
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2012/2013 Academic Session

June 2013

ESA 322/3 – Structural Dynamics
[Dinamik Struktur]

Duration : 3 hours
[Masa : 3 jam]

Please check that this paper contains **TWENTY (20)** printed pages, **THREE (3)** pages appendix and **SIX (6)** questions before you begin the examination.

*[Sila pastikan bahawa kertas soalan ini mengandungi **DUAPULUH (20)** mukasurat bercetak, **TIGA (3)** mukasurat lampiran dan **ENAM (6)** soalan sebelum anda memulakan peperiksaan.]*

INSTRUCTIONS : Answer **SIX (6)** questions.

[ARAHAN : Jawab **ENAM (6)** soalan.]

Answer to each question must begin from a new page.

[Jawapan untuk setiap soalan mestilah dimulakan pada mukasurat yang baru.]

Appendix/Lampiran :

- | | |
|---------------------------------------|--------------------|
| 1. Fundamental Equations in Vibration | [1 page/mukasurat] |
| 2. Laplace Transform Pairs | [1 page/mukasurat] |
| 3. Vibration-Related Formulas | [1 page/mukasurat] |

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

Answer **SIX (6)** questions.
 Jawab **ENAM (6)** soalan.

1. [a] Provide brief answers to the questions (i) to (iii) :
- (i) What is the effect of damping to the frequency of vibration of a free vibrating system?
 - (ii) What is the difference between free vibration and forced vibration?
 - (iii) Explain the following terms of Force Transmissibility Ratio and Displacement Transmissibility Ratio.
 - (iv) Figure 1[a] shows the plot of the Amplitude ratio for a single degree spring mass system under a harmonic force. Based on that figure and appropriate equation show how resonance can be avoided to happen in the system.

(50 marks)

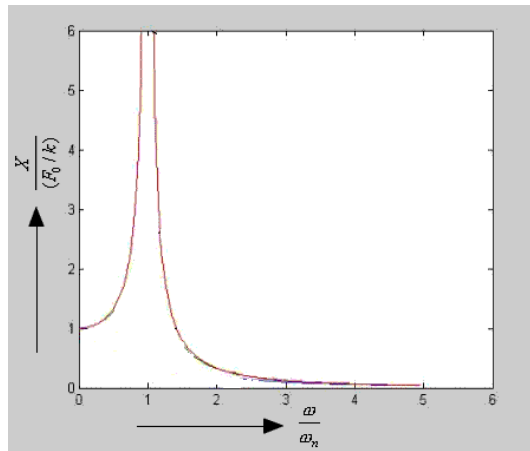


Figure 1[a]

[b] Considers a system as in Figure 1[b]. The system has $k = 100$ N/m, $c = 20$ kg/s, and $m = 90$ kg.

- (i) Draw the free body diagram of the system.
- (ii) Derive an equation of motion for the system for the case where $F(t) = F_o \sin \omega t$ and the surface is friction free.
- (iii) Compute the amplitude of steady state response of the system if the system is initially at rest for values with a harmonic force of 80N at frequency of 0.5Hz.

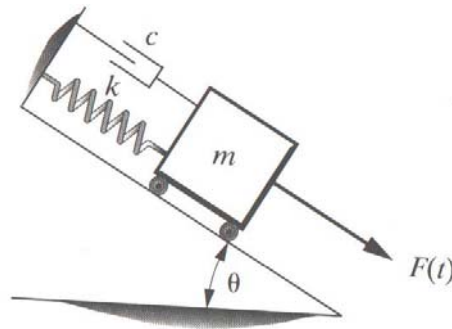
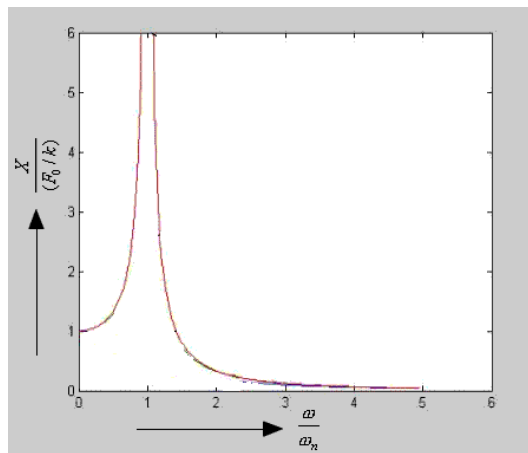


