CHAPTER ONE

INTRODUCTION

1.1 Background

With the recent global alteration of economic emphasis towards Asia and the introduction of One Belt One Road initiative, there will be vast movement of goods and services which will jumpstart the initiation of a more efficient infrastructure networks within the Asian Region. The main transportation modes will nevertheless be sea route, rail and highways. Amid these common modes of transportation, the Asian Highway network is anticipated to play a significant role in the mobility of goods and personnel taking grasp of great production capacity of China, India and the up-and-coming economies around ASEAN countries. Example of One Belt One Road programme is the construction of new highways from China into Laos and Cambodia (Mooi, 2016).

Policies and issues related to the development of the Asian Highway and road transport continue to attract the interest of policymakers and experts attending legislative and expert group meetings and workshop organised by the secretariat. As a part of the initiative to achieve inclusive and sustainable development through regional cooperation and integration in transport in the Asia Pacific region several objective have been set:

(a) Establishment of road safety facility infrastructure standards

This objective emphasizes on *above the ground installations* which include road safety - such as acceleration and deceleration lanes, warning signs, regulatory signs and speed reduction devices and roadside safety features. User-friendliness aspect is highlighted in order to ease drivers' vehicle operations. For that purpose, the predictability of events should be implemented uniformly along the regions.

(b) Development of model intelligent transport systems deployment

Intelligent Transport System (ITS) is a combination of technologies based on the new capabilities offered by modern Information and Communications Technologies (ICT). The deployment of intelligent transport systems permits enhanced traffic management, more fluid traffic flows and higher levels of safety and security. Predictably intelligent transport systems can cope with traffic congestion , reduce traffic accidents and alleviate environmental externalities generated by road transport.

This project provides opportunity to study the experience of selected member countries (in this case Malaysia) in implementing intelligent transport system to improve road infrastructure management and operation. Also to promote the spread use of related technologies through the development of model intelligent transport system of the region.

1.2 Problem Statement

Road injuries and fatalities have been a growing concern in Malaysia. Statistics shows that more than 6,000 motoring fatalities are recorded each year (Pfordten, 2014). The number of death due to road accidents has also increased from 6,286 in 2003 to 6,917 in 2012 and it has been observed that car accidents had contributed to 22% of the total number of fatalities on the road in recent year (Abdul Rahman, 2013). Looking at the details of traffic crashes in 2012, about 23% of fatal crashes were related to speeding (Syed, Mohamed, Musa, Isah, & Wong, 2014). In urban driving environment accidents or collisions usually occur due to human errors such as misjudged distance, loss of control, poor manoeuvring and sudden braking. Drivers tend to misjudge the distance between their cars and the vehicles in front of them during heavy traffic such as rush hour or road congestion due to road works, resulting in collisions when the driver in the vehicles in front of them apply the brakes without warning.

With the rising number of road accidents and fatalities at local road networks, the Malaysian Government has recently announced the implementation of the Automated Awareness Safety System and Kejara Demerit Point System (Mooi, 2016). Observations show that significant progress in reducing road accidents occur in countries that adopt a multi-prong approach to tackling what WHO defines as the five pillars of road safety namely:

(a) road safety management

(b) safer roads

(c) safer vehicles

(d) safer road users

(e) post-crash response

While it is not feasible to address these five elements simultaneously, systems that are expected to be achieved on a short-term basis can only be safer vehicle; as those systems can exploit already existing traffic infrastructures. As proposed by Bertozzi, Broggi, and Fascioli (2000), any on-board system for ITS applications has to meet these important requirements:

- The proposed system must be robust enough to adapt to different conditions and changes of environment, road, traffic, illumination, and weather.
- On-board system for ITS applications requires high level of reliability. Therefore, it is important for the system to undergo extensive phase of testing and validation.
- For commercial reasons the design of an ITS system has to be driven by strict cost criteria that should not cost more than 10% of the vehicle price. Low power consumptions is desired and the system should not disrupt the car performance.
- In order to retain the car design, the system's hardware and sensors have to be kept compact in size.
- The system has to be user-friendly.

Hence, foreseeing a massive and widespread use of autonomous sensing agents, the use of passive sensors, such as cameras, obtains key advantages over the use of active ones (laser-based sensors). Although machine vision could not exceed human sensing possibilities such as absence of fixed illumination, it can, however, help the driver in case of failure, for example in the lack of concentration or drowsy conditions.

1.3 Objectives

1. To develop a prototype of car follower based on monocular camera as a vision sensor.

2. To implement and tune fuzzy logic decision making system using Design of Experiment.

1.4 Project Scope

Since car following system is a wide research area, to keep this project within manageable size the scope of this project has been identified as follows:

1. The prototype of the follower car is represented by 1/10 scale Remote Controlled (RC) car.

2. The system were tested in a controlled environment where the illumination does not heavily fluctuate in order to increase the detection accuracy.

