Second Semester Examination
2017/2018 Academic Session

May/June 2018

MAT516 - Curve and Surface Methods for CAGD
(Kaedah Lengkung dan Permukaan untuk RGBK)

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of FIVE (5) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi FIVE (5) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all THREE** (3) questions.

*[**Arahan:** Jawab **semua TIGA** (3) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

Question 1

Let $B_{i,n}(t) = \frac{n!}{(n-r)!r!}(1-t)^{n-i}t^i$, $t \in [0,1]$ and $\mathbf{B}(t) = \sum_{i=0}^{n} \mathbf{b}_i B_{i,n}(t)$.

(a) Consider a quartic Bézier curve with control points $\mathbf{b}_0(3.0, 3.0)$, $\mathbf{b}_1(4.0, 2.0)$, $\mathbf{b}_2(-1.0, 0.0)$, $\mathbf{b}_3(6.0, 1.0)$, and $\mathbf{b}_4(8.0, 5.0)$.

(i) Evaluate the point $\mathbf{B}(0.6)$ using de Casteljau algorithm.

(ii) Use the triangular array of points evaluated in part (i) to write down the sets of control points, defining the segments $\mathbf{B}_{\text{left}}$ and $\mathbf{B}_{\text{right}}$, obtained when $\mathbf{B}(t)$ is subdivided at $t = 0.6$.

(iii) Determine the first and second derivatives of the curve $\mathbf{B}(t)$.

(b) Consider a quadratic rational Bézier curve defined as

$$\mathbf{R}(t) = \frac{\sum_{i=0}^{2} w_i \mathbf{b}_i B_{i,2}(t)}{\sum_{i=0}^{2} w_i B_{i,2}(t)},$$

with control points $\mathbf{b}_i$ and weights w_i, for $i = 0, 1, 2$. Let the set of weights be $\{w_0 = 1, w_1, w_2 = 1\}$.

(i) Show that when $w_1 = 0$, the curve reduces to a straight line between $\mathbf{b}_0$ and $\mathbf{b}_2$.

(ii) Show that a point $S = \mathbf{R}(0.5)$, which is called the shoulder point of a curve, moves along a straight line from $\frac{\mathbf{b}_0+\mathbf{b}_2}{2}$ to $\mathbf{b}_1$ when w_1 varies from 0 to ∞ (or equivalently when $\frac{w_1}{1+w_1}$ varies from 0 to 1).

[100 marks]

Soalan 1

Biarkan $B_{i,n}(t) = \frac{n!}{(n-r)!r!}(1-t)^{n-i}t^i$, $t \in [0,1]$ *dan* $\boldsymbol{B}(t) = \sum_{i=0}^{n} \boldsymbol{b}_i B_{i,n}(t)$.

(a) Pertimbangkan satu lengkung Bézier kuartik dengan titik kawalan $\boldsymbol{b}_0(3.0, 3.0)$, $\boldsymbol{b}_1(4.0, 2.0)$, $\boldsymbol{b}_2(-1.0, 0.0)$, $\boldsymbol{b}_3(6.0, 1.0)$, *dan* $\boldsymbol{b}_4(8.0, 5.0)$*:*

(i) Nilaikan titik $\boldsymbol{B}(0.6)$ *menggunakan algoritma de Casteljau.*

(ii) Gunakan tatasusunan tiga segi yang dinilaikan dalam bahagian (i) untuk menulis set titik-titik kawalan, yang mentakrifkan $\boldsymbol{B}_{kiri}$ *dan* $\boldsymbol{B}_{kanan}$*, hasil daripada pemecah bahagian* $\boldsymbol{B}(t)$ *pada* $t = 0.5$.

(iii) Tentukan terbitan pertama dan kedua bagi lengkung $\boldsymbol{B}(t)$.

(b) Pertimbangkan satu lengkung Bézier nisbah kuadratik yang ditakrif sebagai

$$\boldsymbol{R}(t)=\frac{\sum_{i=0}^{2} w_i \boldsymbol{b}_i B_{i,2}(t)}{\sum_{i=0}^{2} w_i B_{i,2}(t)},$$

dengan titik kawalan $\boldsymbol{b}_i$ dan pemberat w_i, untuk $i=0,1,2$. Andaikan set pemberat adalah $\{w_0=1, w_1, w_2=1\}$.