Figure 1[b]

(50 marks)

1. Jawab soalan-soalan berikut secara RINGKAS dari soalan (i) hingga soalan (iii) :
- (i) Apakah kesan nisbah redaman ke atas frekuensi getaran sesuatu sistem getaran bebas?
 - (ii) Apakah perbezaan di antara getaran bebas dan getaran paksaan?
 - (iii) Terangkan istilah Nisbah Kebolehpindahan Daya dan Nisbah Kebolehpindahan Anjakan.
 - (iv) Rajah 1[a] menunjukkan plot nisbah amplitud untuk jisim-spring sistem darjah tunggal di bawah daya harmonik. Berdasarkan rajah itu dan persamaan yang sesuai tunjukkan bagaimana resonans boleh dielakkan berlaku dalam sistem tersebut.

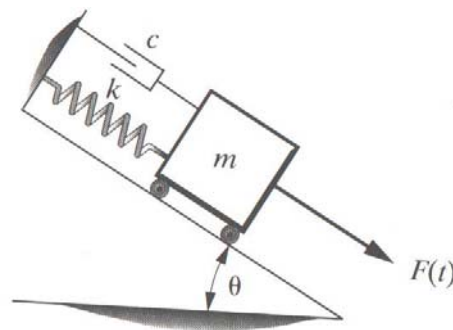


Rajah 1[a]

(50 markah)

[b] Pertimbangkan sebuah sistem seperti di dalam Rajah 1[b]. Sistem tersebut mempunyai nilai $k = 100 \text{ N/m}$, $c=20 \text{ kg/s}$, dan $m=90\text{kg}$

- (i) Lukiskan rajah jasad bebas untuk sistem tersebut.
- (ii) Terbitkan persamaan gerakan untuk sistem tersebut untuk kes $F(t) = F_0 \sin \omega t$ dan permukaan ada bebas geseran.
- (iii) Kirakan amplitud sambutan keadaan mantap sistem tersebut sekiranya sistem tersebut pada mulanya dalam keadaan rehat dan dikenakan daya harmonik pada 80N dalam frekuensi 0.5Hz .



Rajah 1[b]

(50 markah)

2. [a] A 1000 kg aircraft A is taxiing over a runway with velocity 20 km/h. At the next runway, an aircraft B weight 1500kg taxiing at 40 km/h. Assume that suspension system for both aircraft provides an equivalent stiffness of 40 kN/m and damping of 2 kNs/m. Both runway surfaces is approximated as sinusoidal in cross section providing a base motion displacement in meter of:

$$y(t) = 0.02 \sin \omega_b t$$

The runway distance in a cycle is 6 m.

- (i) Draw the equivalent system to model the suspension system of that aircraft. Using the given values for aircraft A and B, determine the effect of speed on the amplitude of displacement of the aircraft as well as the effect of the value of the aircraft's mass
- (ii) Calculate the maximum amplitude of the aircraft A and aircraft B and determine the speed at that maximum amplitude. Explain the phenomena that happen here.
- (iii) Calculate the best choice of the damping coefficient for aircraft A so that the transmissibility is minimized by comparing the magnitude of damping ratio $\zeta = 0.01$, $\zeta = 0.1$ and $\zeta = 0.2$ for the $r = 2$. Explain what will happen if the runway's frequency changes?

(60 marks)

- [b] The tail rotor of a helicopter as in Figure 2[b] can be modeled as a rotating balanced problem. The rotor blade has 20 kg of mass and the tail section is 70 kg. A 450 g mass is stuck on one of the blades at a distance 16 cm from the axis of rotation. The stiffness given for of the system is 10 kNs/m.

- (i) Calculate magnitude of the deflection of the tail section of the helicopter as the tail rotor rotates at 1600 rpm. (Assume a damping ratio of 0.02).
- (ii) Calculate the maximum deflection and speed on that maximum deflection.
- (iii) Based on calculation in (ii), what the pilot supposed to do in avoiding the rotor blade to damage that may cause an accident?