3. The system is restricted to perform car following and collision avoidance only and not lane keeping.

4. The system works offline and the parameters were set based on the data that were obtained from the experiment performed beforehand.

5. The system does not include real time decision making that requires heavy processing power in order to compensate the size and processing capabilities of the microprocessor used.

1.5 Research Contribution

The contribution of this thesis is the design of ground vehicle following system using fuzzy logic controller. Another contribution of this research is the design validation by implementing the system on 1/10 scale hardware ground vehicle prototype.

1.6 Report Outline

As a whole, this report consists of six chapters. The first chapter covers a brief introduction of this project. It was subsequently followed by problem statements, objectives and project scope.

In Chapter 2, literature related to the project was reviewed and presented. The chapter begins with the review of previous works done.

In Chapter 3, the methodology of the project was discussed. Firstly, the project requirements were discussed in detail. The significance of each component chosen was also discussed. Then the project design was explained. An experimental design was presented followed by its data collection procedures. Finally, data analysis, performance evaluation and tests were discussed. This includes pilot tests and integrated test procedures.

Chapter 4 presented the result and discussion of the project. Firstly, the final results were presented. The comparison between two methods of decision making is presented graphically. The lead vehicle detection rate and the system reliability were evaluated and analysed.

The final chapter, which is Chapter 5 concludes the overall report. The limitation of the system was duly presented. Future works and suggestions of improvement were discussed at the end of this chapter.

CHAPTER TWO

LITERATURE REVIEW

In the past decades much research has been carried out to realize the robustness concept of autonomous vehicle (Özgüner, Acarman, & Redmill, 2011). This chapter will discuss past works that had been conducted related to vision system's contribution to autonomous vehicle. This chapter is organized as follows. Firstly the related works will be sorted to types of vision sensors utilized and its corresponding approaches. Then in each section, the strength and drawbacks of each approach or method of related works will be discussed.

2.1 Vision System

There are several fundamental steps in processing image. Figure 2.1 shows the steps involved in image processing. The input/problem domain is obtained through image acquisition method. Depending on the desired application the steps could involve getting image output, image attributes or a combination of both. These processes are executed in order to extract meaningful data.

2.1.1 Vision Sensor

Figure 2.2 shows fundamental steps in image processing from raw input to the final evaluated input. There are several image acquisition methods that are employed.The works of Bharade, Gaopande, and Keskar (2014); Huang, Zhang, Ma, and Yan (2010); Ruyi, Reinhard, Tobi, and Shigang (2011); Tuohy (2010); Zheng (2015) had utilized

