one of the main concerns. A smoothing algorithm that relies on a bootstrap-based thin plate spline function is proposed, which incorporates a $k$-nearest neighbour search and then the point set is projected to the approximated thin plate spline surface. Finally, the proposed method is compared with the Humphrey's Classes Laplacian smoothing algorithm and the algebraic point set surface smoothing algorithm. The results show that the proposed smoothing algorithm has shown the comparable capability of smoothing the noisy point set model and preserving the features of the point set models.

# CHAPTER 1

# INTRODUCTION

Computer aided geometric design (CAGD) is a branch of applied mathematics that studies the algorithms for the smooth curves and surfaces design. Apart from that, it also concerns the mathematical representation and approximation of shapes and other geometric features of objects. CAGD can be related to the field of computer graphics, data structure, numerical analysis, and the theory of approximation. The advantages of the computers allow the products which are available in the automotive and aerospace industries to be initially processed, analysed, manipulated, and visualised within the computer before being entered into the production line. The algorithms can be developed using the programming language such as C/C++, which then will be performed on AutoCAD to display the graphical result. However, the researchers have come out with another alternative which utilise Mathematica and MATLAB to conduct the research instead of the traditional programming languages due to the efficiency and friendly features of both the mathematical computational software.

## 1.1 History of Curves and Surfaces in CAGD

The brief history on the developments of curves and surfaces in CAGD will be provided in this section to propose for some views on the topics that will be discussed in the following sections and chapters. The term "Computer Aided Geometric Design" was devised by R. Barnhill and R. Riesenfled in 1974 in a conference that was held at the University of Utah, United States. The conference was considered as the foundation of the field of CAGD, in which the researchers from United States and Europe were brought together to have a meet (Farin, 2002).

The earliest use of curves in a manufacturing environment was in shipbuilding, which was in early Anno Domini (A.D.) Roman times. Next, the spline function was established from the work of draftsmen who were often needed to draw smooth curves. Hence, the French curve (as shown in Figure 1.1) which is a plastic template that composed of a number of curves of different curvature was used to draw the smooth curves. Apart from that, the elastic long strips of wood were also used to draw the smooth curves which passed through the control points by the weights laid on the draftsman's table and attached to the strips. Around the year 1891, the weights were known as “ducks" while the strips of wood were known as “splines". They are shown in the Figure 1.2. The manual process of drawing the smooth curves was further developed and translated into the mathematical theory (Cheney and Kincaid, 2008).

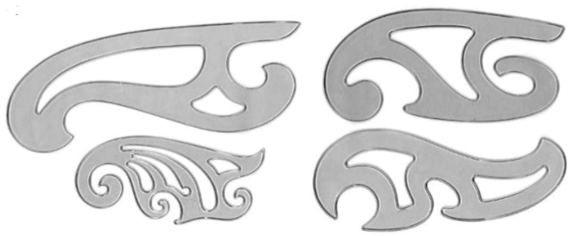


Figure 1.1: Some French curves (Lunday, 2007)

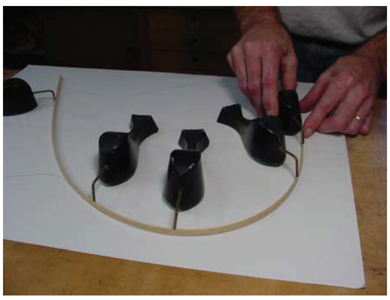
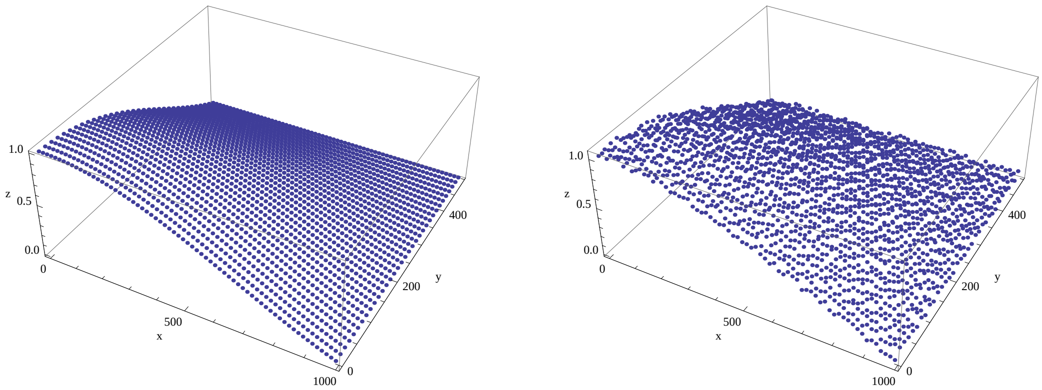


Figure 1.2: The weights (“ducks") and strips of wood (“splines") (Brogan, 2004)