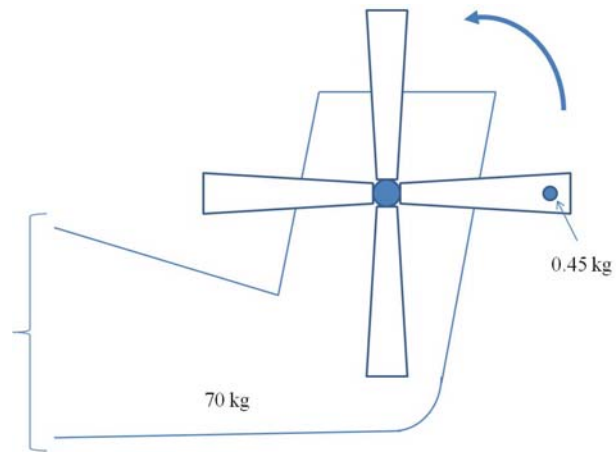


Figure 2[b]

(40 marks)

2. [a] Pesawat A seberat 1000 kg bergerak di atas landasan dengan di landasan yang bersebelahan, Pesawat B seberat 1500kg bergerak pada 40 km/j. Andaikan sistem suspensi untuk kedua-dua pesawat menyediakan kekakuan bersamaan 40 kN/m dan kadar redaman 2 kNs/m. Kedua-dua permukaan landasan yang dianggarkan sebagai sinus dalam keratan rentas menyediakan gerakan anjakan tapak dalam meter:

$$y(t) = 0.02 \sin \omega_b t$$

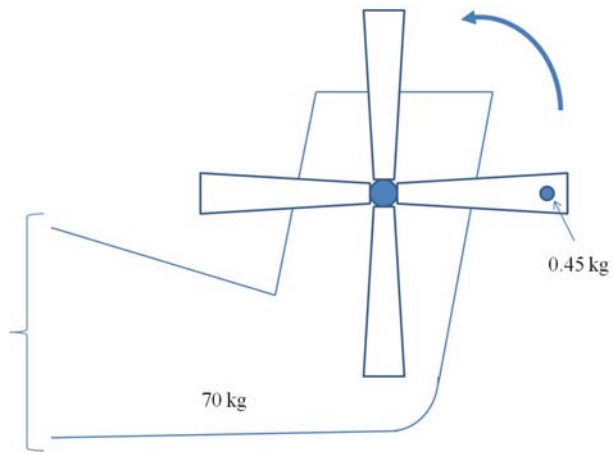
Diberikan jarak landasan untuk satu pusingan adalah 6 meter,

- (i) Lukiskan model sistem yang bersamaan the sistem suspensi pesawat tersebut. Menggunakan nilai-nilai yang telah diberikan untuk pesawat A dan B, tentukan kesan kelajuan dan berat pada amplitud anjakan pesawat. Buktikan melalui pengiraan.
- (ii) Kirakan amplitude maksimum untuk pesawat A dan B dan tentukan kelajuan pada maksimum amplitud. Terangkan fenomena yang berlaku di sini.
- (iii) Kirakan pilihan yang terbaik untuk pekali redaman untuk pesawat untuk membolehkan bolehantaran adalah paling minimum dengan membandingkan magnitud nisbah redaman $\zeta = 0.01$, $\zeta = 0.1$ and $\zeta = 0.2$ for the $r = 2$. Terangkan apa yang akan berlaku sekiranya frekuensi landasan berubah?

(60 markah)

- [b] Rotor ekor helikopter seperti dalam Rajah 2[b] boleh dimodelkan sebagai masalah penyeimbang jisim berputar. Bilah pemutar mempunyai berat sebanyak 20 kg dan bahagian ekor adalah seberat 70 kg. Jisim seberat 450 g terlekat di salah satu daripada bilah pada jarak 16 cm dari paksi putaran. Diberikan kekakuan sistem adalah 10 kNs/m.

- (i) Kirakan magnitud anjakan bahagian ekor helikopter apabila bilah rotor ekor berputar pada 1600 rpm. (Andaikan nisbah redaman adalah 0.02).
- (ii) Kirakan anjakan maksimum dan kelajuan pada anjakan maksimum.
- (iii) Berdasarkan pengiraan pada soalan (ii), Apakah yang sepatutnya dilakukan oleh juruterbang untuk mengelakkan kerosakan pada bilah rotor yang boleh menyebabkan kemalangan?