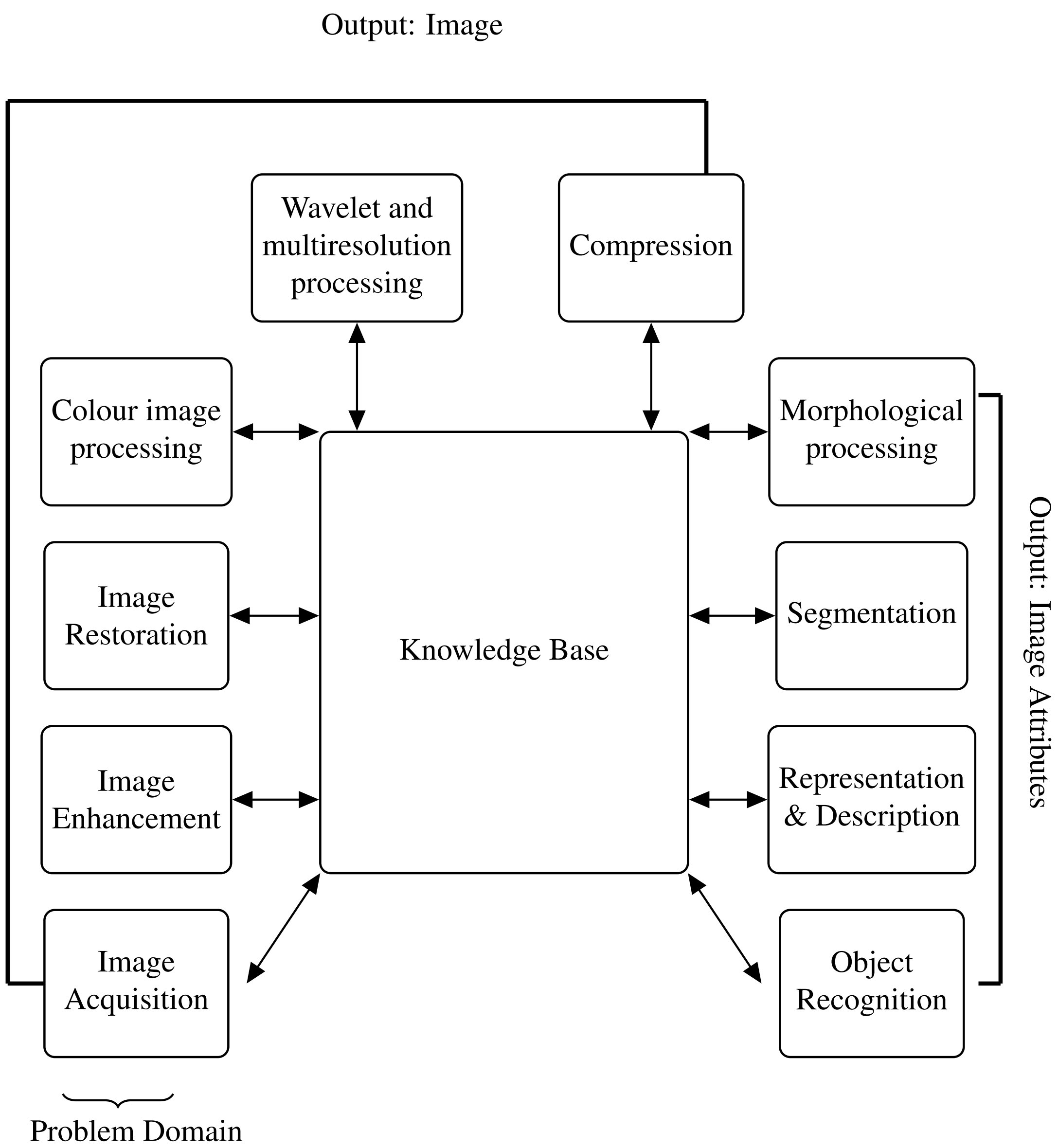

Figure 2.1: Fundamental steps in image processing (adapted from Gonzalez & Woods, 2007)

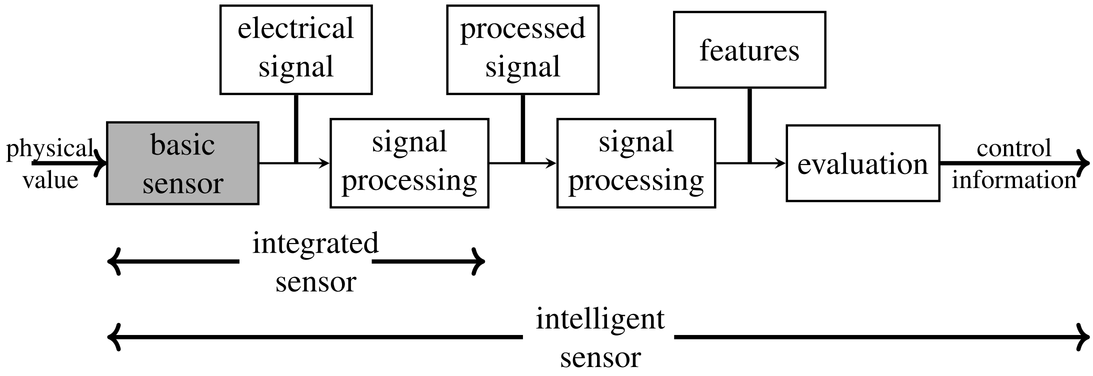

Figure 2.2: Sensor integration levels (adapted from Berns & Ewald, 2009)

a common image acquisition method which is using a camera. The camera is widely used because it is easily available and there are various types of resolution that can be used to suit their needs.

Huang et al. (2010) used more than one camera in their study. With multiple inputs of images from different sides of the vehicle, an algorithm was designed by computing a maximum value of every row for every field and the average of these maximum values of the five fields was then used as the threshold value of each row after integral compensation. The formulae need to be generated after a few repetitive experiments and they might not be robust towards different types of conditions and illumination.

Khalid, Mohamed, and Abdenbi (2013); Klaser, Osorio, and Wolf (2013); Sharma, Jeong, and Kim (2011) on the other hand, utilized stereovision camera as input method. Stereovision-based distance measurement provides reasonably good accuracy for objects within a short distant range, however due to the matching ambiguity, quantization errors, and inaccurate parameters of the camera model, this method has poor accuracy for objects at long distance. Thus, further processing is needed in order to compensate for this shortcoming.