Shipbuilders were the first to mechanise the operation by developing the mathematical model of surfaces to overcome the storage problem of the full-size drawings. In the 1960s, the aircraft and car manufacturers began to use the computers to automate the design of their vehicles. Hence, there was a slow replacement from the traditional method that involved the making of clay models and the production of stamp molds into the mathematical models (Salomon, 2006). The process to design the surface section was eventually more interactive with the aid of computer. The area of computer graphics became well known in the 1960s and 1970s, while some sophisticated software systems were developed in the 1980s for several general use such as in manufacturing, modelling of chemical molecules, geoscience, and modelling of three-dimensional (3D) buildings in the area of architecture. The hardware developments in the 1980s enabled the CAGD techniques to be applied in the computer-generated special effects for movie such as the Jurassic Park film in 1993 (Salomon, 2006). The full length computer-animated film were also produced in some movies such as Toy Story (1995), Finding Nemo (2003), The Incredibles (2004), Frozen (2013), Inside Out (2015), Moana (2016), and many others.

## 1.2 Surface Reconstruction

Surface reconstruction is one of the research areas in computer graphics, which is referred as a reverse engineering problem that is naturally emerged from CAGD. Surface reconstruction is applied in medical imaging, the manufacturing of ship hulls and car bodies, and other free-forms objects such as geologic surface (Pandunata and Shamsuddin, 2013). The point set surface reconstruction is the main concern of the thesis because the point set has gained its popularity in the area of computer graphics due to the advancement of scanning devices. The point set surface reconstruction is defined as the process of obtaining a surface from a set of 3D data points using the computer and the reconstructed surface which is similar or has the best approximation of the origin surface. The set of 3D data points which is obtained from a scanning device such as the 3D scanner is often dense in 3D space, normally the three-dimensional Euclidean space, $\mathbb{R}^3$. Therefore, the set of 3D data points is named as point cloud. Each point in the point cloud is represented in the standard 3D Cartesian coordinate system with $x, y, z$-coordinate, which is capable to indicate its location in 3D space.



(a) The ordered point cloud

(b) The scattered point cloud

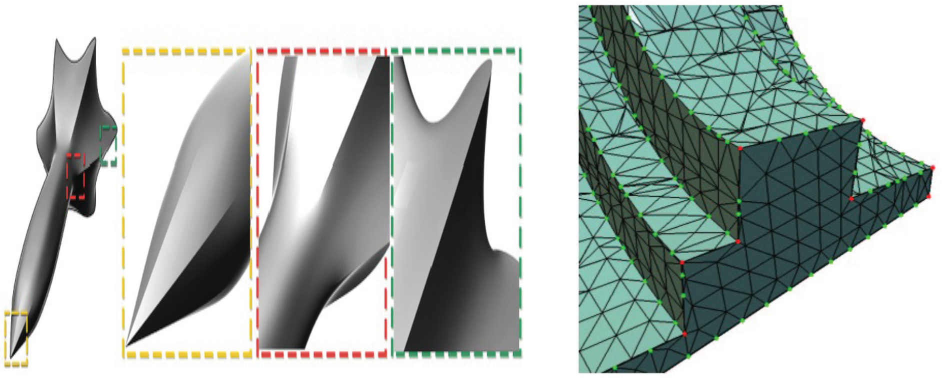
Figure 1.3: The types of point cloud

The point cloud can be categorised into two types, namely the ordered point cloud and the scattered point cloud as shown in Figure 1.3. The ordered point cloud refers to the point cloud which is smooth and organised; whereas the scattered point cloud is known to be unorganised and noisy. Unfortunately, the point cloud is often scattered in the real life practical problem; therefore it is a challenging and difficult task to reconstruct the surface from the scattered point cloud. If the reconstructed surface shows the shape of the original point cloud, thus it is regarded as the outcome of the surface reconstruction.

One of the common point set surface reconstruction pipelines starts from the data acquisition procedure which is obtained from the point cloud from the real life object via the scanning device. Next, the registration is carried out to combine the multiple point clouds which are obtained through different angles. The registered point cloud undergoes the pre-processing step such as simplification, outlier removal, and denoising. The procedure is continued with the normal estimation and normal orientation. After all the procedures are conducted, the point cloud is clean and equipped with the oriented normal, followed by the surface reconstruction (Alliez et al., 2011). Free data set such as Stanford Bunny, Happy Buddha, Dragon, and others can be obtained from the Stanford University's 3D scanning repository. These data sets can be used as samples for the purpose of testing and validating the reconstruction process when the scanning facilities are unavailable.