Rajah 2[b]

(40 markah)

3. [a] One of the most remarkable modern engineering feats is the design and construction of the dynamic vibration absorber in Taipei 101 tower, whose major component is a 700,000 kg orb to protect the tower from earthquakes and typhoons (see Figure 3[a]).

By way of 2-DOF model, free body diagram, and relevant plots, explain the principles of this absorber system.

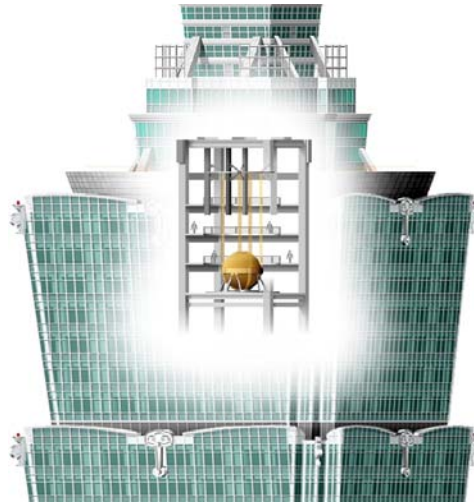


Figure 3[a]

(30 marks)

- [b] A drilling machine shown in Figure 3 [b1] can be modeled as a two degree of freedom system as indicated in the Figure 3 [b2]. The bending stiffness of the column are given by;

$$k_{11} = \frac{768 EI}{7 L^3}; \quad k_{12} = k_{21} = -\frac{240 EI}{7 L^3}; \quad k_{22} = \frac{96 EI}{7 L^3};$$

- (i) Draw a free body diagram of m_1 and m_2 .

Determine,

- (ii) The stiffness matrix and the mass matrix
 (iii) The characteristic equation of the system
 (iv) The natural frequencies of the system in the case of $m_1=30 \text{ kg}$, $m_2=70 \text{ kg}$, $L= 2 \text{ m}$, and $EI=20 \text{ MNm}^2$
 (v) The mode shape of the system (sketch)

- (vi) The amplitude of vibration of m_1 is a transverse harmonic force
 $F_T = 0.1 \sin t$



Figure 3[b1]

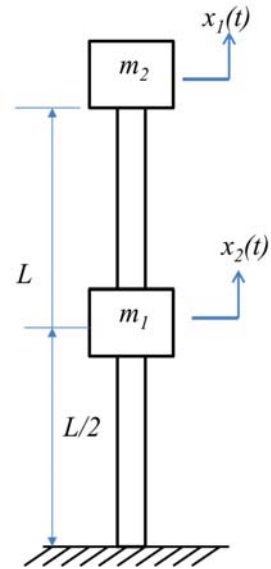
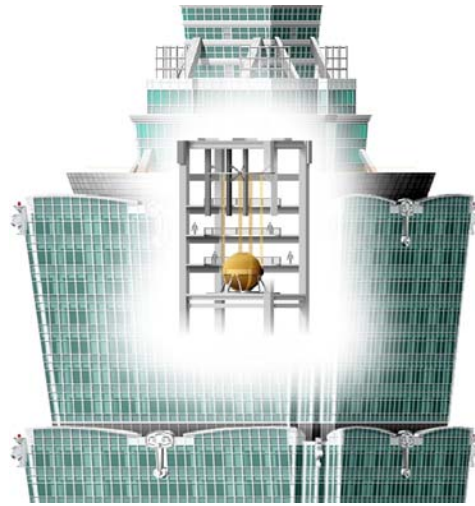


Figure 3[b2]

(70 marks)

3. [a] Salah satu pencapaian kejuruteraan moden yang terunggul ialah rekabentuk dan pembangunan peredam getaran dinamik pada menara Taipei 101 di mana sebuah bebola seberat 700,000 kg digunakan untuk melindungi menara dari gempa bumi dan taufan (lihat Rajah 3[a]).

Dengan menggunakan model 2 darjah kebebasan, gambarajah jasad bebas serta plot-plot yang berkaitan, terangkan prinsip-prinsip sistem redaman ini.