(i) Tunjukkan bahawa apabila $w_1=0$, lengkung tersebut terturun kepada satu garis lurus di antara $\boldsymbol{b}_0$ dan $\boldsymbol{b}_2$.

(ii) Tunjukkan bahawa titik $S=\boldsymbol{R}(0.5)$, yang dipanggil titik bahu lengkung, bergerak sepanjang garis lurus daripada $\frac{\boldsymbol{b}_0+\boldsymbol{b}_2}{2}$ ke $\boldsymbol{b}_1$ apabila w_1 berubah daripada 0 ke ∞ (atau secara setara apabila $\frac{w_1}{1+w_1}$ berubah daripada 0 ke 1).

[100 markah]

Question 2

The B-spline basis functions of degree d, $N_{i,d}(t)$, with knot vector $\{t_0, t_1, ..., t_m\}$ are defined recursively as follows:

$$N_{i,0}(t)=\begin{cases}1, & t\in[t_i,t_{i+1})\\ 0, & \text{otherwise}\end{cases},$$

$$N_{i,d}(t)=\frac{t-t_i}{t_{i+d}-t_i}N_{i,d-1}(t)+\frac{t_{i+d+1}-t}{t_{i+d+1}-t_{i+1}}N_{i+1,d-1}(t),$$

for $i=0,1,...,n$ and $d\geq 1$. The B-spline curve of degree d with control points $\mathbf{b}_0, \mathbf{b}_0, ..., \mathbf{b}_n$ is defined on the interval $[t_d, t_{m-d}]$ by $\mathbf{B}(t)=\sum_{i=0}^{n}\mathbf{b}_i N_{i,d}(t)$.

(a) Suppose $d=3$ with knot vector $\{0,1,2,3,4,5,6,7\}$. Calculate and plot the basis functions, $N_{i,3}(t)$ for $i=0,1,2,3$. [Hint: Use the translation invariance property]

(b) Show that with knot vector $\{0,0,0,0,1,1,1,1\}$, then $N_{i,3}(t)=B_{i,3}(t)$, $i=0,1,2,3$ for $t\in[0,1]$ where $B_{i,3}(t)$ are Bernstein polynomials of degree 3.

(c) Suppose $d=2$ with knot vector $\{0,1,2,3,4,5,6,7\}$ and control points $\mathbf{b}_0(3,2)$, $\mathbf{b}_1(7,-1)$, $\mathbf{b}_2(5,2)$, $\mathbf{b}_3(4,5)$, $\mathbf{b}_4(2,3)$. Apply the de Boor algorithm to evaluate $\mathbf{B}(3.6)$.

(d) A general cubic uniform B-spline in segment i can be written as

$$\mathbf{B}_i(u) = \frac{1}{6}\begin{pmatrix} u^3 & u^2 & u & 1 \end{pmatrix}\begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{pmatrix}\begin{pmatrix} \mathbf{b}_{i-3} & \mathbf{b}_{i-2} & \mathbf{b}_{i-1} & \mathbf{b}_i \end{pmatrix},$$

where $u \in [0,1]$ and $\mathbf{b}_i$ are the control points.

(i) Determine the start and end points of segment $\mathbf{B}_i$.

(ii) Find the tangent vectors at both ends of segment $\mathbf{B}_i$.

(iii) Find the second derivative of segment $\mathbf{B}_i$. Then, show that two adjacent segments $\mathbf{B}_i$ and $\mathbf{B}_{i+1}$ are connected with C^2 continuity.

[150 marks]

Soalan 2

Fungsi asas splin-B darjah d, $N_{i,d}(t)$, dengan vektor simpulan $\{t_0, t_1, ..., t_m\}$ ditakrif secara rekursi seperti berikut:

$$N_{i,0}(t) = \begin{cases} 1, & t \in [t_i, t_{i+1}) \\ 0, & selainnya \end{cases},$$

$$N_{i,d}(t) = \frac{t - t_i}{t_{i+d} - t_i} N_{i,d-1}(t) + \frac{t_{i+d+1} - t}{t_{i+d+1} - t_{i+1}} N_{i+1,d-1}(t),$$

bagi $i = 0, 1, ..., n$ dan $d \geq 1$. Lengkung splin-B berdarjah d dengan titik kawalan $\boldsymbol{b}_0, \boldsymbol{b}_0, ..., \boldsymbol{b}_n$ ditakrifkan pada selang $[t_d, t_{m-d}]$ dengan $\boldsymbol{B}(t) = \sum_{i=0}^{n} \boldsymbol{b}_i N_{i,d}(t)$.

