First Semester Examination
Academic Session 2018/2019

December 2018/January 2019

EME 411 – Numerical Methods for Engineers
[Kaedah Berangka Untuk Jurutera]

Duration : 2 hours
[Masa : 2 jam]

Please check that this paper contains **SEVEN [7]** printed pages including appendix before you begin the examination.

[Sila pastikan bahawa kertas soalan ini mengandungi ***TUJUH [7]*** *mukasurat bercetak beserta lampiran sebelum anda memulakan peperiksaan.]*

INSTRUCTIONS : Answer **ALL FIVE [5]** questions.
*[**ARAHAN** : Jawab **SEMUA LIMA [5]** soalan.]*

Answer Questions In **English** OR **Bahasa Malaysia**.
*[Jawab soalan dalam **Bahasa Inggeris** ATAU **Bahasa Malaysia**.]*

Answer to each question must begin from a new page.
[Jawapan bagi setiap soalan mestilah dimulakan pada mukasurat yang baru.]

In the event of any discrepancies, the English version shall be used.
[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. **Consider a heat conduction problem over the 2D domain below:**

Pertimbangkan masalah kekonduksian haba pada domain 2D di bawah:

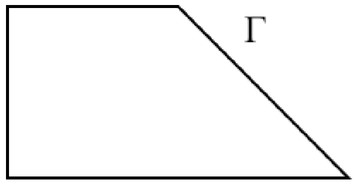

Figure 1
Rajah 1

Suppose the boundary Γ has convective heat transfer

Andaikan sempadan $\boldsymbol{\Gamma}$ terdapat pemindahan haba olakan

$$u + \frac{k}{h}\nabla u \cdot \mathbf{n} = u_\infty$$

where $\boldsymbol{k}, \boldsymbol{h}$, and $\boldsymbol{u}_\infty$ are constants.

di mana k, h, dan u_∞ pemalar.

[a] State which type of boundary condition must be used to model the heat convection and explain the reason why.

Nyatakan jenis syarat sempadan yang mesti digunakan bagi model perolakan haba itu dan terangkan sebabnya.

(**10 marks**/*markah*)

[b] With an aid of a sketch, explain how $(\boldsymbol{\nabla u} \cdot \mathbf{n})$ along Γ can be computed in the FDM formulation.

Dengan bantuan lakaran, terangkan bagaimana $(\nabla u \cdot \boldsymbol{n})$ sepanjang $\boldsymbol{\Gamma}$ boleh dihitung dalam formulasi FDM.

(**5 marks**/*markah*)

2. **Consider the steady state heat conduction in a thin plate with a width of 3 m and height of 2 m with the following boundary conditions:**
   - **<u>Outward</u> heat flux of 100 W/m on the West wall.**
   - **30 °C on all other walls.**

Pertimbangkan aliran haba mantap di dalam sekeping plat dengan lebar 3 m dan tinggi 2 m dan mempunyai syarat-syarat sempadan berikut:
- *Haba ke <u>arah luar</u> bernilai 100 W/m pada dinding Barat.*
- *30 °C pada dinding-dinding lain.*

[a] **Write the complete mathematical statement of the problem.**

Tuliskan penyataan matematik lengkap bagi masalah itu.

(**5 marks**/*markah*)

[b] **Sketch and number the grid points with the step size $h = 1$ m for solving the problem.**

Lakarkan dan nomborkan titik-titik grid dengan saiz langkah $h = 1\ m$ yang digunakan untuk menyelesaikan masalah itu.

(**5 marks**/*markah*)

[c] **For the $u_{i,j}$ on the West wall, derive the FDM equation for this value to include the heat flux.**

Bagi $u_{i,j}$ pada dinding Barat, terbitkan persamaan FDM bagi nilai tersebut berserta fluks haba.

(**8 marks**/*markah*)

[d] **Set up the linear system for this problem and solve for all the temperature values in the grid.**

Binakan sistem linear bagi masalah ini dan selesaikan bagi semua nilai suhu di dalam grid.

(**7 marks**/*markah*)

3. **Discuss at least FOUR(4) aspects of how the formulation of FEM is different from that of FDM.**

Bincangkan sekurang-kurangnya EMPAT(4) aspek bagaimana formulasi FEM adalah berbeza dengan formulasi FDM.

(**12 marks**/*markah*)

4. **Consider the displacement response of a 1 m bar due to the axial forces $P_1 = 60$ N at $x = 0.2$ and $P_2 = 50$ N at $x = 1.0$. The bar is fixed at $x = 0$. Assume $E \cdot A = 100 \times 10^3\ N$.**

Pertimbangkan tindak balas sesaran bagi sebatang bar 1 m oleh daya-daya paksi $P_1 = 60$ N pada $x = 0.2$ dan $P_2 = 50$ N pada $x = 1.0$. Bar itu ditetapkan pada $x = 0$. Andaikan $E \cdot A = 100 \times 10^3$ N.