Surface representation should be identified before the surface reconstruction. Hence, there are two types of surface representation, namely explicit and implicit (Zhao et al., 2001). Mathematically, the explicit surface has the general form which is given by the equation $z = F(x,y)$, whereas the equation for the implicit surface is, $F(x,y,z) = 0$. The examples of explicit representation are parametric surface and triangulated surface. The parametric surfaces include B-spline surface, non uniform rational B-spline (NURBS) surface, and Bézier surface. Meanwhile, Delaunay triangulation and Voronoi diagram are the examples of the triangulated surface. Next, the examples of the implicit representation include the moving least square, radial basis function, and signed distance function. The implicit surface can provide a better topology for arbitrary objects and possess the property to fill up the holes automatically compared to the explicit surface (Xie et al., 2004). The detail reviews on the surface reconstruction techniques are discussed in the work of Lim and Haron (2014).

There are some general issues that will be the concern in surface reconstruction such as the time complexity of the method in handling the point cloud, the existing of the noise and outliers in the point cloud, and the preservation of sharp features of the surface. Note that the feature of the surface refers to a model with a crease, corner, dart, or cusp as shown in Figure 1.4. Finally, a proper procedure is essential to eliminate or at least minimise the issues to obtain a good surface representation.



(a) The close-up views of three parts of sharp features including dart, crease, and cusp, which are bounded by the yellow, red, and green dotted square, respectively (Wang et al., 2016)

(b) The red points denote the corner feature, whereas the green points denote the edge feature (Kobbelt et al., 2001)

Figure 1.4: The types of surface feature

## 1.3 Research Background

B-spline is one of the topics in the context of numerical analysis. The B-spline function is constructed as linear combinations of B-spline with a set of control points. B-splines was studied by Lobachevky as early as the nineteenth century, while Laplace discovered that it possesses the connection with the probability density function (Farin, 2002). The term “B-spline" was coined by Schoenberg (1946), whereby the designation “B" in B-spline stands for basis. Schoenberg’s work dealt with the B-splines over uniform knots, whereas the B-splines over non-uniform was suggested by his colleague, Curry (1947). The importance of B-splines towards the CAGD was significantly enhanced with the discovery of the recursion relation by de Boor (1972), Cox (1972), and Mansfield independently in the same year. The recursion relations were named as de Boor's recursion formula, which is known to be fast and numerically stable. Originally, the B-splines were tedious mathematical approach which is divided differently and numerically unstable. In 1974, Riesenfeld and Gordon applied the de Boor's recursion formula in parametric B-spline curve and realised that the B-spline was the natural generalisation of the de Casteljau recursive formula in Bézier curve evaluation (Farin, 2002). The B-spline curves are further generalised to the non-uniform rational B-splines (NURBS), which had successfully become the standard form of curve and surface in the CAD and CAM industry. There are two schemes of B-spline, namely approximation and interpolation. The approximation scheme of B-spline is selected when there is a presence of noise or insufficient data points, whereas the interpolation scheme is used when the set of data points is in ordered form.

The topic of radial basis functions (RBFs) should also be discussed in this section. Hardy (1971) proposed a new analytical method to fit and represent the irregular surfaces using the multiquadric function. Another RBF, the thin plate splines which were based on the minimum bending energy theory of the thin plate surface were proposed by Duchon (1977). A test on 29 different scattered data interpolation methods in different context which include storage, timing, accuracy, visual pleasantness of the surface, and ease of implementation was carried out. The experimental result revealed that the multiquadric function and thin plate spline were both selected as the best method from the stated context (Franke, 1982). Other RBFs besides the multiquadric function and thin plate spline will be discussed in the coming chapter. Generally, RBFs are widely applied in the area of computer graphics, data processing, and economics (Chen et al., 2014). Similar to the B-splines, there are interpolation and approximation scheme of RBFs. In this thesis, the approximation scheme of B-splines and RBFs are extensively studied due to the fact that the real life models are often contaminated with noise.

Generally, there are two methods to assess the quality and the accuracy of the reconstructed surface. The methods include the subjective method such as visual inspection, or objective method such as error estimation which can be applied to carry out the surface evaluation. Visual inspection refers to the human use of their sense of sight, together with their expertise to carry out the inspection. However, it is important to note that different persons may provide different conclusions, which may unlikely contradict each other. This subjective method can still be served as the initial assessment of the surface. However, the subjectivity of human in assessing the quality of surface should be minimised when the accuracy of the surface becomes an essential issue. For instance, the accuracy of surface fitting is difficult to be determined in the presence of low level of noise. Therefore, the objective method which is the error estimation has to play its role in determining the accuracy of the surface. The statistical methods are being used to minimise the error that arises from the noise in the process of surface reconstruction. On top of that, the statistical methods are powerful approach to analyse and interpret the problem in actuarial science, economics, business, biology, signal processing, quality control, machine learning, and others. The integration of statistical method in the area of CAGD is considered as a great introduction because the statistical approach enables the assessment of the surface to be conducted quantitatively, without having to rely on the visual approach alone. In this thesis, the bootstrap method is selected as the statistical tool because it provides an error estimation of the surface fitting. Efron (1979) introduced a model averaging method called the bootstrap method to estimate the sampling distribution. Multiple data sets are created from the same sample based on the bootstrap procedure. Each new data set consists of $N$ points which are randomly taken from the data sample of $N$ size with replacement. The reuse of the data as a result of repetitive resampling is particularly helpful if the available data is sparse.

