BOUNDARY LAYER SOLUTIONS FOR CONVECTIVE FLOW VIA VARIOUS GROUP TRANSFORMATION METHODS

by

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(Mohammed Jashim Uddin)

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LIST OF ABBREVIATIONS

BVP	Boundary Value Problem
BCs	Boundary Conditions
CH	Convective Heating
DA	Dimensional Analysis
GM	Group Method
IVP	Initial Value Problem
MHD	Magnetohydrodynamic
ODEs	Ordinary Differential Equations
OMA	Order of Magnitude Analysis
PDEs	Partial Differential Equations
PHF	Prescribed Surface Heat Flux
PST	Prescribed Surface Temperature
RKF45	Runge-Kutta-Fehlberg Fourth-Fifth Order Numerical Method

LIST OF SYMBOLS

a	velocity slip parameter
A	viscosity parameter
A_2, A_3	dimensional constants
b	thermal slip parameter
B	unsteadiness parameter
$\vec{B}$	the magnetic induction vector
$B(\bar{x})$	variable magnetic field strength
B_0	constant magnetic field strength
Bi	Biot number
Br	Brinkman number
c	inertia coefficient
$\bar{c}$	dimensional constant
c_1, c_2, c_3	constants
c_4	characteristic stretching intensity $\left(\frac{1}{s}\right)$
C	concentration of species or nanoparticle volume fraction
c_p	specific heat at constant pressure $\left(\frac{J}{kg\ K}\right)$
C_s	heat capacity of solid phase $\left(\frac{J}{kg\ K}\right)$
c_s	the concentration susceptibility
$C_{f\bar{x}}$	skin friction factor
d	mean particle diameter (m)
$D(C)$	variable mass diffusivity $\left(\frac{m^2}{s}\right)$
D	constant mass diffusivity $\left(\frac{m^2}{s}\right)$
D_B	Brownian diffusion coefficient $\left(\frac{m^2}{s}\right)$
D_T	thermophoretic diffusion coefficient $\left(\frac{m^2}{s}\right)$

D_1	thermal slip factor (m)
D_f	Dufour number
D_i	dispersion parameter
D_c	mass diffusivity parameter
e	parameter of the scaling group
$\vec{E}$	electric field vector
Ec	Eckert number
Fr	Froude number
f	dimensionless stream function
g	gravitational acceleration $\left(\frac{m}{s^2}\right)$
f_w	suction/injection parameter
G	parameter of the linear group
Gr	thermal Grashof number
$Gr_{\bar{x}}$	local thermal Grashof number
Gc	mass Grashof number
$Gc_{\bar{x}}$	local mass Grashof number
$h_f(\bar{x})$	variable heat transfer coefficient $\left(\frac{W}{m^2\ K}\right)$
$(h_f)_0$	constant heat transfer coefficient $\left(\frac{W}{m^2\ K}\right)$
h_m	mass transfer coefficient
h_{sf}	latent heat of diffusion $\left(\frac{J}{kg}\right)$
I	inertia parameter
$\vec{J}$	electrical current density
k_p	permeability of the porous medium $\left(m^2\right)$

K	chemical reaction parameter
$\bar{K}$	fluid consistency
$k_0(\bar{x})$	variable reaction rate constant $\left(\frac{1}{S}\right)$
k_0	constant reaction rate constant $\left(\frac{1}{S}\right)$
K_T	thermal diffusion ratio
$k(T)$	temperature dependent thermal conductivity $\left(\frac{W}{mK}\right)$
k_∞	constant thermal conductivity $\left(\frac{W}{mK}\right)$
k_1	Rosseland mean absorption coefficient $\left(\frac{1}{m}\right)$
L	characteristics length (m)
Le	Lewis number
m	power law parameter
m_{FS}	Falkner-Skan power law parameter
m_{in}	index parameter
M	magnetic field parameter
Ma	Mach number
Me	melting parameter
m_w	mass flux $\left(\frac{kg}{m^2\ s}\right)$
n	order of chemical reaction
N	buoyancy ratio parameter for regular fluid
N_1	velocity slip factor $\left(\frac{s}{m}\right)$
$(N_1)_0$	constant velocity slip factor $\left(\frac{s}{m}\right)$
Nr	buoyancy ratio parameter for nanofluid
Nt	thermophoresis parameter
Nb	Brownian motion parameter

$Nu_{\bar{x}}$	local Nusselt number
p	pressure (Pa)
Pr	Prandtl number
Pe	Péclet number
q_r	radiative heat flux $\left(\frac{J}{m^2\ s}\right)$
q_w	heat flux $\left(\frac{J}{m^2\ s}\right)$
Q	generation/absorption parameter
Q_0	heat generation/absorption constant $\left(\frac{J}{m^3\ K\ s}\right)$
R	radiation parameter
Ra	Rayleigh number
$Ra_{\bar{x}}$	local Rayleigh number
Re	Reynolds number
$Re_{\bar{x}}$	local Reynolds number
S	thermal conductivity parameter
Sr	Soret number
Sc	Schmidt number
$Sh_{\bar{x}}$	local Sherwood number
Shr	reduced Sherwood number
$\bar{t}$	dimensional time (s)
T	dimensional temperature within the boundary layer (K)
T_s	temperature of solid porous media (K)
T_f	temperature of the hot fluid (K)
T_m	melting temperature (K)
T_M	mean temperature (K)

$\bar{u}$	dimensional velocity components along $\bar{x}$-axis $\left(\frac{m}{s}\right)$
$\bar{u}_e$	dimensional velocity at the edge of the boundary layer $\left(\frac{m}{s}\right)$
$\bar{u}_w$	dimensional velocity of the plate $\left(\frac{m}{s}\right)$
U_r	reference velocity $\left(\frac{m}{s}\right)$
$\bar{u}_\infty$	free stream velocity $\left(\frac{m}{s}\right)$
V	velocity ratio parameter
$\bar{v}$	dimensional velocity components along $\bar{y}$-axis $\left(\frac{m}{s}\right)$
$\bar{v}_w$	dimensional suction/injection velocity $\left(\frac{m}{s}\right)$
$(\bar{v}_w)_0$	constant dimensional suction/injection velocity $\left(\frac{m}{s}\right)$
$\bar{x}, \bar{y}$	dimensional coordinates along and perpendicular to the plate (m)
X	generator of the Lie group

Greek Letters

α	thermal diffusivity $\left(\frac{m^2}{s}\right)$
α_i	real constants
α_m	molecular diffusivity $\left(\frac{m^2}{s}\right)$
α_d	dispersion thermal diffusivity $\left(\frac{m^2}{s}\right)$
$\tilde{\epsilon}$	porosity of the porous media
$\bar{\epsilon}$	emissivity of the surface
ϵ_{ET}	error tolerance
β	Hartree pressure parameter
β_T	coefficient of thermal expansion $\left(\frac{1}{K}\right)$
β_C	coefficient of mass expansion
Γ	upper incomplete Gamma function, group
Γ_1	group
τ_w	wall shear stress
δ	angle of inclination
$\bar{\delta}$	boundary layer thickness
δ_{ij}	Kronecker delta function
θ	dimensionless temperature
γ	convective heat transfer parameter
$\bar{\gamma}$	dispersion coefficient
$\dot{\bar{\gamma}}$	shear rate
γ_1	ratio of specific heats
μ	dynamic viscosity $\left(\frac{kg}{m\ s}\right)$
$\mu(T)$	temperature dependent dynamic viscosity $\left(\frac{kg}{m\ s}\right)$
μ_e	magnetic permeability of the fluid