[a] **The above problem may be stated with 1-D Poison's equation. Sketch the 1-D domain and state the strong form and the weak form of the problem COMPLETELY (derivation not necessary).**

Masalah di atas boleh dirumuskan dengan persamaan Poisson. Lakarkan domain 1-D dan nyatakan bentuk kuat dan bentuk lemah masalah itu SECARA LENGKAP (terbitan tidak perlu).

(**10 marks**/*markah*)

[b] **Sketch the 1-D mesh of 3 QUADRATIC elements where the nodes are at $x = \{0, 0.2, 0.5, 1.0\}$ m. Label appropriate numbering for the DOFs and elements.**

Lakarkan jejaring 1-D dengan 3 unsur KUADRATIK di mana nod-nod adalah pada $x = \{0,0.2,0.5,1.0\}$ m. Nomborkan semua DOFs dan unsur yang sesuai.

(**5 marks**/*markah*)

[c] **Set up the linear system for the problem. The modified matrix and detailed derivation steps are not necessary.**

Binakan sistem linear bagi masalah itu. Matrik terubahsuai dan langkah-langkah terbitan yang terperinci adalah tidak perlu.

(**15 marks**/*markah*)

[d] **Explain why using 3 LINEAR elements for the same problem also yields the same accuracy as the quadratic elements.**

Terangkan kenapa penggunaan 3 unsur LINEAR bagi masalah yang sama menghasilkan kejituan yang sama seperti unsur-unsur kuadratik.

(**5 marks**/*markah*)

5. **The following is MATLAB code for 1D FEM that is incomplete and incorrect. The code must solve for a mesh of <u>240 elements</u> of varying lengths. Rewrite the code so it is complete. You may only assume:**
- **the nodal coordinates and the element connectivity map have been defined**
- **the functions `getElCoord`, `getLoadVec`, and `getElMat` have been written**

Berikut ialah kod MATLAB bagi FEM 1-D yang tidak lengkap dan salah. Kod itu mesti menyelesaikan bagi jejaring dengan <u>240 unsur</u> yang berbeza panjang. Tuliskan semula kod itu agar ia lengkap. Anda hanya boleh mengandaikan bahawa:
- *koordinat nod dan peta kesalinghubungan unsur telah ditakrifkan*
- *fungsi-fungsi* `getElCoord`, `getLoadVec` *dan* `getElMat` *telah dituliskan.*

```
elementDOF = 2
ndof = 212
K = zeros(ndof,ndof)
bb = zeros(ndof,1)
for e = 1:K
  len = getElCoord(e)
  be = getLoadVec(len)
  ke = getElMat
  for ir from 1 to 2
    irs = elementDOFMap(e,ir)
    bb(irs) = bb(irs) + be(ir)
    for ic from 1 to 2
      ics = elementDOFMap(e,ic)
      K(irs,ics) = K(irs,ics) + K(irs,ics)
    end
  end
end
```

(**13 marks**/*markah*)

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APPENDIX 1
LAMPIRAN 1

Formulas

Forward differences:

$$u_i' = \frac{u_R - u_i}{h} + O(h)$$

Centered Differences:

$$u_i'' = \frac{u_R - 2u_i + u_L}{h^2} + O(h^2) \qquad u_i' = \frac{u_R - u_L}{2h} + O(h^2)$$

2D Stencil for Poisson's equation:

$$-u_{E,j} - u_{W,j} + 4u_{i,j} - u_{i,N} - u_{i,S} = \frac{h^2 f_i}{k}$$

Robin boundary condition

$$p \cdot u + q \cdot \nabla u \cdot \mathbf{n} = g$$

1D heat equation

$$-\alpha \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial t} = Q(x,t)$$

FDM equation with variable material

$$-k_{i-\frac{1}{2}} u_{i-1} + \left(k_{i-\frac{1}{2}} + k_{i+\frac{1}{2}}\right) u_i - k_{i+\frac{1}{2}} u_{i+1} = h^2 f_i$$

Element stiffness matrix:

$$\mathbf{K}^e = \frac{k}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \mathbf{K}^e = \frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\mathbf{K}^e = \frac{k}{l}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 16/3 \end{bmatrix} \qquad \mathbf{K}^e = \frac{EA}{l}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 16/3 \end{bmatrix}$$

Element load vector with point load *P* at ρ_0:

$$\mathbf{b}^{(e)} = P\begin{bmatrix} \xi_1(\rho_0) \\ \xi_2(\rho_0) \end{bmatrix} \qquad \mathbf{b}^{(e)} = P\begin{bmatrix} \xi_1(\rho_0) \\ \xi_2(\rho_0) \\ \xi_3(\rho_0) \end{bmatrix}$$

Basis functions in local coordinate

$$\xi_1(\rho) = \tfrac{1}{2}(1-\rho) \qquad \xi_2(\rho) = \tfrac{1}{2}(1+\rho) \qquad \xi_3(\rho) = \tfrac{1}{2}(1-\rho)^2$$

Local-global coordinates mapping

$$x = \frac{l}{2}\,\rho + \frac{1}{2}\left(x_i + x_j\right)$$