The error estimation of the surface fitting in the presence of noise can be determined with the aid from the bootstrap method, which can minimise the error of a fitting. The information achieved from the bootstrap error estimation is also helpful in the denoising process. The denoising process is defined as the process of removing the noise from the point clouds for the purpose of having a better and organised point clouds. The term denoising is widely used interchangeably with the term smoothing based on the existing literature. There are two types of denoising algorithms, namely mesh and point set denoising. The main difference between the point set denoising with mesh denoising is the absence of connectivity (Ramli, 2012). The aim of the denoising process is not only to remove the noise from the data, but also to preserve the existing features. The detailed mathematical background as well as discussions on the previous works related to this topic will be further elaborated and explored in the coming chapters.

## 1.4 Problem Statements

The point clouds obtained from some of the scanning devices maybe scattered and unorganised instead of being uniformly distributed. Therefore, an appropriate approach to reconstruct the smooth surface from this particular type of point clouds is using the approximation method. According to Mooney and Swift (1999), the interpolation method is considered inappropriate for the data points with large amount of random fluctuation or noise because it interpolates and passes through all the data points which eventually produce an undesired and unsmooth surface with significant errors, thus it is not to be applied in this study. In this thesis, the approximation scheme of the B-spline is studied, whereby the B-spline is defined locally which indicates that a change in the particular parameter will only affect the model locally. The motivation is gained from the previous work on the multilevel B-spline surface fitting from a set of scattered data points. Lee et al. (1997) mentioned that the B-spline approximation algorithm had the capability to process the large amount of data points. Therefore, it is possible to modify the multilevel setting into the single level setting for the purpose of simplifying the algorithm while retaining the comparable result. The multilevel B-spline algorithm is computationally expensive when it is applied to a large number of data points (Lee et al., 2005a). The modified B-spline approximation algorithm is being tested from the aspect of the surface patch fitting. Apart from that, sharp features preservation such as the sharp edge preservation is a general issue for the surface approximation because this feature has the tendency to be smoothed out; therefore, a B-spline sharp edge preservation algorithm is proposed and its efficiency is validated when noise is presented in the sample of scattered data points.

Another approximation method that has been used is known as radial basis function, specifically the thin plate spline. In comparison to the B-spline, the thin plate spline is defined globally, which means that a change in the particular parameter will induce a change to the whole model. The presence of noise in data points will affect the surface approximation for these two types of approximation method; therefore, a comparison is carried out in order to identify which method has the minimum effect towards the noise in the surface approximation.

The approximation scheme of the thin plate spline is studied due to its ability in dealing with the noisy points in the point cloud. The approximation quality of the approximation scheme of the thin plate spline is controlled by the smoothing parameter, which can prevent the problem of underfitting and overfitting. However, the method to select an optimum smoothing is still unknown. Therefore, it has been decided that the training error and a statistical approach known as the bootstrap error estimation will be used to determine the optimum smoothing parameter from a proposed range of smoothing parameters. The experiment to search for the optimum smoothing parameter is conducted for the non-feature and feature point set models with different noise levels.

In this thesis, the point set models will be used throughout this study and knowing that some of the obtained point clouds are contaminated with noise; therefore, point set denoising is also investigated. A new point set denoising algorithm is proposed by extending the result of determining the optimum smoothing parameter of thin plate spline using the statistical approach. The proposed denoising algorithm is applied to denoise the point set model which is corrupted by the moderate level of noise. The proposed denoising algorithm is simple and able to preserve the features of the point set model.

## 1.5 Research Objectives

The purpose of this research is mainly to study the approximation scheme of the B-spline surface as well as the thin plate spline surface. This study further investigates the issue by searching for the smoothing parameter of the thin plate spline as well as the point set denoising method using the thin plate spline. The specific objectives of this research are as follows:

- To introduce a single level B-spline surface approximation algorithm by modifying the existing multilevel B-spline surface approximation algorithm to fit sets of scattered data points. The modified approximation algorithm is validated by observing the surface fitting at different noise levels.
- To propose a B-spline sharp edge preservation algorithm to preserve sharp edges. The proposed algorithm will be validated at different noise levels to observe its efficiency.
- To minimise the effect of the noise on surface approximation by identifying a better surface approximation algorithm from the modified B-spline approximation algorithm and the thin plate spline approximation algorithm through the comparison method.
- To handle the issue of underfitting and overfitting for the approximation scheme of the thin plate spline when fitting the sample with non-feature and feature 3D

data points with different noise levels. The statistical-based error estimation methods are applied in order to search for the optimum smoothing parameter from a proposed interval of smoothing parameters.