ν	coefficient of kinematic viscosity $\left(\frac{m^2}{s}\right)$
σ	variable electric conductivity $\left(\frac{siemens}{m}\right)$
σ_0	constant electric conductivity $\left(\frac{siemens}{m}\right)$
σ_1	Stefan-Boltzman constant $\left(\frac{W}{K^4\,m^2}\right)$
$\bar{\sigma}$	heat capacity ratio
σ_v	tangential momentum coefficient
σ_T	temperature accommodation coefficients
∇	vector differential operator
λ	free/mixed convection parameter
$\bar{\lambda}$	mean free path
η	independent similarity variable
θ	dimensionless temperature function
ϕ	dimensionless concentration/volume fraction function
ψ	stream function $\left(\frac{m^2}{s}\right)$
Ω	porosity parameter
ρ	density $\left(\frac{kg}{m^3}\right)$
ξ_1, ξ_2	infinitesimals for independent variables x, y
τ_1, τ_2, τ_3	infinitesimals for dependent variables ψ, θ, ϕ

Subscripts and superscripts

$'$	differentiation with respect to η
w	condition at the wall
∞	free stream condition

PENYELESAIAN LAPISAN SEMPADAN UNTUK ALIRAN PEROLAKAN MELALUI PELBAGAI KAEDAH PENJELMAAN KUMPULAN

ABSTRAK

Dalam tesis ini, aliran perolakan lamina lapisan sempadan luar dua dimensi dengan pemindahan haba/jisim dengan pelbagai bentuk fizikal dan dengan kehadiran medan magnet, tindak balas kimia, radiasi, pelesapan kelikatan, sumber atau penenggelam haba, penyebaran, peleburan, termoresapan, gerakan Brownian dan pemanasan Joule telah dikaji. Syarat sempadan hidrodinamik yang gelincir atau tak gelincir, serta syarat sempadan haba perolakan atau haba yang gelincir telah diambil kira dalam kajian. Bendalir dianggap Newtonian (biasa dan nano), likat, mampat, hidrodinamik atau magnetohidrodinamik dan mempunyai sifat-sifat fizikal yang tetap atau berubah-ubah. Kedua-dua lapisan sempadan mantap dan tidak mantap telah diambil kira. Pembentangan yang menyeluruh telah diberikan mengenai aplikasi pelbagai transformasi kumpulan (satu parameter dan dua parameter) kepada masalah persamaan lapisan sempadan. Kumpulan transformasi berubah yang baru serta yang sedia ada dibangunkan untuk menjelmakan persamaan pengangkutan kepada bentuk persamaan serupa. Persamaan serupa itu telah diselesaikan secara berangka untuk pelbagai nilai parameter kawalan dengan menggunakan kaedah berangka Runge-Kutta-Fehlberg peringkat keempat kelima. Graf telah diplotkan untuk mempamerkan kesan parameter kawalan ke atas profil halaju, suhu, kepekatan (pecahan isipadu nanopartikel) yang tidak berdimensi, serta ke atas profil faktor geseran, kadar pemindahan haba dan kadar pemindahan jisim yang tidak berdimensi. Data berangka untuk faktor geseran, kadar pemindahan haba dan kadar pemindahan jisim telah disediakan dalam jadual bagi pelbagai nilai parameter kawalan. Medan aliran dan kuantiti lain yang penting secara fizikal telah dipengaruhi dengan ketara oleh parameter kawalan. Perbandingan yang baik telah diperoleh antara keputusan yang dilaporkan dalam tesis ini dengan kajian sebelumnya.

BOUNDARY LAYER SOLUTIONS FOR CONVECTIVE FLOW VIA VARIOUS GROUP TRANSFORMATION METHODS

ABSTRACT

In this thesis, two-dimensional laminar convective external boundary layer flow with heat/mass transfer under various physical configurations and in the presence of magnetic field, chemical reaction, radiation, viscous dissipation, heat source or sink, dispersion, melting, thermophoresis, Brownian motion and Joule heating have been investigated. Velocity slip or no slip boundary conditions, the thermal convective or thermal slip boundary conditions have been taken into consideration. The fluid is assumed to be Newtonian (regular and nano), viscous, incompressible, hydrodynamic or magnetohydrodynamic and has constant or variable physical properties. Both steady and unsteady boundary layers have been taken into account. A thorough presentation of the applications of various transformation group (one parameter and two parameters) to the problem of boundary layer equations is given. New as as well as existing group invariant transformations are developed to transform the transport equations to similarity equations. The similarity equations have been solved numerically by the Runge-Kutta-Fehlberg fourth-fifth order numerical method for various values of the controlling parameters. Graphs have been plotted to exhibit the effects of the controlling parameters on the dimensionless velocity, temperature, concentration (nanoparticles volume fraction) profiles as well as on the the skin friction factor, rate of heat transfer and rate of mass transfer. The numerical data for the skin friction factor, rate of heat and rate of mass transfer have been provided in tables for various values of the governing parameters. The flow field and other quantities of physical interest were significantly influenced by the controlling parameters. Good agreement was found between the results reported in this thesis and published results from the open literature.

CHAPTER 1

GENERAL INTRODUCTION

1.1 Introduction

This thesis is concerned with a theoretical study of the two-dimensional steady and unsteady laminar purely hydrodynamic and magnetohydrodynamic external convective boundary layer flow with heat/mass transfer along a vertical, horizontal, an inclined flat plate and a wedge subject to different boundary conditions. The working fluid is assumed to be incompressible and Newtonian. The contents consist of analysis of twelve distinct problems described in six separate chapters.

1.2 Transport Equations and Boundary Layer

In general, convective heat and mass transfer problems are governed by a system of partial differential equations (PDEs) (linear or nonlinear) with different initial and boundary conditions. Nonlinearities of the governing equations present a special challenge to engineers, mathematicians, computer scientists and physicists. It is often difficult and sometimes even impossible to find their solutions using classical methods such as separation of variables, free parameter and dimensional analysis. Hence, engineers, mathematician, computer scientist, physicists, and applied mathematicians try to find the ways and means to reduce the PDEs into its corresponding ordinary differential equations (ODEs) with their boundary conditions to get the solutions of their problems.

Boundary layer is a very thin region adjacent to the surface over (or under) which fluid is flowing where the viscous, thermal conductivity and mass diffusivity effects are important. The flow outside the boundary layer is known as potential flow, where the viscous, thermal conductivity and mass diffusivity effects, are not significant. The detailed of the boundary layer approximation will be provided in Section 2.7.

