Second Semester Examination
2017/2018 Academic Session

May / June 2018

MSG384 – Introduction to Geometric Modelling
(Pengenalan kepada Pemodelan Geometri)

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **SIX (6)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **ENAM (6)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions : Answer **all four (4)** questions.

*[**Arahan :** Jawab **semua empat (4)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

Question 1

(a) Find a quadratic function in Lagrange form that interpolates three points $(-1,\ 3)$, $(2,\ 2)$ and $(3,\ -1)$.

[50 marks]

(b) Consider a cubic polynomial

$$\boldsymbol{P}(t) = \boldsymbol{C}_0 H_0(t) + \boldsymbol{C}_1 H_1(t) + \boldsymbol{C}_2 H_2(t) + \boldsymbol{C}_3 H_3(t), \qquad t \in [0,\ 1],$$

where $H_i(t)$, $i = 0,\ 1,\ 2,\ 3$, indicate Hermite basis functions and $\boldsymbol{C}_i$ are the related coefficients. Suppose

$$\boldsymbol{P}(0) = \boldsymbol{C}_0, \qquad \boldsymbol{P}(1) = \boldsymbol{C}_1,$$

$$\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t}(0) = \boldsymbol{C}_2, \qquad \frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t}(1) = \boldsymbol{C}_3,$$

find all the functions $H_i(t)$.

[50 marks]

Soalan 1

(a) Cari suatu fungsi kuadratik dalam bentuk Lagrange yang menginterpolasi tiga titik $(-1,\ 3)$, $(2,\ 2)$ *dan* $(3,\ -1)$.

[50 markah]

(b) Pertimbangkan satu polinomial kubik

$$\boldsymbol{P}(t) = \boldsymbol{C}_0 H_0(t) + \boldsymbol{C}_1 H_1(t) + \boldsymbol{C}_2 H_2(t) + \boldsymbol{C}_3 H_3(t), \qquad t \in [0,\ 1],$$

yang mana $H_i(t)$, $i = 0,\ 1,\ 2,\ 3$, *menandakan fungsi asas Hermite dan* $\boldsymbol{C}_i$ *ialah pekali berkaitan. Andaikan*

$$\boldsymbol{P}(0) = \boldsymbol{C}_0, \qquad \boldsymbol{P}(1) = \boldsymbol{C}_1,$$

$$\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t}(0) = \boldsymbol{C}_2, \qquad \frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t}(1) = \boldsymbol{C}_3,$$

cari semua fungsi $H_i(t)$.

[50 markah]

Question 2

Let the Bernstein polynomials of degree n be defined by

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!} t^i (1-t)^{n-i}, \qquad t \in [0,\ 1], \qquad \text{for } i = 0,\ 1,\ \ldots,\ n.$$

(a) Present the curve below in the form of cubic Bézier

$$y(t) = 3B_0^2(t) + 2B_1^2(t) + 4B_2^2(t), \qquad t \in [0,\ 1].$$

[50 marks]

(b) Given two polynomials

$$\boldsymbol{P}(t) = \begin{pmatrix} -2 \\ a \end{pmatrix} + \begin{pmatrix} b \\ -2 \end{pmatrix} t + \begin{pmatrix} 0 \\ 2 \end{pmatrix} t^2,$$

$$\boldsymbol{Q}(t) = \begin{pmatrix} 0 \\ c \end{pmatrix} B_0^2(t) + \begin{pmatrix} a \\ 4 \end{pmatrix} B_1^2(t) + \begin{pmatrix} 3 \\ 6 \end{pmatrix} B_2^2(t),$$

where $t \in [0,\ 1]$. Suppose they join with geometric continuity G^1, determine the values a, b and c.

[50 marks]

Soalan 2

Katakan polinomial Bernstein berdarjah n ditakrif sebagai

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!} t^i (1-t)^{n-i}, \qquad t \in [0,\ 1], \qquad \textit{bagi } i = 0,\ 1,\ \ldots,\ n.$$

(a) Tunjukkan lengkung di bawah dalam bentuk Bézier kubik

$$y(t) = 3B_0^2(t) + 2B_1^2(t) + 4B_2^2(t), \qquad t \in [0,\ 1].$$

[50 markah]

(b) *Diberi dua polinomial*

$$\boldsymbol{P}(t) = \begin{pmatrix} -2 \\ a \end{pmatrix} + \begin{pmatrix} b \\ -2 \end{pmatrix} t + \begin{pmatrix} 0 \\ 2 \end{pmatrix} t^2,$$

$$\boldsymbol{Q}(t) = \begin{pmatrix} 0 \\ c \end{pmatrix} B_0^2(t) + \begin{pmatrix} a \\ 4 \end{pmatrix} B_1^2(t) + \begin{pmatrix} 3 \\ 6 \end{pmatrix} B_2^2(t),$$

yang mana $t \in [0, 1]$. *Andaikan mereka bergabung dengan keselanjaran geometri* G^1, *tentukan nilai-nilai* a, b *dan* c.

[50 markah]

Question 3

Let $\boldsymbol{u} = (u_0, u_1, \ldots, u_{n+k})$ be a non-decreasing knot vector where n and k are positive integers with $n \geq k - 1$. The normalised B-spline basis functions of order k are defined recursively by

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad \text{for } k > 1$$

and

$$N_i^1(u) = \begin{cases} 1, & u \in [u_i, u_{i+1}) \\ 0, & \text{otherwise} \end{cases}$$

where $i = 0, 1, \ldots, n$.