Rajah 3[a]

(30 markah)

- [b] Sebuah mesin penggerudian yang ditunjukkan dalam Rajah 3[b2] boleh dimodelkan sebagai sistem dua darjah sistem kebebasan seperti yang ditunjukkan dalam Rajah 3[b]. Kekakuan lentur tiang diberikan sebagai,

$$k_{11} = \frac{768 EI}{L^3}; \quad k_{12} = k_{21} = -\frac{240 EI}{L^3}; \quad k_{22} = \frac{96 EI}{L^3};$$

- (i) Lukis rajah jasad bebas bagi m_1 dan m_2

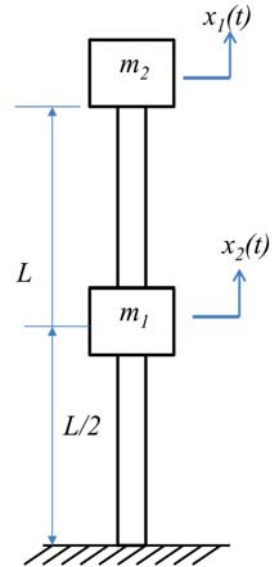
Tentukan,

- (ii) Matriks kekakuan dan matriks jisim
 (iii) Persamaan ciri sistem
 (iv) Frekuensi-frekuensi tabii sistem bagi kes $m_1=30$ kg, $m_2=70$ kg, $L= 2$ m, dan $EI=20$ MNm²
 (v) Bentuk-bentuk mod sistem (lakarkan)

- (vi) Amplitud getaran jisim m_1 jika daya harmonik melintang $F_T = 0.1 \sin t$



Rajah 3[b1]



Rajah 3[b2]

(70 markah)

4. Figure 4 shows strutted-left-wing modeled as slender beam with length L and mass m mounted on spring K_1 at the root and K_2 at the strut. The beam has a centre of gravity located at L_1 from the left spring, K_1 . Derive the equations of motion using:

- [a] (i) Newton's law of motion
 (ii) Lagrange's equation

(80 marks)

- [b] List down the steps needed to determine the value of L_1 where the heaving motion is decoupled from the rotating motion (you do not have to solve the equation)

x_1 and x_2 are the vertical motion of beam at K_1 and K_2 . Use θ to represent the angular motion of the beam.

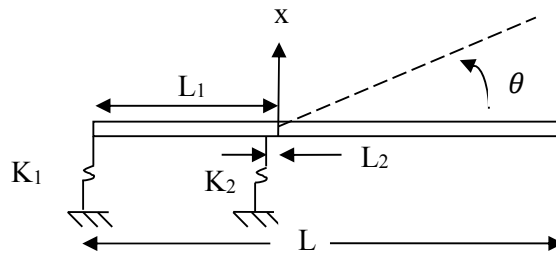


Figure 4

(20 marks)

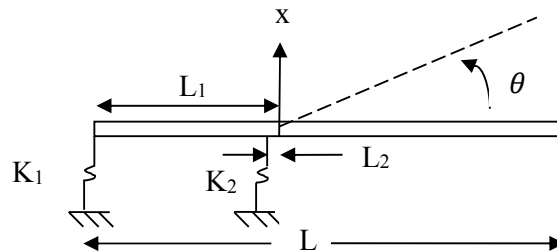
4. Rajah 4 menunjukkan sayap kiri berstrut dimodel sebagai rasuk langsing dengan panjang L dan jisim m disokong oleh spring dengan K_1 di akar dan K_2 di strut. Rasuk itu mempunyai pusat graviti terletak pada jarak L_1 daripada spring disebelah kiri, K_1 . Terbitkan persamaan gerakan menggunakan:

- [a] (i) Hukum gerakan Newton
(ii) Persamaan Lagrange

(80 markah)

- [b] Senaraikan langkah-langkah yang perlu untuk menentukan nilai L_1 di mana gerakan melambung tidak digandingkan dengan gerakan memutar (anda tidak perlu menyelesaikan persamaan ini)

x_1 dan x_2 adalah gerakan menegak rasuk di K_1 dan K_2 . Gunakan θ bagi mewakili gerakan sudut rasuk.



Rajah 4

(20 markah)

...8/-

5. The Laplace Transform can be used to determine the transfer function of a dynamic system and it is one of the methods apart from the solution in time domain to determine the response of the system for any given input.

[a] In your opinion in solving vibration problems, which method is preferable and why?