1.3 Solution Techniques of Boundary Layer Equations

To solve the boundary layer equations, it is better to reduce them into simplified forms. In doing so, the first step is to study the possible similarity form of the equations and the corresponding similarity equations. Similarity independent variable is the combination of original independent variables. The objectives of seeking similarity solutions are twofold. Firstly, the PDEs for a given problem are reduced to the ODEs. By this means, it is possible to obtain a number of analytical or numerical solutions. Secondly, the results obtained by similarity solutions may be directly useable in the technical arena (Na, 1979; Ames, 1972; Seshadri and Na, 1985). A vast literature of similarity solutions has appeared in the arena of fluid mechanics, convective heat and mass transfer and aerodynamics. Most existing solutions, in the technical arena, are similarity solutions in the sense that the pertinent boundary layer equations along with relevant boundary conditions under suitable transformations are reduced to a set of ODEs in terms of similarity variable. Similarity variables may be derived by dimensional arguments, by sophisticated group theoretic method, by method of free parameter or by separation of variables. Among them, the group theoretic method which includes the dimensional analysis as special case is the most powerful, sophisticated and systematic to generate similarity transformations of the transport equations (Ames, 1972; Hansen, 1964; Seshadri and Na, 1985). In the case of group theory, the similarity solution is the invariant solution of initial and boundary value problems. Group invariant transformations do not change the structural form of the equations under investigation. Of late, the group-theoretic approach to PDEs or ODEs with auxiliary conditions is widely applied in various fields of mathematics, mechanics, and theoretical physics and many results published in these area demonstrates that group theory is an efficient tool for solving intricate problems formulated in terms of differential equations (Jalil et al., 2010; Bluman et al., 2009). Numerical methods for the solutions of nonlinear ODEs are important and nowadays several software packages such as Maple, Mathematica and Matlab are available to obtain such solutions.

1.4 Heat and Mass Transfer Analysis of Regular Steady Flow

Transfer of heat and mass from various geometries located in non-porous or porous media occured in lots of engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and underground energy transport (Bergman et al., 2011). In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously (Nield and Bejan, 2006; Nakayama, 1995). The study of hydrodynamic flow with the applications of magnetic field, also known as magnetohydrodynamic (MHD) flows is significant for industrial, technological, and geothermal applications, such as high-temperature plasmas, cooling of nuclear reactors, liquid metal fluids, MHD accelerators and power generation systems. The rate of cooling and the desired properties of the end product can be controlled by the use of electrically conducting fluid and application of magnetic fields. The idea of MHD is that magnetic fields can induce currents in a moving conductive fluid, which create forces on the fluid, and also change the magnetic field itself (Herdricha et al., 2006; Hoernel, 2008). The study of MHD flow with slip boundary conditions and concentration dependent mass diffusivity is not available in the literature.

Thermal radiation effect is important in systems consisting of solid and gases. Thermal radiation within these systems is usually the result of emission by the hot walls and the gas-particle mixture. This radiation undergoes complex interaction with the system, primary due to absorption and scattering processes. Radiation must be considered in calculating thermal effects in many engineering processes occurring at high temperatures, such as nuclear power plants, gas turbines, propulsion devices for aircraft, missiles, satellites and space vehicles, and various devices for space technology (Qatanani and Schulz, 2004). In the case of radiative heat transfer process, the fluid is electrically conducting as it is ionized due to the high working temperature (Aboeldahab and Azzam, 2005).

There are three types of convective flow which are free, forced and combined. Free convective flow occurs in atmospheric and oceanic circulation, electronic machinery, heated or cooled enclosures, electronic power supplies etc. It has many applications such as its influence on operating temperatures of power generating and electronic devices (Bejan, 2004; Bergman et al., 2011). It plays a great role in thermal manufacturing applications and is important in establishing the temperature distribution within buildings as well as heat losses or heat loads for heating, ventilating and air conditioning systems (Bejan, 2004).

The are some situations where fluid flow is caused by external agent which is known as forced convection. Forced convective flows have many practical applications in various branches of industry, such as the metallurgical, chemical, mechanical, electrical and food industries. The heat and mass transfer on surfaces such as those of boilers, heating and smelting furnaces, heat exchangers, condensers, and other types of industrial and civic equipments is often caused by various forms of forced convection under large temperature and concentration differences (Al-Sanea, 2004).

A situation where forced and free convection are of comparable magnitude, known as mixed or combined convective flow. In nature as well as in many technological devices, situations arise where forced and free convective flow act in a simultaneous manner to establish the flow regime, the temperature and the concentration fields around a heated or cooled permeable or impermeable plate or a stretching sheet. Example of mixed convection include flow in electronic equipment cooled by a fan and flows in the ocean (because of tide) and in the atmosphere (because of wind flow) (Moulic and Yao, 2009; Sengupta et al., 2011).

In reality, the presence of pure air or water is scarce. Some foreign mass may be present either naturally or mixed with the air or water. There are lots of transport process existing in nature and in industrial applications in which heat and mass transfer is a consequence of buoyancy effects caused by diffusion of heat and chemical species. The study of such processes is useful for improving a number of chemical technologies, such as polymer production and food processing (Chamkha and Nakhi, 2008). Chemical reactions can occur in processes such as drying, distribution of the temperature and moisture over agricultural fields, damage of crops due to freezing, energy transfer in a wet cooling tower and flow in a desert cooler (Pal and Mondal, 2011). Chemical reactions can be classified as either homogeneous or heterogeneous processes. A homogeneous chemical reaction is one that occurs uniformly throughout a given phase. On the other hand, a heterogeneous chemical reaction takes place in a restricted area or within the boundary of a phase (Patil and Kulkarni, 2008). A reaction between two gases, two liquids or two solids is homogeneous. A reaction between a gas and a liquid, a gas and a solid or a liquid and a solid is heterogeneous (Levenspiel, 1999). A few representative fields of interest in which combined heat and mass transfer plays an important role are in design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, food processing and cooling towers (Seddeek and Salem, 2005).

1.5 Heat and Mass Transfer Analysis of Regular Unsteady Flow

Processes of double-diffusive transport phenomena are associated with buoyancy driven flows induced by the combined temperature and concentration gradients. It occurs in nature and industry (Pop and Ingham, 2001). In nature, such flows are encountered in oceans, lakes, shallow coastal water, and the atmosphere. Double-diffusive convection also occurs in the sun where temperature and helium diffusions take place at different rates. Convection in magma chambers and sea-wind formations are among other manifestations of double-diffusive convection in nature (Mojtabi and Charrier-Mojtabi, 2005). In industry, the examples of this type of convection are the chemical processes, crystal growth, the dispersion of contaminants through water-saturated soil, the underground disposal of nuclear wastes, the formation of microstructures during the cooling of molten metals, solidification, food processing, and migration of impurities in non-isothermal material processing applications (Nield and Bejan, 2006).

Thermophoresis is a phenomenon which causes small particles to be driven away from a hot surface and towards a cold one. Technological problems include particle deposition onto a surface from a condensing vapor-gas mixture, a semi-conductor wafer in the electronic industry, blade surface of gas turbines, and problems for nuclear reactor safety (Kandasamy et al., 2011). Thermophoresis is the dominant mass transfer mechanism in the modified chemical vapor deposition process used in the fabrication of optical fiber and also important in view of its relevance accidents by radioactive particle deposition in nuclear reactors (Seddeek and Salem, 2005).