2.1.2 Camera Calibration

Internal or intrinsic parameters are contained in the calibration matrix, which can be parameterized by: focal length, aspect ratio, skew, and the location of the offset of the principal point in the image. The unknown parameters are f_x and f_y (camera focal lengths) and (u_0, v_0) which are the optical centres expressed in pixels coordinates. The principle offset of the image varies between camera model and can be obtained from calibration matrix (Hugemann, 2010). The general camera matrix is shown in Eq. (2.3) consisting of optical center at x-coordinate in pixel (u_0) and optical center at x-coordinate in pixel (v_0). The equations for a_x, a_y, radial distortion (x-plane) (x_{dr}), radial distortion (y-plane) (y_{dr}), tangential distortion (x-plane) (x_{dt}), and tangential distortion (y-plane) (y_{dt}) are shown in Eq.(2.1)-(2.7) .

$$a_x = \frac{f_x}{d_x} \tag{2.1}$$

$$a_x = \frac{f_y}{d_y} \tag{2.2}$$

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \underbrace{\begin{bmatrix} a_x & 0 & u_0 \\ 0 & a_y & v_0 \\ 0 & 0 & 1 \end{bmatrix}}_{camera\ matrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{2.3}$$

$$x_{dr} = x(1 + k_1 r^2 + k_2 r^4 + k_3 r^6) \tag{2.4}$$

$$y_{dr} = y(1 + k_1 r^2 + k_2 r^4 + k_3 r^6) \tag{2.5}$$

$$x_{dt} = x + [2p_1 xy + p_2(r^2 + 2x^2)] \tag{2.6}$$

$$y_{dt} = y + [p_1(r^2 + 2y^2) + 2p_2xy] \tag{2.7}$$

where r^2 (Frank, Stachniss, Grisetti, Arras, & Burgard, 2010; Hugemann, 2010) is shown in Eq. (2.8) :

$$r^2 = (x - u_0)^2 + (y - v_0)^2 \tag{2.8}$$

2.1.3 Image Processing Sequences

For image processing, the algorithm can be designed by importing libraries from OpenCV. OpenCV is a computer vision and machine learning software library. It was developed as an open source to provide foundation for computer vision purposes and to provide function aid of the use of machine perception in commercial products (itseez, 2014).

There are times when the image need to be pre-processed so that it can provide meaningful input for further detection and recognition. Three noise removing filters such as mode,mean and median filter and two morphological operation which are dilation and erosion will be discussed in this section.

For each mode, mean and filter, the surrounding neighbourhood pixel (eg. 3x3 kernel window) is sequentially examined. For mode filter the most common pixel will replace the central pixel. The flaw with this filter is that the edges might appear jagged after the operation. In mean filter case, the magnitude of the central pixel is substituted by the averaged pixel of the neigbourhood pixel. The improved version of mean filter is the median filter; which is less susceptible to extreme fluctuation of pixels (Ghosh, 2013, p. 10.6).

Morphological operation in image processing context is the means for obtaining image components that are functional in characterising and describing the shape region such as boundaries, skeleton and convex hull (Gonzalez & Woods, 2007; Krigg, 2014). Morphological operations are usually performed on binary images. Dilation is a morphological operation that thickens or expands the white region. It is useful when the region of interest (ROI) is too small for further processing or when there are broken parts that need to be joined as an object. Erosion on the other hand is a morphological operation that removes excessive white pixels. Erosion is useful for removing small white noises or separating two connected regions (Mordvintsev & Rahman, 2014).

2.1.4 Colour Space Conversion

The establishment of colour space is intended to aid the modeling of colours in a specific way. Basically, a colour space is a way to describe each colour in a form of coordinate system. For digital image processing application, the most commonly used in the hardware oriented models is the *RGB* model. Depending on the application, the image colour space can be converted. There are several types of colour space other than *RGB* and the ones will be discussed in this section are *HSV* colour space and YC_bC_r colour space. In this section the conversion of the aforementioned colour spaces will be reviewed. It should be noted that the following conversion follows the OpenCV library notation.

2.1.4(a) HSV Colour Space

HSV model is more apt than the *RGB* model for numerous image processing operations. *H*, stands for hue denotes the dominant pure colour as perceived by an observer while S, saturation denotes the level to which that pure colour has been dispersed by white light. Since colour and value, V are non-correlated, it is feasible to manipulate one without affecting the other (Ghosh, 2013, p. 2.14).

While there are other colour space like $L*A*B$ that could be used to implement colour segmentation it is reported by Bora, Gupta, and Khan (2015) that HSV colour space performed better than the former. The conversion is represented in Eq. (2.9)-(2.13).

$$Vmax \leftarrow (R,G,B) \tag{2.9}$$

$$S \leftarrow \begin{cases} \dfrac{V - min(R,G,B)}{V} & , if\ V \neq 0 \\ 0 & , otherwise \end{cases} \tag{2.10}$$

$$H \leftarrow \begin{cases} 60(G-B)/V - min(R,G,B) & if\ V = R \\ 120 + 60(B-R)/(V - min(R,G,B)) & if\ V = G \\ 240 + 60(R-G)/(V - min(R,G,B)) & if\ V = B \end{cases} \tag{2.11}$$