(10 marks)

[b] Formulate the transfer function of a single degree of freedom system with mass m , stiffness k and damping c when subjected to a general forcing function, $f(t)$

(40 marks)

[c] For the value of $m=1$ kg, $c=10$ Ns/m and $k=21$ N/m, determine the poles of the system. What is the significance of these values?

(30 marks)

[d] Draw the block diagram of this system.

(20 marks)

5. *Kaedah jelmaan Laplace boleh digunakan untuk menentukan rangkap pindah bagi sebuah sistem dinamik dan ianya merupakan satu kaedah selain daripada penyelesaian di dalam domain masa untuk menentukan tindakbalas sistem apabila dikenakan sebarang bentuk daya.*

[a] Pada pendapat anda di dalam menyelesaikan masalah getaran, kaedah manakah yang lebih mudah digunakan dan kenapa?

(10 markah)

[b] Terbitkan rangkap pindah bagi sebuah system satu darjah kebebasan dengan jisim m , kekakuan k dan redaman c apabila dikenakan satu fungsi daya umum $f(t)$

(40 markah)

[c] Bagi nilai-nilai $m=1$ kg, $c=10$ Ns/m and $k=21$ N/m, tentukan nilai kutub-kutub sistem. Apakah kepentingan nilai ini?

(30 markah)

[d] Lukis rajah blok bagi sistem ini.

(20 markah)

6. [a] Formulate the 2-D divergence speed, v_d based on Figure 6[a].

Use :

$L = \text{lift vector}$

$c = \text{wing chord}$

$\theta = \text{elastic twist angle}$

$AC = \text{aerodynamic center}$

$K = \text{wing torsional stiffness}$

$M_0 = \text{wing pitching moment about AC}$

$$C_L = C_{L,0} + \frac{\partial C_L}{\partial \alpha}(\alpha + \theta)$$

(30 marks)

- [b] What is the preferred way to increase divergence speed, v_d ? With respect to aerodynamic and flexural centers, what condition is required for a wing to be stable at all speed against divergence?

(20 marks)

- [c] Figure 6[b] shows a model of wing section flutter. For determining flutter speed, derive the instantaneous equations of vertical force and moment equilibrium in terms of ω , y_0 and α_0 which represent simple harmonic motion.

Use :

$$y = y_0 e^{i\omega t}$$

$$\alpha = \alpha_0 e^{i\omega t}$$

$k = \text{flexural stiffness}$

$k_\theta = \text{torsional stiffness}$

$m = \text{wing mass}$

$I_O = \text{moment of inertia about O}$

$L = L_y y + L_{\dot{y}} \dot{y} + L_{\ddot{y}} \ddot{y} + L_\alpha \alpha + L_{\dot{\alpha}} \dot{\alpha} + L_{\ddot{\alpha}} \ddot{\alpha} = \text{total aerodynamic lift}$

$M = M_y y + M_{\dot{y}} \dot{y} + M_{\ddot{y}} \ddot{y} + M_\alpha \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_{\ddot{\alpha}} \ddot{\alpha} = \text{total nose-up moment}$

(30 marks)

- [d] Describe a technique to determine the flutter speed on an actual aircraft. **(20 marks)**

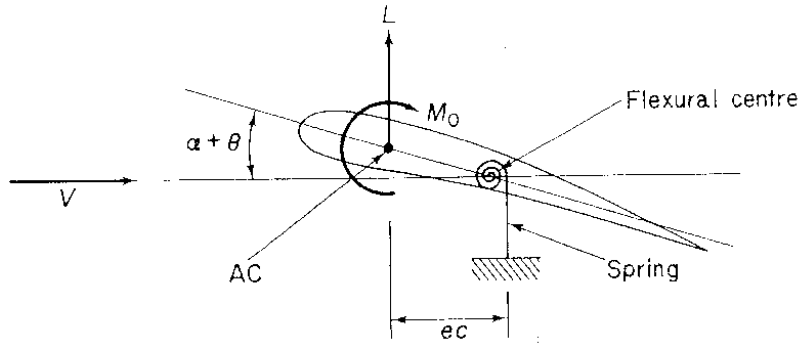


Figure 6[a]