1.6 Nanoparticles Volume Fraction Analysis of Nanofluid

In many industries and engineering applications such as power generator, manufacturing and transportation, fluid heating and cooling are important. Usually, fluids are used to transport the generated heat. However, conventional heat transfer fluids (e.g., water, ethylene glycol, engine oil etc) have poor thermal conductivity. They require high velocities or complex geometries to efficiently transfer the generated heat from a given surface to the fluid. Nanofluid is suspension of nanoparticles in the base fluid. The thermal properties of nanoparticles are superior to those of conventional fluids. Due to small size and very large specific surface area of the nanoparticles, nanofluids have better thermophysical properties such as high thermal conductivity, minimal clogging in flow passages, long term stability and homogeneity. Improved thermo-physical properties can enhance the efficiency and the performance of the nanofluid and hence reduce the operating cost, as expected in the industries (Khanafer and Vafai, 2011). Nanofluids show diverse applications in a variety of areas due to their superior thermo physical properties (thermal conductivity, thermal diffusivity, viscosity, and convective heat transfer coefficients, minimal clogging in flow passages, long term stability and homogeneity): industrial cooling applications (e.g., electronics cooling, vehicle cooling, transformer cooling, super powerful and small computers cooling and so on), advanced nuclear systems, nuclear reactors, transportation industry (e.g., an automobiles, trucks, and airplanes), microelectromechanical systems, microelectronic fuel cell, hybrid power engine, biomedical applications (nano-drug delivery, cancer therapeutics, cryopreservation), process industries: materials and chemicals, detergent, food and drink and oil gas amongst others (Rana and Bhargava, 2012; Wong and Leon, 2010; Khanafer and Vafai, 2011). Nanobiotechnology is also a fast developing field of research and application in many domains such as in medicine, pharmacy, cosmetics and agro-industry (Schaefer, 2010). Advances in nanoelectronics, nanophotonics, and nanomagentics have seen the arrival of nanotechnology as a distinct discipline in its own right (Sharma, 2010).

1.7 Problem Statement

The present study focuses on the problems of convective (forced, free, mixed) heat/mass transfer external laminar boundary layer flow of a viscous incompressible flow of Newtonian (conventional and nanofluid) fluids along flat plates and wedge located in a porous and non-porous media under the influence of various effects. These include magnetic field, variable viscosity, variable thermal conductivity, variable mass diffusivity, porosity, variable velocity slip, variable thermal slip, buoyancy force, thermal radiation, heat generation/absorption, chemical reaction, thermophoresis, Brownian motion, melting effect, and suction/injection effects. Various boundary conditions such as no slip, slips, thermal convective boundary conditions are taken into account. Both steady and unsteady flow have been studied.
The thesis focuses on the following problems.

1. Lie Group Analysis of Boundary Layer Flow along a Vertical Plate with Variable Mass Diffusivity, Velocity Slip and Thermal Convective Boundary Conditions.

2. MHD Boundary Layer Flow in Porous Media along a Vertical Plate with Variable Mass Diffusivity and Velocity Slip.

3. MHD Forced Convective Flow along a Moving Permeable Radiating Vertical Flat Plate.

4. MHD Forced Convective Flow along a Permeable Radiative Wedge with Temperature Dependent Viscosity and Thermal Conductivity.

5. Combined Heat and Mass Transfer Analysis by Free Convective Flow along a Moving Permeable Vertical Flat Plate with Convective Boundary Condition.

6. Heat Transfer Analysis by Free Convective Flow along a Moving Horizontal Flat Plate with Convective Boundary Condition.

7. Mixed Convective Boundary Layer Flow past a Permeable Vertical Flat along with Slip Boundary Conditions.

8. MHD Mixed Convective Flow along a Permeable Inclined Radiating Flat Plate with the Temperature-Dependent Thermal Conductivity, Variable Reactive Index, and Heat Generation/Absorption.

9. Impact of Melting and Dispersion on Unsteady Boundary Layer Flow past a Vertical Plate in Porous Media.

10. Melting Effects on Combined Heat and Mass Transfer past a Vertical Plate in Porous Media.

11. Free Convection Boundary Layer Flow past a Heated upward facing Horizontal Flat Plate Embedded in a Porous Media Filled by a Nanofluid and with a Convective Boundary Condition.

12. MHD Boundary Layer Slip Flow of a Nanofluid past a Convectively Heated Stretching Sheet with Heat Generation/Absorption.

Researchers normally apply the conventional no slip boundary conditions at the wall surface over which fluid flow but there are some situations where no slip conditions lead to unrealistic behavior-for example, the spreading of a liquid on a solid substrates, corner flow and the extrusion of polymer melts from a capillary tube (Thompson and Troian, 1997). No slip condition must be replaced by slip condition when fluid flows around microfluidic and nanofluidic (Nguyen and Wereley, 2009).

The difference between the fluid velocity at the wall and the velocity of the wall itself is directly proportional to the shear stress. The proportional factor is called the slip length. The velocity slip boundary condition is $|u| = u_w + N_1 \left|\frac{\partial u}{\partial y}\right|$, where u_w is the velocity of the surface, $N_1 > 0$ is the velocity slip factor (Hak, 2002). The corresponding thermal slip boundary condition is $T = T_w + D_1 \frac{\partial T}{\partial y}$, where T_w is the wall temperature of the surface, $D_1 > 0$ is the thermal slip factor (Hak, 2002). For gaseous flow the slip condition of the velocity and the jump condition of the temperature are $|u|_{\text{wall}} = \bar{\lambda}\frac{2-\sigma_v}{\sigma_v}\left|\frac{\partial u}{\partial y}\right| + \frac{3\mu}{4\rho T_{\text{gas}}}\left|\frac{\partial T}{\partial x}\right|$ and $T_{\text{wall}} = \frac{2-\sigma_T}{\sigma_T}\frac{2\gamma_1}{\gamma_1+1}\frac{\bar{\lambda}}{Pr}\frac{\partial T}{\partial y}$, where σ_v, and σ_T are the tangential momentum coefficient, the temperature accommodation coefficients, $\bar{\lambda}$ is the mean free path, γ_1 is the ratio of specific heats.

1.8 Objectives and Methodology

Mathematical model is required to explain real world situation in detail for useful analysis and interpretation. The main objectives of the thesis are:

(i) to formulate new mathematical models of the problem of convective boundary layer flow along flat plates and wedge in nonporous and porous media subject to the various effects and various boundary conditions,

(ii) to develop new as well as existing similarity transformations and corresponding similarity solutions of the transport equations using various group transformation methods and solve them numerically,

(iii) to investigate the effects of the pertinent controlling parameters on the dimensionless velocity, temperature and concentration/nanoparticle volume fraction profiles as well as on the dimensionless friction factor, rate of heat and rate of mass transfer at the wall.

The detail of the objectives will be provided in each chapters separately.