If $H < 0$ then $H \leftarrow H + 360$. Initial output is:

$$0 \leq V \leq 1, 0 \leq S \leq 1,\ 0 \leq H \leq 360 \tag{2.12}$$

In OpenCV context the H is not measured from 0° to 360°. The values are converted to destination data type (8-bit). Since 360°is too big, it is halved so that the range can be fit from 0 to 255:

$$0 \leq V \leq 255,\ 0 \leq S \leq 255,\ 0 \leq H \leq 180 \tag{2.13}$$

2.1.4(b) YCrCb Colour Space

YC_bC_r colour space (sometimes abbreviated as YCC colour space) is employed for the representation of colour vectors in digital video recording (Koschan & Abidi, 2008). While Y is the luminance component, C_b and C_r are the blue and red chroma components respectively. The conversion of YC_rC_b colour space is shown in Eq. (2.14)-(2.16).

$$Y \leftarrow 0.299 \cdot R + 0.587 \cdot G + 0.114 \cdot B \tag{2.14}$$

$$Cr \leftarrow (R - Y) \cdot 0.713 + \kappa \tag{2.15}$$

$$Cb \leftarrow (B - Y) \cdot + \kappa \tag{2.16}$$

where κ is a constant fixed depending on type of images :

$$\kappa = \begin{cases} 128 & 8 \ bit \ images \\ 32768 & 16 \ bit \ images \\ 0.5 & floating \ point \ images \end{cases} \tag{2.17}$$

2.1.4(c) Histogram Backprojection

Histogram backprojection is a way of recording how well the pixels of the given image fit the distribution of pixel in the histogram model. The histogram of a feature is calculated and it is then used to find this feature in an image OpenCV (2015a). Example of histogram backprojection is shown in Figure 2.3.

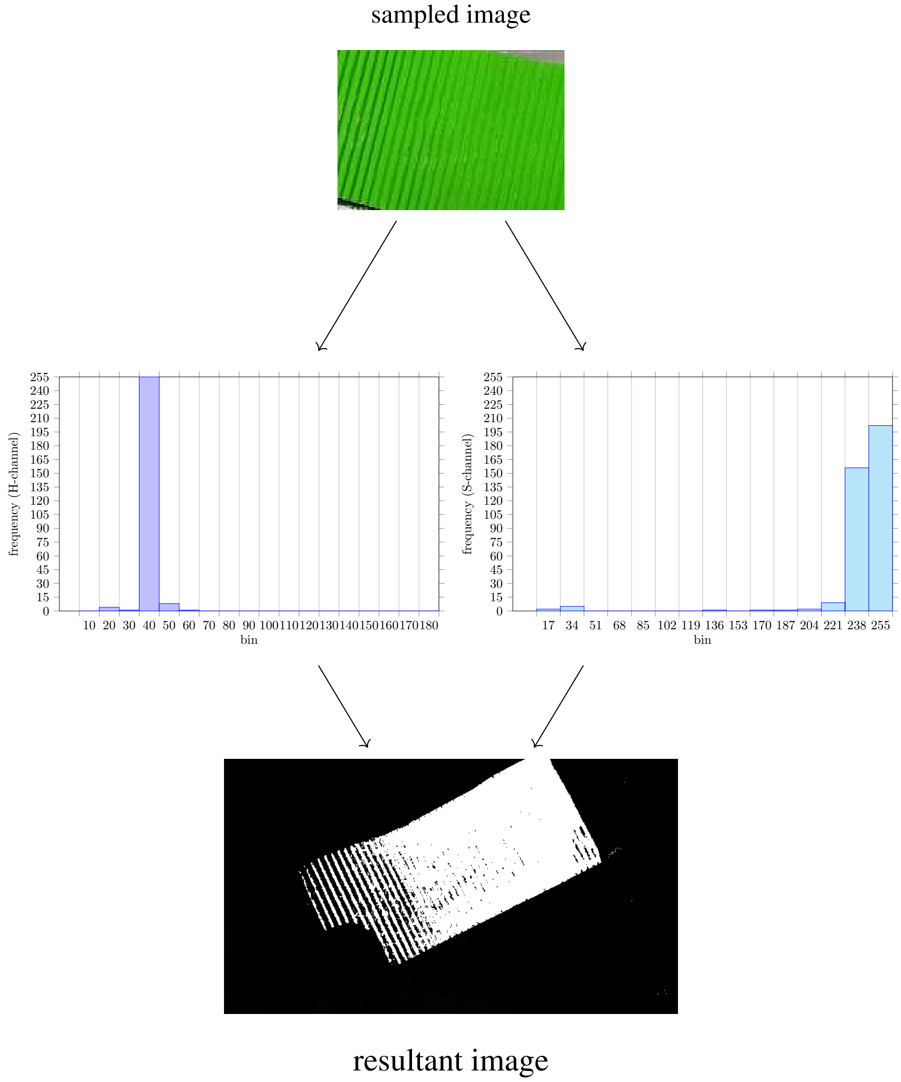

Figure 2.3: Example of histogram backprojection

2.1.5 Feature Detection and Description

Descriptor in image processing term is the way the segmented image could be described in a meaningful way. The method discussed in this section are moments, intelligent descriptors and inverse perspective mapping.