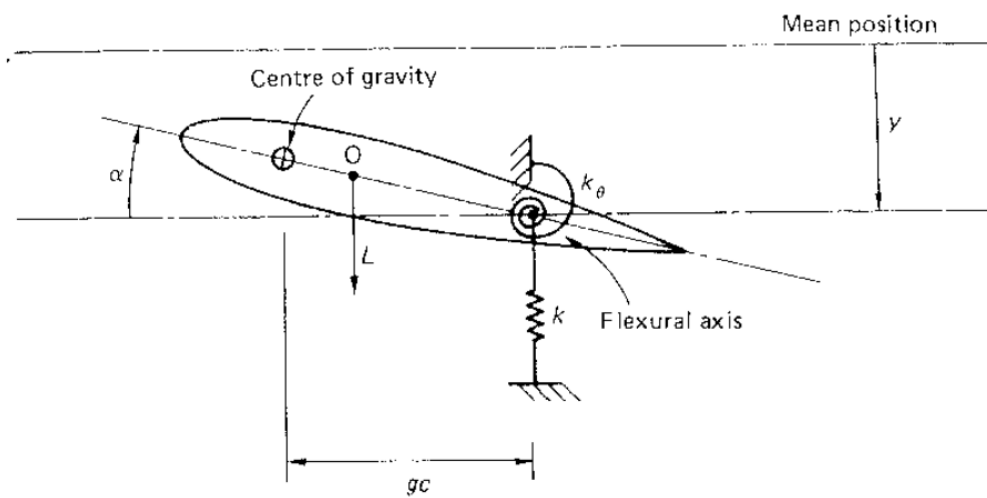


Figure 6[b]

6. [a] Terbitkan laju kecapahan, v_d 2-D berdasarkan Rajah 6[a].

Gunakan :

L = lift vector

c = wing chord

θ = elastic twist angle

AC = aerodynamic center

K = wing torsional stiffness

M_0 = wing pitching moment about AC

$$C_L = C_{L,0} + \frac{\partial C_L}{\partial \alpha} (\alpha + \theta)$$

(30 markah)

- [b] Apakah cara terbaik untuk meningkatkan laju kecapahan, v_d ? Berkaitan dengan pusat aerodinamik dan lenturan, apakah keadaan yang diperlukan oleh sayap untuk stabil pada semua laju?

(20 markah)

- [c] Rajah 6[b] menunjukkan model kibraran keratan sayap . Untuk menentukan laju kibraran, terbitkan persamaan ketikaimbangan daya tegak dan momen dalam terma ω , y_0 dan α_0 yang mewakili pergerakan harmonik mudah.

Gunakan :

$$y = y_0 e^{i\omega t}$$

$$\alpha = \alpha_0 e^{i\omega t}$$

k = flexural stiffness

k_θ = torsional stiffness

m = wing mass

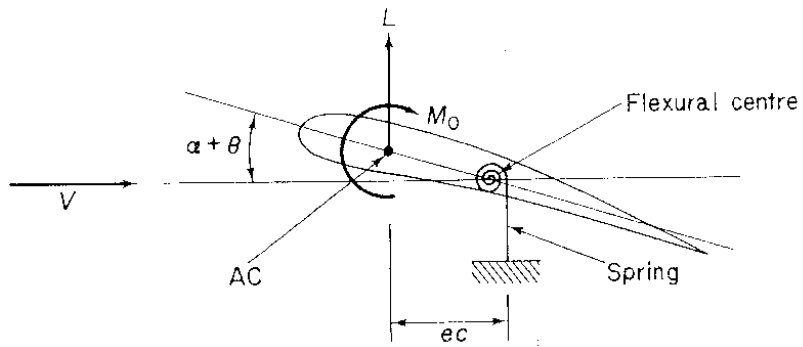
I_O = moment of inertia about O

$L = L_y y + L_{\dot{y}} \dot{y} + L_{\ddot{y}} \ddot{y} + L_\alpha \alpha + L_{\dot{\alpha}} \dot{\alpha} + L_{\ddot{\alpha}} \ddot{\alpha}$ = total aerodynamic lift