The methodologies are:

(i) transform the transport equations including pertinent boundary conditions into dimensionless form using suitable dimensionless variables,

(ii) use appropriate group transformation method to develop the similarity transformations and hence the similarity solutions,

(iii) solve the similarity equations numerically and draw the graphs of the dimensionless velocity, temperature, concentration/nanoparticle volume fraction, friction factor, heat transfer rates and mass/nanoparticle volume fraction transfer rates to show the effect of various parameters on them.

1.9 Scope and Importance

The present study is confined to the problem of two-dimensional steady and unsteady external laminar boundary layer flow with convective heat/mass transfer of a viscous, incompressible, electrically conducting and nonconducting Newtonian fluid along a flat plates (horizontal, vertical or an inclined) or wedge in porous and non-porous media. Some prescribed parameters, such as a permeability, viscosity, thermal conductivity, mass diffusivity, power law index, velocity slip, thermal slip, thermal convective, radiation, dissipation, Joule heating, heat generation/absorption, magnetic field, buoyancy ratio, chemical reaction, order of chemical reaction, velocity ratio and thermophoresis, Brownian motion, melting, dispersion, unsteadiness, suction/injection are included in the study of the boundary layers behaviors. For porous media, the governing PDEs are approximated by Darcy and non-Darcy flow models. The nanofluids model incorporate the thermophoresis and Brownian motion effects. The similarity transformations generated by group methods are used to transform the transport equations into similarity equations with boundary conditions. The numerical scheme, the Runge-Kutta-Fehlberg fourth-fifth (RKF45) order method is used to solve the similarity equations.

The study finds applications in boundary layer control, in the polymer industry, cooling problems, solar power technology, nuclear reactors, polymeric materials processing and to reduce the drag in aerodynamics etc. The details applications of each problems are given in the relevant chapters.

1.10 Structure of the Thesis

This thesis consists of an introductory chapter and eight main chapters. Chapter 1 provides a general introduction, applications and several physical features involved in the study of convective heat/mass transfer external boundary layer flow (steady and unsteady) of regular fluid and nanofluid in porous media or non-porous media. Chapter 2 presents the basic concepts on convective heat and mass transfer and various group-theoretic transformation methods.

Chapter 3 discusses boundary layer flow with heat and mass transfer. Section 3.4 investigates Lie group analysis of boundary layer flow. Section 3.5 analyzes MHD flow with combined effect of radiation and magnetic field with Joule and dissipation effect via scaling group of transformation. Concentration dependent mass diffusivity and velocity slip boundary condition have been taken into account to formulate the problems.

In Chapter 4, forced convective flow with radiation and thermal convective boundary condition has been investigated. Section 4.4 focuses on similarity solutions of forced convective flow over a moving vertical flat plate in the moving free stream via linear group of transformation. Section 4.5 examines forced convective flow over a wedge with the temperature dependent viscosity and thermal conductivity via scaling group of transformation. In formulating the problems the fluid is assumed to be an electrically conducting. The thermal convective boundary condition and thermal radiation effect are taken into account.

Chapter 5 focuses on the investigation of free convective boundary layer flow of a moving flat plate. Section 5.4 presents heat and mass transfer analysis by free convection flow along a vertical flat plate in the quiescent free stream. Group method followed by boundary layer concept has been applied to develop similarity transformation. Section 5.5 deals with heat transfer analysis of free convection flow along a horizontal moving flat plate via one parameter deductive group method. The thermal convective boundary condition has been used in formulating both problems.

The aim of the Chapter 6 is to study mixed convective flow over vertical and an inclined flat plate in the moving free stream. In Section 6.4, Lie group analysis has been used to study heat and mass transfer analysis of mixed convective flow over a permeable vertical plate with the velocity and thermal boundary conditions. In Section 6.5, scaling group of transformation has been used to study heat and mass transfer by MHD mixed convective flow over a convectively heated permeable inclined radiating flat plate with heat generation or absorption taking into account the temperature dependent thermal conductivity and higher order homogenous chemical reaction.

Chapter 7 concentrates on the study of unsteady flow in porous media. Section 7.4 deals with the study of the effects of melting, thermal dispersion and inertia on unsteady combined convective flow over a vertical plate embedded in a saturated non-Darcy porous media for both aiding and opposing external flows. Two parameters group method developed by Hamad (2011) has been applied to explore new similarity transformations. Section 7.5 investigates simultaneous heat and mass transfer with double-diffusion effect, by unsteady combined convective flow near a vertical plate embedded in a porous media saturated by Newtonian fluid with dispersion and melting effect taking into account concentration dependent mass diffusivity. New similarity transformations and corresponding similarity solutions of the governing equations have been obtained by the group method followed by boundary layer concept.

Chapter 8 covers boundary layer flow of a nanofluid. Section 8.4 deals with free convection flow from a convectively heated upward facing horizontal flat plate embedded in a porous media filled with a nanofluid. Section 8.5 deals with MHD boundary layer slip flow of a nanofluid over a convectively heated stretching sheet with heat generation/absorption effects. In both cases, linear group of transformation have been used to explore the similarity transformations and corresponding similarity equations.

In Chapters 3 to 8, the related previous works are reviewed in brief in the literature review section. In each chapter the similarity equations have been solved numerically by the Runge-Kutta-Fehlberg fourth-fifth (RKF45) order numerical method . Finally, Chapter 9 consists of conclusions of the thesis and possible further works in this field.

CHAPTER 2
BASIC CONCEPTS

2.1 Introduction

This chapter deals with the basic properties of the fluid, the basic transport equations and their reduction to Prandtl boundary layer equations. The order of magnitude analysis of the basic equations are performed to neglect small order terms. The important dimensionless numbers used in this thesis are also included. This chapter also includes basic concepts of various group methods.

2.2 Newtonian and Non-Newtonian Fluids

Newton's law of viscosity states that shear stress is proportional to the tangential velocity gradient. Fluids obeying Newton's law of viscosity are known as Newtonian fluids, i.e.,

$$\tau_{y,x} = \mu \frac{du}{dy} = \mu \dot{\tilde{\gamma}} \tag{2.2.1}$$

where τ is the shear stress, μ is the dynamic coefficient of viscosity, x indicates the direction of the shear stress, y the direction of the velocity gradient, $\dot{\tilde{\gamma}}$ is the shear rate. For Newtonian fluid, the dynamic viscosity is independent of the shear rate. Equation (2.2.1) is known as a constitutive equation. If $\tau_{y,x}$ is plotted against $\dot{\tilde{\gamma}}$, the result is a linear relation whose slope is the dynamic viscosity μ. Examples of Newtonian fluid are: water, air, benzene, ethyl alcohol, most oils and most solutions of low molecular weight. There are many fluids which do not obey the Newton's law of viscosity, i.e., equation (2.2.1). These fluid are termed as non-Newtonian fluids. Such fluids include honey, gel, drilling mud, mayonnaise, peanut butter, toothpaste, egg whites, liquid soaps, multi grade engine oils, various suspensions such as coalwater or coaloil slurries, food products, inks, glues, soaps, polymer solutions, etc. Non-Newtonian fluids show various behaviors: time-independent (Bingham plastic, pseudo-plastic and dilatant fluid), time-dependent (thixotropic and rheopectic fluids), viscoelastic fluid (e.g., egg white). Viscoelastic fluids possess

