Second Semester Examination
2017/2018 Academic Session

May/June 2018

MGM511 - Linear Algebra
[Aljabar Linear]

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of EIGHT (8) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN (8) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all FOUR** (4) questions.

*[**Arahan:** Jawab **semua EMPAT** (4) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

Question 1

(a) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

(i) Find conditions on a, b, c and d such that $AB = BA$.

(ii) Give an example of matrix B satisfying condition in (i).

(b) A matrix C is said to be orthogonal if $C^T = C^{-1}$.

(i) Show that $\det(C) = \sqrt{1}$.

(ii) Show that the following matrix is orthogonal:

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

(c) Consider the upper triangular matrix

$$U = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}.$$

Show that $\det(U) = a_{11}a_{22}a_{33}a_{44}$.

[100 marks]

Soalan 1

(a) *Andaikan* $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ *dan* $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

(i) *Dapatkan syarat untuk* a, b, c *dan* d *sedemikian bahawa* $AB = BA$.

(ii) *Beri satu contoh matriks B yang memenuhi syarat di dalam (i).*

(b) *Suatu matriks C dikatakan ortogonal jika* $C^T = C^{-1}$.

(i) *Tunjukkan bahawa* $\det(C) = \sqrt{1}$.

(ii) *Tunjukkan bahawa matriks berikut adalah ortogonal:*

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

(c) *Pertimbangkan matriks segitiga atas*

$$U = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}.$$

Tunjukkan bahawa $\det(U) = a_{11}a_{22}a_{33}a_{44}$.

[100 markah]

Question 2

(a) Consider the following system of equations:

$$\begin{aligned} 2x_1 + x_2 - 6x_4 - 2x_5 &= 0, \\ -3x_1 + x_2 - 5x_3 + 4x_4 + 3x_5 &= 0, \\ -2x_1 + 2x_2 - 6x_3 + 2x_5 &= 0. \end{aligned}$$

(i) Write the coefficient matrix A of the system.

(ii) Solve the system using the Gauss-Jordan elimination.

(iii) State a basis for the column space of A.

(iv) State a basis for the row space of A.

(v) State a basis for the null space of A.

(vi) State the rank and nullity of A.

(vii) Find a basis for the orthogonal complement to the subspace W of $\mathbb{R}^5$ spanned by the vectors $(2,1,0,-6,-2), (-3,1,-5,4,3), (-2,2,-6,0,2)$.

(b) Let $Q\mathbf{x} = \mathbf{0}$ be a homogeneous system of n linear equations in n unknowns, and let P be an invertible $n \times n$ matrix. Show that $Q\mathbf{x} = \mathbf{0}$ has only the trivial solution if and only if $(PQ)\mathbf{x} = \mathbf{0}$ has only the trivial solution.

[100 marks]

Soalan 2

(a) *Pertimbangkan sistem persamaan linear berikut:*

$$\begin{aligned} 2x_1 + x_2 - 6x_4 - 2x_5 &= 0, \\ -3x_1 + x_2 - 5x_3 + 4x_4 + 3x_5 &= 0, \\ -2x_1 + 2x_2 - 6x_3 + 2x_5 &= 0. \end{aligned}$$

(i) *Tuliskan matriks pekali A untuk sistem ini.*

(ii) *Selesaikan sistem ini menggunakan kaedah Gauss-Jordan.*

(iii) *Nyatakan asas bagi ruang lajur A.*

(iv) *Nyatakan asas bagi ruang baris A.*

(v) *Nyatakan asas bagi ruang nol A.*

(vi) *Nyatakan pangkat dan kenolan A.*

(vii) *Dapatkan asas bagi pelengkap berortogon subruang W dalam* $\mathbb{R}^5$ *yang direntang oleh vektor* $(2,1,0,-6,-2), (-3,1,-5,4,3), (-2,2,-6,0,2)$.

(b) *Andaikan* $Q\mathbf{x} = \mathbf{0}$ *suatu sistem homogen dengan n persamaan linear dan n anu, dan andaikan P matriks tersongsangkan* $n \times n$. *Tunjukkan bahawa* $Q\mathbf{x} = \mathbf{0}$ *hanya mempunyai penyelesaian remeh jika dan hanya jika* $(PQ)\mathbf{x} = \mathbf{0}$ *hanya mempunyai penyelesaian remeh.*

[100 markah]

Question 3

(a) Let S be the set of pairs of positive real numbers and define the addition and scalar multiplication operations as follows:

$$\mathbf{u}+\mathbf{v} = (u_1, u_2) + (v_1, v_2) = (u_1v_1, u_2v_2)$$
$$k\mathbf{u} = k(u_1, u_2) = (u_1^k, u_2^k).$$

Assume that S is a vector space.