- To denoise the 3D point set models corrupted by the moderate noise levels by designing a point set denoising algorithm based on the optimum smoothing parameter of the thin plate spline which are obtained using the statistical method. The performance of the denoising algorithm is justified by the real life point set model at different noise levels. A comparison is carried out between the point set denoising algorithm with other denoising algorithms.

## 1.6 Research Methodology

The following research methodology is used to achieve the research objectives:

- The related literature need to be reviewed as well as the mathematics background needs to be studied thoroughly in order to modify the existing multilevel B-spline algorithm. The B-spline approximation method that exists in the multilevel B-spline algorithm is further to be modified to become single-level B-spline algorithm. The observation is not limited to the B-spline surface approximation from a set of scattered data points, but also the effect of noise on the surface fitting.

- The variance-based detection method is extended and integrated in the proposed B-spline sharp edge preservation algorithm. The effect of noise in preserving the sharp edges is also being observed. The graphical results from the first and second objectives are displayed by writing the programming codes using Mathematica 9.

- The bicubic B-spline surface that is fitted by the modified B-spline approximation algorithm is compared with the approximation scheme of the thin plate spline surface fitting at different noise levels in order to observe the effect of noise towards these two surface approximations. A small value of the smoothing parameter is being selected, and this parameter controls the quality of approximation of the thin plate spline. The subjective method that involves the visual inspection is being used to observe the graphical results generated from Mathematica 9. The visual inspection is used to assess the results in the second and third objectives. Therefore, a survey will be served as the validation tool. A total of 21 respondents are invited to respond and give their feedback. The respondents are limited to the academic staffs and students from Universiti Sains Malaysia who have the knowledge in computer aided geometric design.

- The existing smoothing parameter in the thin plate spline will enable the approximation quality to be controlled, that is to prevent the problem of underfitting and overfitting. Therefore, it is important to search for the optimum smoothing parameter of the thin plate spline through the statistical method which allows for an objective opinion to be provided by giving the error of estimation for the surface approximation. The training error and bootstrap leave-one-out error of the point set model at different noise levels are determined using MATLAB R2012a. The numerical values that were achieved are tabulated and plotted to search for the optimum smoothing parameter. The specific regions of the point set model are selected using the $k$-nearest neighbour search algorithm to justify the result, which is then fitted by the thin plate spline with the optimum smoothing parameter and other smoothing parameters. The experiment is carried out for both point set model with and without features.

- In the context of application, specifically the denoising process for the noise corrupted 3D point sets, a point set denoising algorithm is designed by applying the result gained from the optimum smoothing parameter. The point set model with moderate noise levels is selected to test the performance of the proposed algorithm. The proposed algorithm does not involve all the data points in the point cloud; hence the features of the model can be preserved. The sample of data points with neighbourhood of size $k$ is selected from the point cloud, which is then fitted by the thin plate spline surface with the optimum smoothing parameter. Next, the sample of data points is projected to the surface, and similar procedure is carried out for other selected neighbourhoods. The proposed algorithm eventually updates the position of the data points which leads to the smoothing effect. The codes are programmed in Mathematica 9 and the results are visualised through MeshLab 1.3.2. The proposed algorithm is also compared to other denoising algorithms.

The flow chart of the research objectives and the research methodology is shown in Figure 1.5.