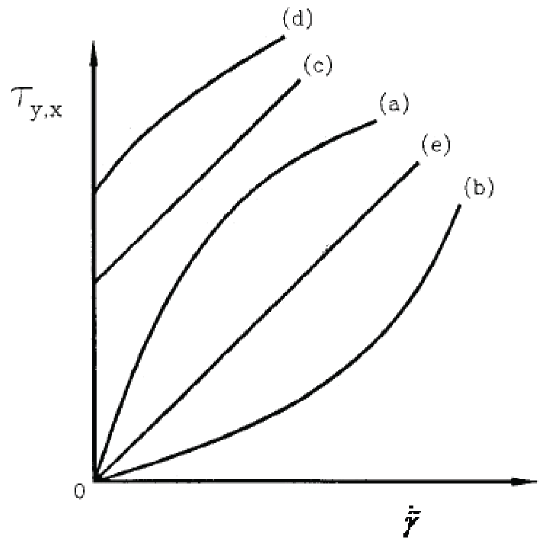

Figure 2.1: Flow curves of purely viscous, time-independent fluids: (a) pseudoplastic; (b) dilatant; (c) Bingham plastic; (d) Herschel-Bulkley; (e) Newtonian.

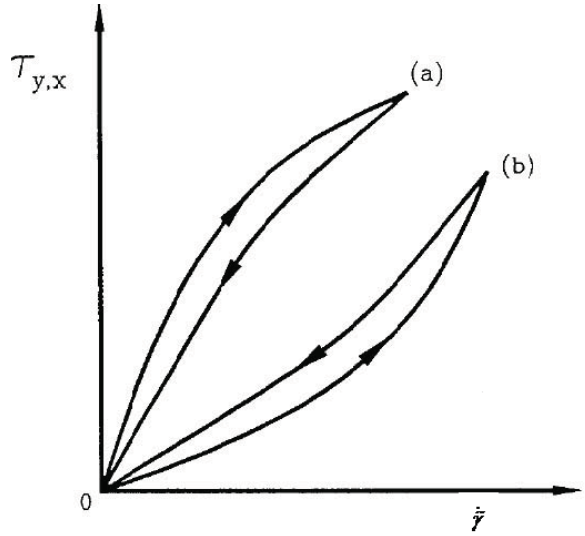

Figure 2.2: Flow curves for purely viscous, time-dependent fluids: (a) thixotropic; (b) rheopectic.

both viscous and elastic properties, have received considerable attention because of their ability to reduce both drag and heat transfer in channel flows. Purely viscous time-dependent fluids are those in which the shear stress in a function only of the shear rate but in a more complicated manner than that described in equation (2.2.1). Figure 2.1 exhibits the characteristics of purely viscous time-independent fluids. In the Figure 2.1, (a) and (b) are fluids where the shear stress depends only on the shear rate but in a nonlinear way. Fluid (a) is called pseudoplastic (or shear

thinning), and fluid (b) is called dilatant (or shear thickening). Curve (c) is one which has an initial yield stress after which it acts as a Newtonian fluid, called Bingham plastic, and curve (d), called Herschel-Bulkley, also has a yield stress after which it becomes pseudoplastic. Curve (e) depicts a Newtonian fluid (Kreith and Goswami, 2005). Figure 2.2 shows flow curves for two common classes of purely viscous time-dependent non-Newtonian fluids. The behavior of such fluids depends upon the time-dependent rate at which the shear stress is applied. Curve (a) shows a pseudoplastic time-dependent fluid and curve (b) a dilatant time-dependent fluid. They are called, respectively, thixotropic and rheopectic fluids and are complicated by the fact that their flow curves are difficult to characterize for any particular application. Several model are used to explain behavior of non-Newtonian fluid. Some of this models are; power law fluids, Sisko fluids, Ellis fluids, Prandtl fluids Williamson fluids, Sutterby fluids, Reiner-Rivlin fluids, Bingham plastic, Eyring fluids, Powell-Eyring fluids, Reiner-Philippoff fluids etc. (Patel and Timol, 2012).
Ostwald-de-Waele model (power-law model) is the most popular model.
Ostwald-de-Waele model is

$$\tau = \bar{K} \left| \dot{\bar{\gamma}} \right|^{n_{pl}-1} \dot{\bar{\gamma}} \tag{2.2.2}$$

Here, $\bar{K}$ is the fluid consistency and n_{pl} is the flow index (power law parameter). Note that if $n_{pl} = 1$, the fluid becomes Newtonian and $\bar{K}$ becomes the dynamic viscosity μ. Because of its simplicity, the power law constitutive equation has been most often used in rheological studies, but at times it is inappropriate because it has several inherent flaws and anomalies.

2.3 Nanofluids

Nanofluid is a suspension of ultrafine solid nanoparticles such as small amounts of metal, nonmetal or nanotubes with sizes normally less than 100 nm in diameter with the base fluid such as water, ethylene glycol, and oil. Nanofluids exhibit enhanced thermophysical properties such as thermal conductivity, thermal diffusivity, viscosity and convective heat transfer coefficients compared to those of base fluids.

Nanoparticles are made from various materials, such as oxides (Al_2O_3, CuO), nitrides (AlN, SiN), carbides (SiC, TiC), semiconductors (TiO_2, SiC), metals (Cu, Ag), carbon nanotubes and composite materials such as alloyed nanoparticles or nanoparticle core-polymer shell composites. Nanofluids aim to achieve the maximum possible thermal properties at the minimum possible concentrations (preferably 1% by volume) by uniform dispersion and stable suspension of nanoparticles (preferably$<$ 10 nm) in base fluids (Wang and Mujumdar, 2007). With the ongoing technology development of miniaturization of both microelectro mechanical and microfluidic devices, there is a growing interest in higher thermal conductivities of heat transfer fluids in industry. The idea of improving heat transfer performance of fluids with the inclusion of solid particles was first introduced by Maxwell (1904). However, suspensions involving milli-or microsized particles create problems, such as fast sedimentation, clogging of channels, high pressure drop, and severe erosion of system boundaries. Choi (1995) has shown that these problems can be overcome by using nanofluids. Modern material technologies can be used to manufacture nanometer-sizes particles. Due to small sizes and very large specific surface areas of the nanoparticles, nanofluids have superior thermophysical properties like high thermal conductivity, minimal clogging in flow passages, long term stability, and homogeneity. Nanotechnology can be treated as one of the most significant driving forces for the next major industrial revolution of current century.

