Second Semester Examination
2017/2018 Academic Session

May / June 2018

MGM562 - Probability Theory
(Teori Kebarangkalian)

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **EIGHT (8)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **EIGHT (8)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions : Answer **all six (6)** questions.

*[**Arahan :** Jawab **semua enam (6)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

Question 1

Let X be a function defined as follows:

$$f_X(x) = \begin{cases} (0.75^x)(0.25) & x = 0,1,2,\ldots \\ 0 & \text{otherwise.} \end{cases}$$

(a) For $k = 1,2,\ldots,5$, sketch the graph of $\frac{f_X(k)}{f_X(k-1)}$ versus k.

[4 marks]

(b) Given the following events, C and D where both are sets of non-negative integer numbers such that, $C = \{x \in \mathbb{Z} | x \in \text{odd number}\}$ and $D = \{x \in \mathbb{Z} | x < 10\}$. Find

(i) *P(C).*
(ii) *P(D).*
(iii) $P(C \cap D)$.
(iv) $P[(D - C)|C']$.

[14 marks]

Soalan 1

Biarkan X sebagai suatu fungsi yang ditakrifkan seperti berikut:

$$f_X(x) = \begin{cases} (0.75^x)(0.25) & x = 0,1,2,\ldots \\ 0 & selainnya. \end{cases}$$

(a) *Bagi* $k = 1,2,\ldots,5$, *lakarkan graf bagi* $\frac{f_X(k)}{f_X(k-1)}$ *melawan k*

[4 markah]

(b) *Diberi peristiwa berikut,* C *dan* D *yang mana kedua-duanya merupakan set-set integer bukan negatif, iaitu,* $C = \{x \in \mathbb{Z} | x \in nombor\ ganjil\}$ *dan* $D = \{x \in \mathbb{Z} | x < 10\}$. *Cari*

(i) $Kb(C)$.
(ii) $Kb(D)$.
(iii) $Kb(C \cap D)$.
(iv) $Kb[(D - C)|C']$.

[14 markah]

Question 2

Let X be a continuous random variable with density function

$$f(x) = \frac{3}{2500}x^2(10-x)I_{[0,10]}(x)$$

(a) Find the mean and variance of X.

[6 marks]

(b) Use Markov's Theorem to show that the probability of X exceeds 8 is less than 0.75.

[2 marks]

(c) Find the exact probability of X exceeds 8.

[4 marks]

Soalan 2

Biarkan X sebagai pembolehubah rawak dengan fungsi ketumpatan kebarangkalian

$$f(x) = \frac{3}{2500}x^2(10-x)I_{[0,10]}(x)$$

(a) Cari min dan varians bagi X.

[6 markah]

(b) Gunakan Teorem Markov untuk menunjukkan bahawa kebarangkalian X melebihi 8 adalah kurang daripada 0.75.

[2 markah]

(c) Cari kebarangkalian X melebihi 8.

[4 markah]

Question 3

Let X be a random variable having the distribution as follows:

$$f_X(x;\alpha,\theta) = \frac{\alpha\theta^{\alpha}}{(x+\theta)^{\alpha+1}} I_{[0,\infty)}(x)$$

Let $Y = \ln\left(\frac{X+\theta}{\theta}\right)$.

(a) Solve the following problems;

(i) Identify the relationship of X and Y.
(ii) Find $\frac{dX}{dY}$.
(iii) Find the domain of Y.

[6 marks]

(b) From (a) obtained, state the distribution of Y.

[3 marks]

(c) Find the moment generating function of Y.

[3 marks]

(d) Let $\alpha = 3$ and $\theta = 1$, by using (c), find the mean and variance of X.

[9 marks]

Soalan 3

Biarkan X sebagai pembolehubah rawak yang mempunyai taburan seperti berikut:

$$f_X(x;\alpha,\theta) = \frac{\alpha\theta^{\alpha}}{(x+\theta)^{\alpha+1}} I_{[0,\infty)}(x)$$

Biarkan $Y = \ln\left(\frac{X+\theta}{\theta}\right)$.

(a) Selesaikan masalah-masalah berikut;

(i) Kenal pasti kaitan antara X dan Y.
(ii) Cari $\frac{dX}{dY}$.
(iii) Cari domain bagi Y.

[6 markah]

(b) Daripada (a), nyatakan taburan bagi Y.

[3 markah]

(c) Cari fungsi penjana momen bagi Y.

[3 markah]

(d) Biarkan $\alpha = 3$ *dan* $\theta = 1$, *dengan menggunakan (c), cari min dan varians bagi X.*

[9 markah]

Question 4

Find $P\left[X \leq E(X) + 0.75\sqrt{Var(X)}\right]$ where the random variable X has the moment generating function as follows:

(a) $(2 - e^t)^{-3}$.

[9 marks]

(b) $M_X(t) = \exp(4t + 8t^2)$.