(i) Determine the zero vector $\mathbf{0}$ in S.

(ii) Verify that $k\mathbf{0} = \mathbf{0}$ holds.

(iii) Let $\mathbf{u} = (u_1, u_2)$ be in S. Find $-\mathbf{u}$ such that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$.

(b) Let $V = \mathbb{M}_{2\times 2}$ and $W = \left\{ A \in V : A = \begin{bmatrix} x & -x \\ y & z \end{bmatrix} \right\}$.

(i) Show that W is a subspace of V.

(ii) Find a basis for W.

(c) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 + 2x_3 \\ 2x_1 + x_2 + x_3 \\ x_1 + x_2 \end{bmatrix}.$$

(i) Show that T is a linear transformation.

(ii) Find the standard matrix A for T.

(iii) Find the determinant of the standard matrix A.

(iv) Determine whether T is one-to-one.

[100 marks]

Soalan 3

(a) *Andaikan S set yang mengandungi pasangan nombor nyata positif dengan operasi penambahan dan pendaraban skalar seperti berikut:*

$$\mathbf{u}+\mathbf{v} = (u_1,u_2)+(v_1,v_2) = (u_1v_1,u_2v_2)$$
$$k\mathbf{u} = k(u_1,u_2) = (u_1^k,u_2^k).$$

Anggap bahawa S ialah ruang vektor.

(i) *Tentukan vektor sifar,* $\mathbf{0}$ *dalam S.*

(ii) *Tentusahkan bahawa* $k\mathbf{0} = \mathbf{0}$ *adalah benar.*

(iii) *Andaikan* $\mathbf{u} = (u_1,u_2)$ *dalam S. Dapatkan* $-\mathbf{u}$ *sedemikian* $\mathbf{u}+(-\mathbf{u}) = \mathbf{0}$.

(b) *Andaikan* $V = \mathbb{M}_{2\times 2}$ *dan* $W = \left\{A \in V : A = \begin{bmatrix} x & -x \\ y & z \end{bmatrix}\right\}$.

(i) *Tunjukkan bahawa W merupakan subruang untuk V.*

(ii) *Dapatkan asas untuk W.*

(c) *Andaikan* $T : \mathbb{R}^3 \to \mathbb{R}^3$ *ditakrifkan sebagai*

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1-2x_2+2x_3 \\ 2x_1+x_2+x_3 \\ x_1+x_2 \end{bmatrix}.$$

(i) *Tunjukkan bahawa T merupakan transformasi linear..*

(ii) *Dapatkan matriks piawai A untuk T.*

(iii) *Dapatkan penentu bagai matriks piawai A.*

(iv) *Tentukan sama ada T adalah satu-dengan-satu.*

[100 markah]

Question 4

(a) Find the best least squares fit by a quadratic curve to the points

$$(-2,0), \quad (-1,0), \quad (0,1), \quad (1,0), \quad (2,0).$$

(b) Let W be spanned by the vectors

$$\mathbf{u}_1 = (1,1,1,1), \qquad \mathbf{u}_2 = (1,2,4,5), \quad \mathbf{u}_3 = (1,-3,-4,-2).$$

(i) Use the Gram-Schmidt process to transform the above basis into an orthogonal basis of W with the assumption that the Euclidean inner product is defined on it.

(ii) Convert the orthogonal basis obtained in part (ii) into an orthonormal basis.

[100 marks]

Soalan 4

(a) Dapatkan penghampiran kuasa dua terkecil terbaik dengan lengkung kuadratik untuk titik-titik

$$(-2,0), \quad (-1,0), \quad (0,1), \quad (1,0), \quad (2,0).$$

(b) Biar W direntang oleh vektor-vektor

$$\mathbf{u}_1 = (1,1,1,1), \qquad \mathbf{u}_2 = (1,2,4,5), \quad \mathbf{u}_3 = (1,-3,-4,-2).$$

(i) Dengan menggunakan proses Gram-Schmidt, ubah asas di atas kepada asas ortogon untuk W dengan andaian bahawa hasil darab terkedalam Euclidean tertakrif padanya.

(ii) Tukar asas ortogon yang diperoleh di bahagian (ii) kepada asas ortonormal.

[100 markah]

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