2.4 Heat Transfer

Heat transfer is thermal energy transfer due to a temperature difference. Heat transfer is of particular interest to engineers, who attempt to understand and control the flow of heat through the use of thermal insulation (thermal design), heat transfer enhancement (heat exchangers design), temperature control, and other devices (Cengel, 2006). In thermal insulation, the ultimate objective is to minimize heat transfer rate while in heat exchanger design, the objective is to improve the thermal contact between the heat exchanging entities, that is, to minimize the temperature difference, because in this way the rate of entropy generation is reduced. This can be done by changing the flow patterns of the two streams and the shapes of the solid surfaces bathed by fluid streams. In the temperature control design problem, the heat transfer rate and flow configuration must vary in such way that ambient temperature does not exceed a certain ceiling value. There are three types of thermal energy transport: (i) conduction, (ii) convection and (iii) radiation. In various types of studies related to heat transfer or thermal transport, considerable effort has been directed in the convection mode, in which heat transfer process takes place with the motion of the fluid.

2.4.1 Conduction

Conduction is the process of heat transfer by molecular motion, supplemented in some cases by the flow of free electrons, through a body (solid) from a region of high temperature to a region of low temperature. Heat transfer by conduction also takes place across the interface between two bodies in contact when they are at difference temperature. Conduction is greater in solids, where atoms are in contact. In liquids (except liquid metals) and gases, the molecules are usually further apart, giving a lower chance of molecules colliding and passing on thermal energy. The heat transfer rate must be vanished when the medium is equilibrium.
Conduction heat transfer is described macroscopically by Fourier's law, which is

$$\vec{q} = -k(T)Ar\nabla T, \tag{2.4.1}$$

where $\vec{q}$ is the quantity of heat transmitted, T is the temperature, $k(T)$ is the thermal conductivity, Ar is the area through which the heat is flowing, $\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$ is the vector differential operator. The magnitude of thermal conductivity for a given substance very much depends on its microscopic structure and also tends to vary somewhat with temperature. Metals are usually the best conductors of thermal energy. However, fluids (liquids and gasses), except liquid metals, are not typically good conductors. This is due to the large distance between atoms in a gas:

fewer collisions between atoms means less conduction. As density decreases so does conduction. Conductivity of gases increases with temperature but only slightly with pressure near and above atmospheric. Conduction does not occur at all in a perfect vacuum.

2.4.2 Convection

Heat convection is the term applied to process involved when the energy is transferred from a surface to a fluid flowing over it as a result of a difference between the temperatures or concentration of the surface and the fluid. In convection, therefore, there is always a surface, a fluid flowing relative to this surface and a temperature difference between the surface and the fluid. Convection heat transfer occurs extensively in practice. The cooling of the cutting tool during a machining operation, the cooling of the electronic components in a computer, the generation and condensation of steam in a thermal power plant, the heating and cooling of buildings, cooking, and the thermal control of a re-entering spacecraft, all, for example, involve convective heat transfer. Convective heat transfer is divided broadly into three basic process, (i) free convection, (ii) forced convection, and (iii) mixed convection.

2.4.2.1 Free Convection

Free convection flow arises when buoyancy forces due to density difference occur and these acts as a driving forces. In this case fluid motion caused entirely by buoyancy force that arises due to density changes resulting from the temperature variations or concentration differences of the flow. Buoyancy forces may act in different force fields, the gravitational field being most common, the centrifugal force field, the Coriolis force field and the electromagnetic force field are also found in nature. It occurs in atmospheric circulation, nuclear reactor, cooling system, electronic power supplies and so forth. One example of free convection is the flow generated by fire. Free convection flow velocities are generally much smaller than those associated with the forced convection and heat transfer rate is generally smaller. In brief, one can say that free convection flow results from the action of body forces on the fluid, that is, forces which are proportional to the mass or density of the fluid. A heated body cooling in the ambient air produces free convection flow in the region surrounding it. Considerable effort has been given to the convective heat transfer because of its importance in technical applications in which the relative motion of the fluid provides an additional mechanism for the transfer of energy and of the material, the latter being a more important consideration in cases where mass transfer due to concentration difference occurs. Free convective heat transfer is important only when there exist no external flow. It is expected from dimensional analysis that for large Reynolds number (Re) (i.e., for large flow velocity) and small Grashof number (Gr), the influence of free convection on heat transfer can be neglected. On the other hand, for large Grashof number and small Reynolds number the free convection would be a dominating factor. The free convection process is present in various physical phenomena such as fire engineering, combustion modeling, nuclear energy, heat exchangers, petroleum reservoir etc.

2.4.2.2 Forced Convection

If the fluid motion arises due to an external agent, such as the externally imposed flow of a fluid stream by a fan, a blower, the winds or the motion of the heated objects itself, the process is known as forced convection. It occurs in electronic devices which are not classified as heat exchangers, such as furnaces with artificial draft and regenerators. Examples are central heating, air conditioning, heat exchangers etc.

Buoyancy causes variations in the velocity and temperature fields of forced convection flows leading to the variations in the Nusselt number and the wall shear stress or friction coefficient, parameters that are important for most engineering problems. For case of upward forced convection over a flat plate with surface heated to a temperature higher than the surrounding temperature, the buoyancy forces aid convective motion whereas if the surrounding temperature is greater than surface temperature, buoyancy forces oppose the flow. Here Reynolds number Re is the important parameter.

2.4.2.3 Mixed Convection

In nature, there are some situations where forced and free convection act simultaneously in establishing the flow and temperature or concentration field near the heated or cooled plate body. That is, if the relative importance of the forced and free convection are of comparable order; the phenomena may be termed as mixed convection flow. The laminar boundary layer flow due to mixed convection has received considerable attention for both steady and unsteady situation in evaluating flow parameters for technical purposes. Mathematically, if the relative importance of the forced and free convection is of comparable order $Re \approx Gr^2$; the phenomena may be termed as mixed convection flow. Here Re is the Reynolds number and Gr is the Grashof number. In recent years, the transfer of heat to and from enclosed or partially enclosed regions by means of free convection or by a combination of natural and forced convection has taken a new significance in the field of aeronautics, automatic power, electronics and chemical and electrical engineering.

2.4.3 Radiation

Radiation is heat transfer in the form of electromagnetic waves. All substances, solid bodies as well as liquids and gases, emit radiation as a result of their temperature and they are also capable of absorbing such energy. No medium is necessary for radiation to occur; radiation works even in and through a perfect vacuum. The energy from the sun travels through the vacuum of space before warming the earth. Also, the only way that energy can leave earth is by being radiated to space. Radiation can be viewed either in terms of electromagnetic waves or in terms of transport of photons. The medium involved can be either nonparticipating or participating. Nonparticipating media include outer space and atmospheric air over short distances, in which photons can travel almost unimpeded from one surface to another. Radiation heat exchange between such surfaces depends only on surface temperatures, surface radiation properties and the geometry of the configuration. Participating media include combustion gases containing H_2O and CO_2, as well as gases containing aerosols such as dust, soot and small liquid droplets. Radiation exchange between surfaces separated by a participating medium depends also on the radiation properties of the medium. In the case of a participating gas species, the emissivity and absorptivity depend strongly on temperature (Jones, 2000). The radiation heat transfer phenomenon is described macroscopically by a modified form of the Stefan-Boltzmann law, which is

$$\bar{Q} = \bar{\epsilon}\sigma T_s^4, \tag{2.4.2}$$

where $\bar{Q}$ is total emission, σ_1 is the Stefan-Boltzmann constant and $\bar{\epsilon}$ is a radiative property of the surface termed emissivity that characterizes how effectively the surface radiates with values in the range of $0 \leq \bar{\epsilon} \leq 1$ and T_s is the absolute temperature. This property is called the emissivity of the surface. The Stefan-Boltzmann constant σ_1 is 5.669×10^{-8}. Thermal radiation takes place according to the fourth power of the absolute temperature of the surface. Radiation heat transfer is important at high temperature such as those found in combustion systems, solar energy and nuclear reactors.