2.1.5(a) Moments

Moments is the weighted average of the pixel intensity inside an image. Moments are represented as shown in Eq.(2.18) in OpenCV library.

$$mu_{ji} = \sum_{x,y} (array(x,y) \cdot (x-\bar{x})^j \cdot (y-\bar{y})^i) \tag{2.18}$$

The moments of a contour are defined in the same way but computed using the Green's Theorem (OpenCV, 2015b). Moments are useful to determine the mass centre. The mass centre $(\bar{x},\bar{y})$ is represented as:

$$\bar{x} = \frac{m_{10}}{m_{00}}, \bar{y} = \frac{m_{01}}{m_{00}} \tag{2.19}$$

Huang et al. (2010); Khalid et al. (2013) proposed using Hough transform to find the extreme lines of the road. While Huang et al. (2010) utilized the road line marking to estimate the midpoint of the road so that the vehicle could stay inside the lane without swerving laterally, Khalid et al. (2013) used built in library Open Source Computer Vision Library (OpenCV) as an alternative in order to detect contours or edges.

Another method of feature extraction proposed by Sharma et al. (2011) is by using Scale Invariant Feature Transform (SIFT). SIFT is a method for extracting distinctive invariant features from images that can be used to perform reliable matching between different views of an object or scene (Lowe, 2004). However, this requires a lot of processing resources to implement. To reduce the computation time Self-Organising Map (SOM) was used by Sharma et al. (2011) to improve the matching process.

2.1.6 Tracking & Navigation

For tracking purposes, Khalid et al. (2013); Le, Combs, and Yang (2013); Teoh and Bräunl (2012) adopted Kalman filter in their study for optimal estimation of the state of a dynamic system, on the basis of noisy measurements and an uncertain model of the system dynamics. A Kalman filter was used in the tracking to predict the locations of objects in the future video frames. Kalman filter is advantageous in the tracking process because the position of a moving feature point in the next frame could be estimated. The uncertainty of the estimation could also be assessed by the Kalman filter. The processing time could also be optimized as Kalman filter reduces the search area for re-detecting an object. This also leads to lesser false positives. Othe method of point estimation would be exponential smoothing. Exponential smoothing is a known method to produce a smoothed time series. Other methods like single moving averages treat past observation equally but exponential smoothing allocates exponentially less weights as the observation gets older. In other words, current observations get more priority in forecasting than the older observations. There are three types of exponential smoothing but in this research the method is focused on the triple exponential smoothing where the trend and the periodicity of the time series are taken into consideration. There are two types of triple exponential smoothing which are additive and multiplicative Janert (2010). There are two variations to this method that differ in the nature of the seasonal component. The additive method is preferred when the seasonal variations are roughly constant through the series, while the multiplicative method is preferred when the seasonal variations are changing proportional to the level of the series. Equation (2.20)-(2.21) shows the difference between these two types:

$$additive \quad \begin{cases} s_i & = \alpha_s(x_i - p_{i-k}) + (1-\alpha_s)(s_{i-1} + t_{i-1}) \\ t_i & = \beta_s(s_i - s_{i-1}) + (1-\beta_s)t_{i-1} \\ p_i & = \gamma_s(x_i - s_i) + (1-\gamma_s)p_{i-k} \\ x_{i+h} & = s_i + ht_i + p_{i-k+h} \end{cases} \tag{2.20}$$

$$multiplicative \quad \begin{cases} s_i & = \alpha_s \dfrac{x_i}{p_{i-k}} + (1-\alpha_s)(s_{i-1} + t_{i-1}) \\ t_i & = \beta_s(s_i - s_{i-1}) + (1-\beta_s)t_{i-1} \\ p_i & = \gamma_s \dfrac{x_i}{s_i} + (1-\gamma_s)p_{i-l} \\ x_{i+h} & = (s_i + ht_i)p_{i-k+h} \end{cases} \tag{2.21}$$

where s_i is smoothed signal, t_i is smoothed trend, p_i is periodic component, and x_{i+h} is forecast of i^{th} data. The smoothing weightage parameter (α_s),trend weightage parameter (β_s), and seasonality weightage parameters (γ_s) are constants that is estimated to reduce mean square error.