[6 marks]

Soalan 4

Cari $Kb\left[X \leq E(X) + 0.75\sqrt{Var(X)}\right]$ *yang mana pembolehubah rawak X mempunyai fungsi penjana momen seperti berikut:*

(a) $(2 - e^t)^{-3}$.

[9 markah]

(b) $M_X(t) = \text{eks}(4t + 8t^2)$.

[6 markah]

Question 5

Let X has a gamma distribution, with the following probability density function,

$$f_X(x; \alpha, \theta) = \frac{1}{\Gamma(\alpha)} x^{\alpha-1} \left(\frac{1}{\theta}\right)^{\alpha} e^{-\frac{1}{\theta}x} I_{[0,\infty)}(x)$$

If $\theta = 2$, X is said to have chi-squared distribution with the mean to be its number of degrees of freedom.

(a) State the mean, variance and moment generating function of X.

[4 marks]

(b) If $\alpha = 8$,

(i) State the number of degrees of freedom for X.

(ii) Find $f_X(14)$.

(iii) Find the value of x, so that $F_X(x) = 0.05$.

[9 marks]

Soalan 5

Biarkan X mempunyai taburan gamma, dengan fungsi ketumpatan kebarangkaliannya seperti berikut,

$$f_X(x;\alpha,\theta) = \frac{1}{\Gamma(\alpha)} x^{\alpha-1} \left(\frac{1}{\theta}\right)^{\alpha} e^{-\frac{1}{\theta}x} I_{[0,\infty)}(x)$$

Jika $\theta = 2$, X dikatakan mempunyai taburan khi-kuasa dua dengan minnya sebagai bilangan darjah kebebasan.

(a) *Nyatakan min, varians dan fungsi penjana momen bagi X.*

[4 markah]

(b) *Jika $\alpha = 8$,*

(i) *Nyatakan bilangan darjah kebebasan bagi X.*

(ii) *Cari $f_X(14)$.*

(iii) *Cari nilai x, supaya $F_X(x) = 0.05$.*

[9 markah]

Question 6

Let the joint variables of X and Y have the function

$$f(x,y) = \begin{cases} 6[1-(x+y)] & x > 0, y > 0 \text{ and } x+y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that the function above is a joint probability density function.

[3 marks]

(b) Show that X and Y are identical.

[5 marks]

(c) What is $Cov(X,Y)$?

[8 marks]

(d) Find $Var(X+Y)$.

[5 marks]

Soalan 6

Biarkan pembolehubah tercantum X dan Y mempunyai fungsi

$$f(x,y) = \begin{cases} 6[1-(x+y)] & x>0, y>0 \; dan \; x+y<1 \\ 0 & selainnya. \end{cases}$$

(a) *Tunjukkan bahawa fungsi di atas adalah suatu fungsi ketumpatan kebarangkalian tercantum.*

[3 markah]

(b) *Tunjukan bahawa X dan Y adalah secaman.*

[5 markah]

(c) *Apakah $Kov(X,Y)$?*

[8 markah]

(d) *Cari $Var(X+Y)$.*

[5 markah]

Appendix

X	$f_X(x)$	$E(X)$	$Var(X)$	$M_X(t)$
$bin(n,p)$	$\binom{n}{x}p^x(1-p)^{n-x}$, $x=0,1,\dots,n$	np	$np(1-p)$	$(pe^t+1-p)^n$
$Po(\lambda)$	$\frac{e^{-\lambda}\lambda^x}{x!}$, $x=0,1,2,\dots$	λ	λ	$e^{\lambda(e^t-1)}$
$Geo(p)$	$p(1-p)^x$, $x=0,1,2,\dots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{1-(1-p)e^t}$
$NB(r,p)$	$\binom{x+r-1}{x}p^r(1-p)^x$, $x=0,1,2,\dots$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{p}{1-(1-p)e^t}\right]^r$
$U(a,b)$	$\frac{1}{b-a}$, $a<x<b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb}-e^{ta}}{t(b-a)}$
$N(\mu,\sigma^2)$	$\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$, $-\infty<x<\infty$	μ	σ^2	$\exp\left(\mu t+\frac{\sigma^2t^2}{2}\right)$
$\exp(\theta)$	$\frac{1}{\theta}e^{-x/\theta}$, $x>0$	θ	θ^2	$(1-\theta t)^{-1}$
$\Gamma(\alpha,\theta)$	$\frac{1}{\Gamma(\alpha)}\frac{x^{\alpha-1}}{\theta^\alpha}e^{-x/\theta}$, $x>0$	$\alpha\theta$	$\alpha\theta^2$	$(1-\theta t)^{-\alpha}$

$(a+b)^n = \sum_{x=0}^{n} {}^nC_x a^x b^{n-x}$	$g(x) = \sum_{m=0}^{\infty} \frac{g^{(m)}(x_0)(x-x_0)^m}{m!}$	$Var(aX \pm bY) = a^2Var(X) \pm 2abCov(X,Y) + b^2Var(Y)$
$P(X-Y) = P(X \cap \bar{Y})$		

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