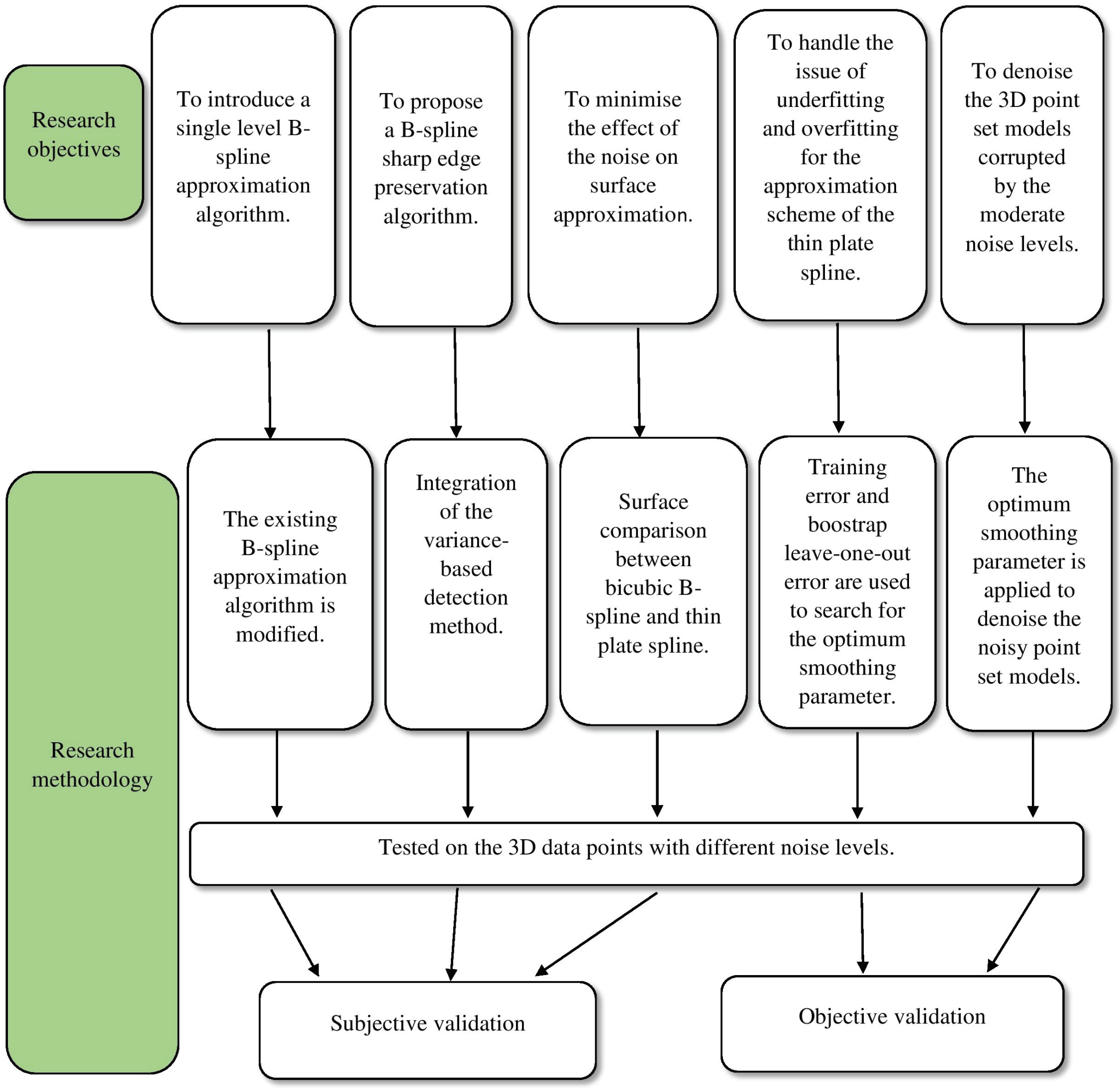



Figure 1.5: The flow chart of the research objectives and the research methodology

## 1.7 Thesis Organisation

The organisation of this thesis is as follows:

**Chapter 1**: The history of the curves and surfaces is briefly described, which is then followed by the basic introduction of the surface reconstruction. The research background, problem statements, research objectives, research methodology, and thesis organisation are also provided in this chapter.

**Chapter 2**: The related literature review and mathematical background are discussed in this chapter. The literature review is included to introduce the basic understanding on the works that have been conducted previously. The mathematical background such as B-spline, radial basis function, *k*-nearest neighbour search algorithm, normal estimation using principal component analysis, bootstrap method, and others are discussed because they act as the main theoretical framework for this thesis. The aim of these theoretical parts is to enhance the understanding of the subsequence chapters.

**Chapter 3**: The modified version of B-spline approximation algorithm together with the graphical results are described in detail. The effect of different levels of noise on the modified B-spline surface fitting algorithm and the sharp edge preservation are investigated in this chapter. Next, the modified B-spline surface fitting algorithm is further examined by comparing it with the thin plate spline surface fitting in the context of fitting and noise. The smoothing parameter of the thin plate spline is manually selected with a small value. In the next chapter, the search for the optimum smoothing parameter is further studied.

**Chapter 4**: The statistical approach is used to determine the optimum smoothing parameter of the thin plate spline. Firstly, the experiment is carried out for the point set model without features because it can be served as the initial observation or ground truth before proceeding with the point set model with features. The training error and bootstrap leave-one-out error estimation are used as the tool to calculate the error of the fitting by the thin plate spline for the point set model at different noise levels. The numerical values of these errors are tabulated and plotted, which enables the optimum smoothing parameter to be determined. The validation is carried out by selecting a region of the point set model which is fitting using the thin plate spline with the optimum and other smoothing parameter values. The similar experiment is repeated for the point set model with features to observe the real life application.

**Chapter 5**: The proposed point set denoising algorithm is discussed in this chapter. The proposed algorithm makes use of the optimum smoothing parameter to denoise the point set models that are contaminated by a moderate noise level. Apart from that, the comparisons and validations are conducted for other existing denoising algorithms by looking at their denoising efficiency in the presence of noise at a moderate level.

**Chapter 6**: The conclusion of this thesis and the possible directions for future research are discussed in this chapter.

# CHAPTER 2

# BACKGROUND THEORY AND LITERATURE REVIEW

In this chapter, the mathematical backgrounds that will be used frequently in the following chapters are presented and the relevant literature is reviewed.