(a) Suppose a B-spline curve

$$\boldsymbol{P}(u) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} N_0^3(u) + \begin{pmatrix} 2 \\ 4 \end{pmatrix} N_1^3(u) + \begin{pmatrix} 4 \\ 1 \end{pmatrix} N_2^3(u)$$

is defined with $\boldsymbol{u} = (0, 1, 2, 3, 4, 5)$, find the point $\boldsymbol{P}(3)$.

[50 marks]

(b) Suppose $\boldsymbol{u} = (-2, -1, 0, 1, 2, 3)$ and

$$\boldsymbol{P}(u) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} N_0^3(u) + \begin{pmatrix} 2 \\ 4 \end{pmatrix} N_1^3(u) + \begin{pmatrix} 4 \\ 1 \end{pmatrix} N_2^3(u), \qquad u \in [0, 1],$$

find the point on the curve which gives maximum coordinate-y.

[50 marks]

Soalan 3

Katakan $\boldsymbol{u} = (u_0,\ u_1,\ \ldots,\ u_{n+k})$ *ialah suatu vektor simpulan tak menyusut yang mana* n *dan* k *ialah integer positif dengan* $n \geq k - 1$. *Fungsi asas splin-B ternormal berperingkat* k *ditakrif secara rekursi oleh*

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad \textit{bagi } k > 1$$

dan

$$N_i^1(u) = \begin{cases} 1, & u \in [u_i,\ u_{i+1}) \\ 0, & \textit{sebaliknya} \end{cases}$$

yang mana $i = 0,\ 1,\ \ldots,\ n$.

(a) *Andaikan lengkung splin-B*

$$\boldsymbol{P}(u) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} N_0^3(u) + \begin{pmatrix} 2 \\ 4 \end{pmatrix} N_1^3(u) + \begin{pmatrix} 4 \\ 1 \end{pmatrix} N_2^3(u)$$

ditakrif dengan $\boldsymbol{u} = (0,\ 1,\ 2,\ 3,\ 4,\ 5)$, *cari titik* $\boldsymbol{P}(3)$.

[50 markah]

(b) *Andaikan* $\boldsymbol{u} = (-2,\ -1,\ 0,\ 1,\ 2,\ 3)$ *dan*

$$\boldsymbol{P}(u) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} N_0^3(u) + \begin{pmatrix} 2 \\ 4 \end{pmatrix} N_1^3(u) + \begin{pmatrix} 4 \\ 1 \end{pmatrix} N_2^3(u), \qquad u \in [0,\ 1],$$

cari titik lengkung yang memberi koordinat-y *maksimum.*

[50 markah]

Question 4

(a) Consider a tensor-product Bézier surface

$$z(x,\ y) = 2B_2^2(x)B_1^2(y) + 3B_1^2(x)B_1^2(y) + 2B_1^2(x)B_2^2(y),$$

where $B_s^2(t)$, $t \in [0,\ 1]$, indicate the Bernstein polynomials of degree 2.

(i) Find the relevant control points of this Bézier surface.

(ii) Find the normal vector to the surface at $(x,\ y) = (0.5,\ 0.5)$.

[50 marks]

(b) A bilinearly blended Coons patch $\boldsymbol{S}(u, v)$ is defined in Cartesian space with four boundaries

$$\boldsymbol{S}(u,\ 0)=\begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}, \qquad \boldsymbol{S}(u,\ 1)=\begin{pmatrix} u \\ 1 \\ 4u-u^2 \end{pmatrix}, \qquad u\in[0,\ 1],$$

$$\boldsymbol{S}(0,\ v)=\begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}, \qquad \boldsymbol{S}(1,\ v)=\begin{pmatrix} 1 \\ v \\ 4v-v^2 \end{pmatrix}, \qquad v\in[0,\ 1].$$

Calculate the point $\boldsymbol{S}(0.5,\ 0.5)$.

[50 marks]

Soalan 4

(a) *Pertimbangkan satu permukaan Bézier produk tensor*

$$z(x,\ y)=2B_2^2(x)B_1^2(y)+3B_1^2(x)B_1^2(y)+2B_1^2(x)B_2^2(y),$$

yang mana $B_s^2(t)$, $t\in[0,\ 1]$, *menandakan polinomial Bernstein berdarjah* 2.

(i) *Cari titik-titik kawalan yang berkaitan bagi permukaan Bézier ini.*

(ii) *Cari vektor normal kepada permukaan tersebut pada* $(x,\ y)=(0.5,\ 0.5)$.

[50 markah]

(b) *Satu tampalan Coons teraduan dwilinear* $\boldsymbol{S}(u, v)$ *ditakrif dalam ruang Cartesan dengan empat sempadan*

$$\boldsymbol{S}(u,\ 0)=\begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}, \qquad \boldsymbol{S}(u,\ 1)=\begin{pmatrix} u \\ 1 \\ 4u-u^2 \end{pmatrix}, \qquad u\in[0,\ 1],$$

$$\boldsymbol{S}(0,\ v)=\begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}, \qquad \boldsymbol{S}(1,\ v)=\begin{pmatrix} 1 \\ v \\ 4v-v^2 \end{pmatrix}, \qquad v\in[0,\ 1].$$

Kirakan titik $\boldsymbol{S}(0.5,\ 0.5)$.

[50 markah]

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