2.5 Mass Transfer

Transfer of a material through a fluid-fluid interface or a fluid-solid interface is called mass transfer. Mass transfer occur when there is concentration difference. If the concentration near the interface is not uniform, mass transfer takes place due to the effect of mass diffusion. Mathematically mass diffusion can be described by the diffusion equation. This equation is derived from Fick's law, which states that the net movement of diffusing substance form higher concentration to lower concentration, per unit area of cross section (the flux) is proportional to the concentration gradient. The constant of proportionality is the diffusion coefficient, which depends on the diffusing species and the material through which diffusion occurs. An analogous statement of Fick's law, for heat instead of concentration, is Fourier's law (Bergman et al., 2011). Mass transfer is closely related to diffusion. It may be predetermined in its shape by the specific situation, e.g., the surface of a pond from which evaporation occurs or it may be determined by the physical process itself, e.g., evaporation of a liquid in droplet from or of gas vapor bubbles in a liquid (Bergman et al., 2011). Convective mass transfer problems play an important role in nature and in many engineering problems. For example, evaporation of water at the surface of the oceans, cooling towers of power stations, ablation cooling for reentry space vehicles, combustion processes or the dissolving of a sugar cube in a cup of coffee (Kays et al., 2005).

2.6 Fundamental Transport Equations and Boundary Conditions

The analysis of convection momentum, heat and mass transfer relies on the applications of basics laws: conservation of mass, momentum, energy and species. Since the objective of the thesis is the determination of the velocity, temperature and concentration distribution, the basic laws must be formulated in an appropriate form. The fundamental equations of flow of viscous compressible fluids are the equation of continuity (conservation of mass), motions (conservation of momentum), energy (conservation of energy) and species (conservation of species).

2.6.1 Fundamental Equations in Vector Form

The four basic equations, namely, continuity, Navier-Stokes, energy and mass diffusion, in dimensional form, are (White, 2006)

$$\frac{\partial \bar{\rho}}{\partial \bar{t}} + \nabla \cdot \left(\bar{\rho} \vec{\bar{V}} \right) = 0, \tag{2.6.1}$$

$$\bar{\rho} \frac{D \vec{\bar{V}}}{D \bar{t}} = \bar{\rho} \vec{\bar{g}} - \nabla \bar{p} + \nabla \left[\bar{\mu} \left(\frac{\partial \bar{u}_i}{\partial \bar{x}_j} + \frac{\partial \bar{u}_j}{\partial \bar{x}_i} \right) + \delta_{ij} \bar{\lambda} \nabla \cdot \vec{\bar{V}} \right], \tag{2.6.2}$$

$$\bar{\rho} \frac{Dh}{D\bar{t}} = \frac{D\bar{p}}{D\bar{t}} + \nabla \cdot \left(\bar{k} \nabla T \right) + \left[\bar{\mu} \left(\frac{\partial \bar{u}_i}{\partial \bar{x}_j} + \frac{\partial \bar{u}_j}{\partial \bar{x}_i} \right) + \delta_{ij} \bar{\lambda} \nabla \cdot \vec{\bar{V}} \right] \frac{\partial \bar{u}_i}{\partial \bar{x}_j}, \tag{2.6.3}$$

$$\frac{DC}{D\bar{t}} = \nabla \cdot \left(\bar{D} \nabla C \right), \tag{2.6.4}$$

where for a linear Newtonian fluid, the viscous stresses is $\left[\bar{\mu} \left(\frac{\partial \bar{u}_i}{\partial \bar{x}_j} + \frac{\partial \bar{u}_j}{\partial \bar{x}_i} + \delta_{ij} \bar{\lambda} \nabla \cdot \vec{\bar{V}} \right) \right]$. Here $\vec{\bar{V}} = \bar{u}\vec{i} + \bar{v}\vec{j} + \bar{w}\vec{k}$ is the velocity of the fluid particle, $D = \frac{\partial}{\partial \bar{t}} + \bar{u}\frac{\partial}{\partial \bar{x}} + \bar{v}\frac{\partial}{\partial \bar{y}} + \bar{w}\frac{\partial}{\partial \bar{z}}$

is the material derivative, $\bar{\lambda}$ is the coefficient of second viscosity, $\bar{\mu}$ is the coefficient of dynamic viscosity, $\bar{k}$ is the thermal conductivity, $\bar{D}$ is the mass diffusivity, and δ_{ij} is the Kronecker delta defined as

$$\delta_{ij} = \begin{cases} 1, & \text{when } i = j, \\ 0, & \text{when } i \neq j. \end{cases} \tag{2.6.5}$$

Here $h = \bar{e} + \frac{\bar{p}}{\bar{\rho}}$ is the enthalpy, $\bar{e}$ is the internal energy, and $dh = c_p dT + (1 - \beta T)\frac{d\bar{p}}{\bar{\rho}}$. The equations (2.6.1)-(2.6.5) are based on the assumptions, (i) the fluid is a continuum, (ii) the particles are essentially in thermodynamic equilibrium, (iii) heat conduction follows Fourier's law, (iv) there are no internal heat sources (e.g., radiation, Joule's heat), (v) stress tensor is symmetric, (vi) the fluid is isotropic (there is no locally preferred direction), Newtonian, (vii) the Stokes hypothesis is valid (i.e., $3\bar{\lambda} + 2\bar{\mu} = 0$), (viii) concentration C of single diffusing species is low and no chemical reaction occurs, (ix) mass diffusion follows Fick's law, (x) force producing electrical and magnetic field are excluded and (xi) body force is only due to gravity.

The last term of right hand side of equation (2.6.3) is the viscous dissipation term (positive). It is the work done by the viscous stresses. In low speed flow this term will usually be negligible. It is important for gases at extremely low temperature. The equations (2.6.1)-(2.6.5) involve eight variables, namely $\bar{p}$, $\vec{\bar{V}}$, T, $\bar{\rho}$, h, $\bar{\mu}$, $\bar{D}$, $\bar{k}$ of which three variables, namely $\bar{p}$, $\vec{\bar{V}}$, T are assumed to be primary variables. The remaining variables $\bar{\rho}$, h,$\bar{\mu}$, $\bar{D}$, $\bar{k}$ are assumed to be functions of $\bar{p}$, T. The dependencies of the quantities $\bar{\rho}$, h, $\bar{\mu}$, $\bar{D}$, $\bar{k}$ on pressure $\bar{p}$ are generally very small and may be neglected.

2.6.2 Boundary Conditions

Various boundary conditions may be imposed as follows:

1. Hydrodynamic Boundary Conditions

   (a) No slip