## 2.1 Background Theory

### 2.1.1 Curve continuity

The concern of this section is the continuity at the joint of two neighboring curve segments; hence it is important to know how the individual segments can be connected. The following definitions taken from Pressley (2010) are necessary to be discussed before the types of continuity are introduced.

**Definition 2.1** *A parametrised curve in* $\mathbb{R}^n$ *is a map* $\gamma : (\alpha, \beta) \to \mathbb{R}^n$*, for some* $\alpha, \beta$ *with* $-\infty \leq \alpha < \beta \leq \infty$*. The symbol* $(\alpha, \beta)$ *denotes the open interval such that*

$$(\alpha, \beta) = \{t \in \mathbb{R} \mid \alpha < t < \beta\}.$$

The parametrisation of a curve is not unique as there is infinitely number of different parametrisation. This concept can be formalised based on the following definitions:

**Definition 2.2** *Let* $(\tilde{\alpha}, \tilde{\beta})$ *and* $(\alpha, \beta)$ *being open intervals in* $\mathbb{R}$*. A parametrised curve* $\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^n$ *is a reparametrisation of a parametrised curve* $\gamma : (\alpha, \beta) \to \mathbb{R}^n$ *if there* *is a smooth bijective map* $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ *(the reparametrisation map) such that the inverse map* $\phi^{-1} : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$ *is also smooth and*

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t})) \quad \forall \tilde{t} \in (\tilde{\alpha}, \tilde{\beta}).$$

*Note that* $\phi$ *has a smooth inverse,* $\gamma$ *is a reparametrisation of* $\tilde{\gamma}$*:*

$$\tilde{\gamma}(\phi^{-1}(t)) = \gamma(\phi(\phi^{-1}(t))) = \gamma(t) \quad \forall t \in (\alpha, \beta).$$

The reparametrisation results are geometrically identical, which means that the two curves have the same image.

The concept of continuity can be introduced in detail based on the two definitions. The two types of continuity to be considered are as follows (Salomon, 2006):

- Parametric continuity, $C^n$- It is known as $n$th-order parametric continuity, where $n$ usually is 0, 1, and 2. The algebra is used to describe the smoothness of the parameter's value along the curve. It is natural in analysis, but leads to an unsatisfactory model of smoothness which is too stiff in practical cases. In addition, it is not invariant with respect to the reparametrisation. Furthermore, the curve is viewed as a function rather than a shape.
- Geometric continuity, $G^n$- It is called as $n$th-order geometric continuity, where $n$ normally is 0, 1, and 2. It is a less restrictive form compared to the parametric continuity because it can be defined using only the shape of the curve, but the choice of parametrisation does not affect the outcome. In addition, it is invariant

under any transformation of the parameter.

If two consecutive segments meet at a point, the total curve is said to have $G^0$ geometric continuity. Next, if the directions of the tangent vectors of the two segments are the same at the join point, then it is said to have $G^1$ continuity. In general, a curve has $G^n$ at join point if every pair of the first $n$ derivatives of the two segments has the same direction at the join point.

If the same derivatives also have identical magnitudes at the join point, then the curve is said to have $C^n$ parametric continuity at the join point. $C^0$, $C^1$, and $C^2$ referred as point, tangent, and curvature continuity, respectively. Apart from that, $C^{-1}$ indicates the curve which includes discontinuities. Since $C^n$ is more restrictive than $G^n$, thus a curve that has $C^n$ continuity at a join point ensures the $G^n$ continuity, but not vice-versa. Therefore, a curve which tangent vector and curvature vector are continuous everywhere is said to have $G^2$ continuity.

### 2.1.2 B-splines

There are number of ways to define the B-splines. They are defined as divided differences of truncated power function by Curry and Schoenberg (1966), blossoming by Ramshaw (1987) and recurrence formula by de Boor (1972), Cox (1972), and Mansfield. The recurrence formula is widely used because it is suitable for computer implementation (Kong, 2013). Some of the advantages of B-spline curve include the B-spline curve that features local control and any desired degree of continuity without having to depend on the number of control points. There is an approach to control the shape of the curve which is known as knots. Let $T = \{t_0, t_1, t_2, \ldots, t_n\}$ be a nondecreasing sequence of real numbers, $t_0 \leq t_1 \leq t_2 \leq ... \leq t_n$. Each $t_i$ is called a knot and $\boldsymbol{t} = (t_0, t_1, t_2, \ldots, t_n)$ is known as the knot vector.