(a) Andaikan $d = 3$. Kira dan lakar fungsi-fungsi asas, $N_{i,3}(t)$ bagi $i = 0, 1, 2, 3$ dengan vektor simpulan $\{0, 1, 2, 3, 4, 5, 6, 7\}$. [Petunjuk: Gunakan sifat invarians translasi]

(b) Tunjukkan bahawa dengan vektor simpulan $\{0, 0, 0, 0, 1, 1, 1, 1\}$, maka $N_{i,3}(t) = B_{i,3}(t)$, $i = 0, 1, 2, 3$ bagi $t \in [0, 1]$ yang mana $B_{i,3}(t)$ adalah polinomial Bernstein darjah 3.

(c) Andaikan $d = 2$ dengan vektor simpulan $\{0, 1, 2, 3, 4, 5, 6, 7\}$ dan titik kawalan $\boldsymbol{b}_0(3, 2)$, $\boldsymbol{b}_1(7, -1)$, $\boldsymbol{b}_2(5, 2)$, $\boldsymbol{b}_3(4, 5)$, $\boldsymbol{b}_4(2, 3)$. Gunakan algoritma de Boor untuk menilai $\boldsymbol{B}(3.6)$.

(d) Suatu splin-B seragam kubik am dalam segmen i boleh ditulis sebagai

$$\boldsymbol{B}_i(u) = \frac{1}{6}\begin{pmatrix} u^3 & u^2 & u & 1 \end{pmatrix}\begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{b}_{i-3} & \boldsymbol{b}_{i-2} & \boldsymbol{b}_{i-1} & \boldsymbol{b}_i \end{pmatrix},$$

yang mana $u \in [0,1]$ *dan* $\boldsymbol{b}_i$ *adalah titik-titik kawalan.*

(i) Tentukan titik mula dan titik akhir segmen $\boldsymbol{B}_i$.

(ii) Cari vektor tangen pada kedua-dua hujung segmen $\boldsymbol{B}_i$.

(iii) Cari terbitan kedua segmen $\boldsymbol{B}_i$. *Seterusnya, tunjukkan bahawa dua segmen* $\boldsymbol{B}_i$ *dan* $\boldsymbol{B}_{i+1}$ *yang bersebelahan bersambung dengan keselanjaran* C^2.

[150 markah]

Question 3

Determine the control points and weights of the rational Bézier surface obtained by extruding the cubic rational Bézier curve with control points $\mathbf{b}_0(2,3,0)$, $\mathbf{b}_1(1,5,2)$, $\mathbf{b}_2(1,7,-1)$, $\mathbf{b}_3(2,9,-3)$, and weights $u_0 = 1, u_1 = 2, u_2 = 3, u_3 = 1$, in the direction $\mathbf{n} = (\frac{1}{3}, \frac{2}{3}, -\frac{2}{3})$ through distance $\delta = 6$ units. Then, write down the general equation of the resulting surface.

[50 marks]

Soalan 3

Tentukan titik kawalan dan pemberat bagi permukaan Bézier nisbah yang didapati dengan menolak satu lengkung Bézier nisbah kubik dengan titik kawalan $\boldsymbol{b}_0(2,3,0)$, $\boldsymbol{b}_1(1,5,2)$, $\boldsymbol{b}_2(1,7,-1)$, $\boldsymbol{b}_3(2,9,-3)$, *dan pemberat* $u_0 = 1, u_1 = 2, u_2 = 3, u_3 = 1$, *pada arah* $\boldsymbol{n} = (\frac{1}{3}, \frac{2}{3}, -\frac{2}{3})$ *menerusi jarak* $\delta = 6$ *unit. Seterusnya, tuliskan persamaan am permukaan yang terbentuk.*

[50 markah]

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