Bharade et al. (2014) on the other hand utilized regression analysis to derive the equation needed to predict the distance of the tracked object under various conditions. While the system was shown to have close distance prediction of the object regardless of its size it has unsatisfactory processing time. Linear regression is beneficial as it is faster in comparison to neural networks. Linear regression models are simple and require minimum memory to implement, so they work well on embedded controllers that have limited memory space (Caraciolo, 2011).

Other studies by Yen, Wang, and Chien (2015) and Lien, Hsia, and Su (2015) employed the Camshift algorithm method; an upgraded version of mean-shift algorithm. The mean-shift algorithm is a robust method of finding local extrema in the density distribution of a data set. This is an affluent process for continuous distributions which is essentially a hill climbing method applied to a density histogram of data.

Figure 2.4 shows how mean-shift tracker works. Assume B1 is the initial bounding box. Its centre is marked with blue. However, the true centroid point inside bounding box B1 is located differently (marked by red). Therefore, the mean-shift tracker will reiterate the centre of the bounding box so that it can match with the previous centroid. As it iterates it will finally obtain bounding box with maximum pixel distribution which is B2 with its desired centre marked with light blue. Supposedly the centroid and the desired centre match or only differ with slightest error. Camshift (continuous adaptive mean-shift) differs from the mean-shift in a way that the search window adapts itself in size (Bradski & Kaehler, 2008).

Figure 2.5 shows the flowchart of Camshift. The highlighted box is an additional step provided by Camshift. Should there be well-segmented distributions (say object features that stay compact), then this algorithm will automatically adjust itself for the size of the object as the object moves closer to and further from the camera. For added accuracy in an occluded surrounding, Lien et al. (2015) paired up the Camshift tracking with Kalman filter.

Hill climbing is a mathematical optimization technique which belongs to the family of local search. It is a heuristic function that estimates how close a given state is to a

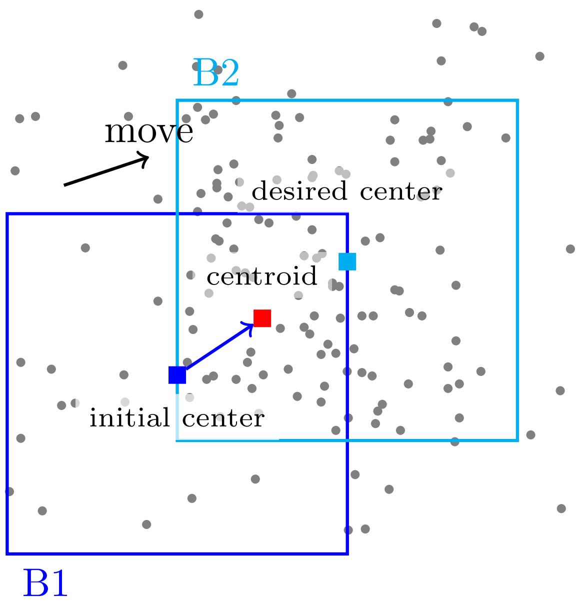

Figure 2.4: Meanshift

goal state. The algorithm will start with a random solution to a problem, loop (iterate) itself so that it can acquire an improved solution by incrementally changing a single element of the solution. If the alteration generates a better solution, an incremental alteration is made to the new solution, repeating until no more adjustments can be found (Rich, Knight, & Nair, 2010).

2.1.7 Distance Estimation

A method that can be used to estimate the distance of the tracked object is the inverse perspective mapping method. Inverse perspective mapping is a method to translate the front view captured by the camera to a bird's eye view. It can be used either to detect lane (Ruyi et al., 2011) or to calculate the distance between the camera and the lead vehicle (LV)/object (Bharade et al., 2014; Tuohy, 2010; Tuohy, O'Cualain, Jones, & Glavin, 2010). Inverse perspective mapping is useful because it eliminates the problem where the input image captured from a monocular camera suffers from