The equation of B-spline is defined as follows:

Given the knot vector, $\boldsymbol{t}$ such that

$$\boldsymbol{t} = (t_0, t_1, t_2, \ldots, t_{n-1}, t_n, t_{n+1}, \ldots, t_{n+k}).$$

The associated B-spline basis functions of order $k$ (degree $k-1$), which is denoted as $N_i^k(t)$, is defined recursively by

$$N_i^1(t) = \begin{cases} 1, & t_i \leq t < t_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

for $k = 1$ and

$$N_i^k(t) = \frac{t - t_i}{t_{i+k-1} - t_i} N_i^{k-1}(t) + \frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} N_{i+1}^{k-1}(t)$$

for $k > 1$ and $i = 0, 1, 2, \ldots, n$.

If the knot vectors are equally spaced that is $t_{i+1} - t_i =$ constant, for all $i$, then it produces uniform knot vectors and the resulting B-spline are called uniform B-splines. The example of uniform knot vector is $(0, 1, 2, 3, 4, 5)$. If the knot vectors are non-equally spaced but subjected to the constraint such that $t_i \leq t_{i+1}$, for all $i$, then it is a non-uniform knot vector. The example of non-uniform knot vector is $(0, 2, 5, 6, 6, 11, 20)$. Another type is known as open uniform knot vectors which has $k$ equal knot values at each end. An open knot vector can be generated based on the values for $n$ and $k$ as shown below:

$$t_i = \begin{cases} t_0, & 0 \le i \le k-1 \\ t_i \ : \ t_{j+1} - t_j = \text{constant}, k-1 \le j \le n, & k \le i \le n \\ t_{n+k}, & n+1 \le i \le \ n+k. \end{cases}$$

Figure 2.1 shows the plot of the first four B-spline basis functions, $N_i^k(t)$ on the same axes.

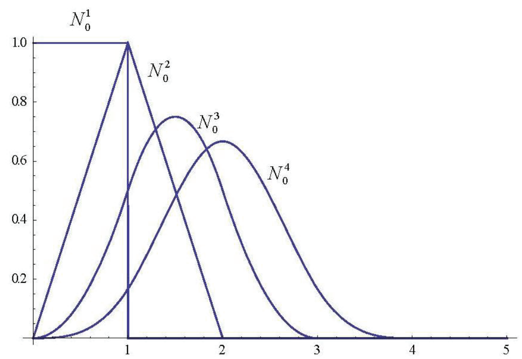



Figure 2.1: The uniform B-spline basis functions for $1 \le k \le 4$ and $i = 0$

Figure 2.2 shows the plot of the uniform B-spline basis functions, $N_i^k(t)$ of order $k = 3$.

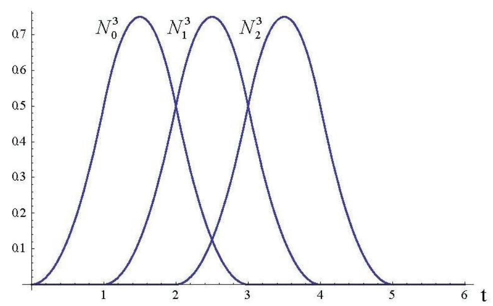



Figure 2.2: The uniform B-spline basis functions of order $k = 3$ and $0 \le i \le 2$

The general (uniform and non-uniform) B-spline curve of order $k$ (degree $k$-1) can be defined as

$$Q(t) = \sum_{i=0}^{n} \mathbf{P}_i N_i^k(t), \qquad t_{k-1} \leq t \leq t_{n+1}.$$

The vectors $\mathbf{P}_i \in \mathbb{R}^2$ (or $\mathbb{R}^3$) for $i$ such that $0 \leq i \leq n$, are known as de Boor points or control points of the curve. The polygon formed by connecting the de Boor points with the line segments is called the de Boor polygon or control polygon of the curve. The curve $Q(t)$ is in the parametric form. Each control point is multiplied by its basis that is $N_i^k(t)$ and in the range of knot values $[t_{k-1}, t_{n+1}]$. The basis blends the contributions of the different control points. In addition, any terms of the form $\frac{0}{0}$ or $\frac{x}{0}$ in the calculation of the basis functions are assumed to be zero. Each basis function $N_i^k(t) > 0$ when it has support on the interval $(t_i, t_{i+k})$, whereas it is zero outside its support. The partition of unity, that is the basis are barycentric such that $\sum_{i=0}^{n} N_i^k(t) = 1$ for $t \in [t_{k-1}, t_{n+1}]$. The basis $N_i^k(t)$ has parametric continuity $C^{k-2}$ at each simple knot $t_i$. The knot vector $(t_0, t_1, \ldots, t_{n+k})$ consists of $n+k+1$ nondecreasing real numbers $t_i$. A few more properties of B-spline curve include:

i) The B-spline curve can be plotted by varying the parameter $t$ over the range of knot values $[t_{k-1}, t_{n+1}]$.

ii) Each segment of the curve (between two consecutive knot values) depends on $k$ control points, which implies the local control property of the curve.

iii) Any control point participates in at most $k$ segments.

iv) The curve and its first $k-2$ derivatives are continuous over the entire range. The non-uniform B-splines can have discontinuities.