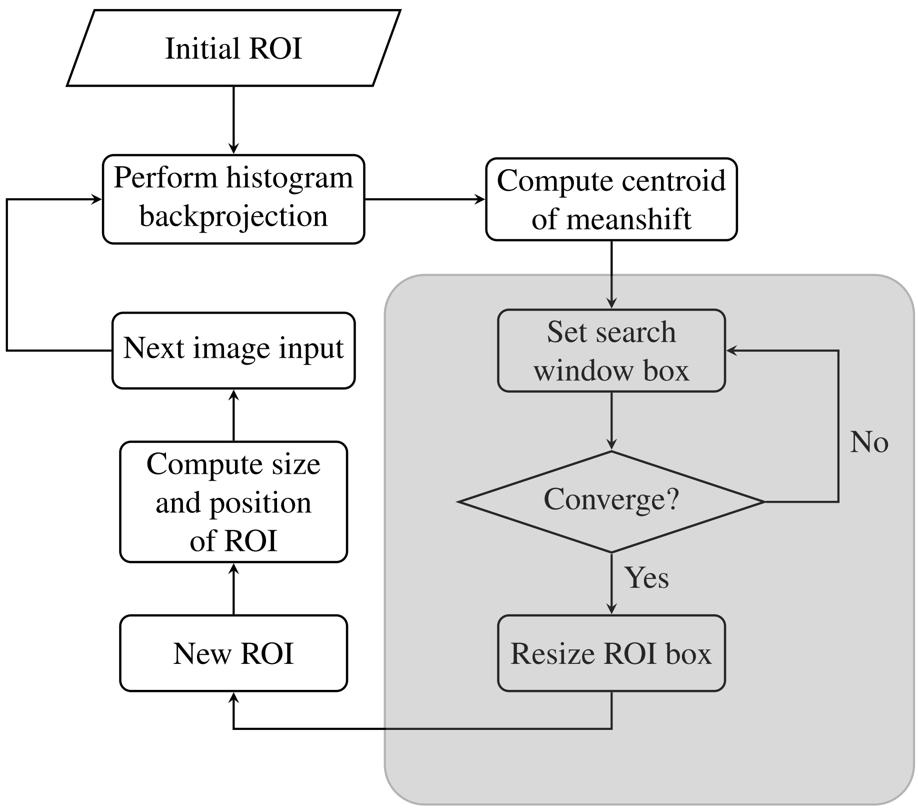

Figure 2.5: Camshift Tracker Flowchart

high degree of uncertainty in object distance estimation due to the nonlinear relation between object height and its actual distance from the camera. However, this method was reported to suffer from excessive memory consumption which make it impractical to be used in embedded systems. Studies by Jiangwei, Lisheng, Lie, Rongben, and Bibing (2004); Li and Wang (2012); Lu, Wang, and Chu (2012); Zhan, Huang, and Wu (2008) employed distance approximation using pinhole method. Pinhole method is the method exclusive to monocular vision sensor. The proposed system has been tested in various situations and has been extremely efficient in distance estimation and object detection. However it is extremely sensitive to changes in the environmental conditions. The tolerance to changes light intensity can be improved by observing its effect on predicted distance value.

2.2 Fuzzy Logic Control System

Fuzzy logic is determined as a set of mathematical principles for knowledge representation based on degrees of membership rather than on crisp membership of classical binary logic. In fuzzy logic, the two-valued truth set of boolean logic is replaced by a multi-valued one, usually the unit interval [0,1]. Truth sets taking values in this range are said to be normalized (Meehan & Joy, 1998; Negnevitsky, 2011).

A fuzzy control system is centred on a set of fuzzy “if-then” rules of behaviour that gather the input(s) obtained from the environment which was then mapped in accordance to the rules devised and provide a solution as an output.

Fuzzy control is useful in situations where there is no acceptable mathematical model for the plant or if there are experienced human operators who can adequately control the plant. On top of that, should there be large ambiguity or unknown disparity in plant parameters and structures, fuzzy control system can be applied to devise qualitative control rules in terms of vague and fuzzy sentences (Trillas & Eciolaza, 2015).

In these cases, fuzzy control can be considered as a model-free approach and it does not require a mathematical model of the objective plant. It is referred to as a knowledge-based control approach, and it makes an effective use of all available information related to the system, from sensors which provide numerical measurements of key variables to human experts who provide linguistic descriptions about the system and control instructions (Thanh, Thanh, The, Hung, & Xuan, 2014; Trillas & Eciolaza, 2015).

2.2.1 Fuzzy Control Methods

The two most commonly used fuzzy methods are Mamdani method and Sugeno method. Fuzzy control using Mamdani method is a more interactive and natural method. However, Mamdani type fuzzy inference requires a significant amount of computational burden. Sugeno method, on the contrary is well-matched with optimization and adaptive techniques, which makes it popular in control problem solving (Negnevitsky, 2011).

Research by Adewuyi (2012); Agarwal and Bose (2014); Kaur and Kaur (2012); Kaur and Verma (2015) support that Sugeno-based fuzzy logic has an advantage compared to Mamdani-based fuzzy logic controller. A simulation test comparing Sugeno method with Mamdani method for robot navigation by Kaur and Verma (2015) shows that the robot that employed Sugeno method managed to complete its task relatively faster compared to Mamdani method.

2.2.2 The Representation of Membership Functions

Some commonly used fuzzy sets are triangle shaped fuzzy sets ($\bigwedge$) and trapezoidal fuzzy membership functions ($\prod$). Triangle shaped fuzzy sets are defined as shown in Equation (2.22); where x is the crisp input, $\bigwedge_A$ is the lower limit of the triangle , $\bigwedge_B$ is the middle limit of the triangle and $\bigwedge_C$ is the upper limit of the triangle.