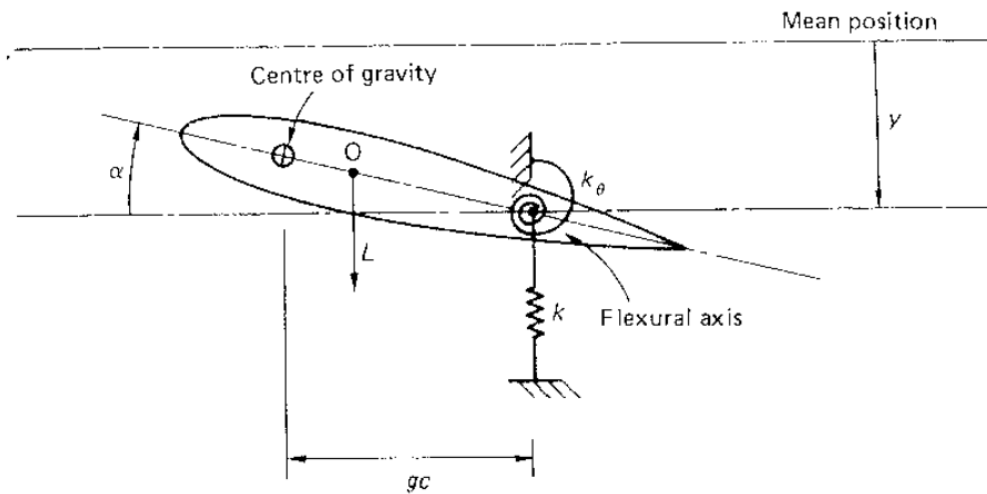
$M = M_y y + M_{\dot{y}} \dot{y} + M_{\ddot{y}} \ddot{y} + M_\alpha \alpha + M_{\dot{\alpha}} \dot{\alpha} + M_{\ddot{\alpha}} \ddot{\alpha}$ = total nose-up moment

(30 markah)

- [d] Terangkan satu teknik untuk menentukan laju kibraran pada pesawat sebenar. **(20 markah)**



Rajah 6[a]



Rajah 6[b]

Fundamental Equations in Vibration

$$1. \zeta = \frac{c}{2m\omega_n};$$

$$2. x(t) = Ce^{-\zeta\omega_n t} \sin(\omega_d t + \psi), \quad \omega_d = \sqrt{1 - \zeta^2}\omega_n$$

$$C = \sqrt{x_0^2 + \frac{(\dot{x}_0 + \zeta\omega_n x_0)^2}{(1 - \zeta^2)\omega_n^2}} \quad ; \quad \psi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}\omega_n x_0}{\dot{x}_0 + \zeta\omega_n x_0}$$

$$3. \text{ For } F(t) = me\omega^2 \sin \omega t$$

$$X = \frac{me\omega^2}{\sqrt{(k - M\omega^2)^2 + (c\omega^2)^2}}, \quad \phi = \tan^{-1} \left[\frac{c\omega}{k - M\omega^2} \right]$$

$$\frac{F_T}{F_0} = \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2}$$

$$4. \text{ For } y = Y \sin \omega t,$$

$$\frac{X}{Y} = \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2} \quad \phi = \tan^{-1} \left[\frac{2\zeta r^3}{1 + (4\zeta^2 - 1)r^2} \right]$$

$$\frac{F_T}{kY} = r^2 \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2}$$

$$5. \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \frac{1}{\Delta(\omega)} \quad ; \quad \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$6. \text{ For cantilever beam with load } P \text{ at the free end, } \delta_{\max} = PL^3(3EI)^{-1}$$

$$7. \text{ For cantilever beam with uniform load } w, \delta_{\max} = wL^4(8EI)^{-1}$$

LAPLACE TRANSFORM PAIRS

	$F(s)$	$f(t)$
1	1	$\delta(t) = \text{Dirac delta function}$
2	$\frac{1}{s}$	$u(t) = \text{unit step function}$
3	$\frac{1}{s^2}$	t
4	$\frac{1}{s^n} \quad (n = 1, 2, \dots)$	$\frac{t^{n-1}}{(n-1)!}$
5	$\frac{1}{s-a}$	e^{at}
6	$\frac{a}{s^2+a^2}$	$\sin at$
7	$\frac{s}{s^2+a^2}$	$\cos at$
8	$\frac{a}{s^2-a^2}$	$\sinh at$
9	$\frac{s}{s^2-a^2}$	$\cosh at$
10	$\frac{1}{(s^2+a^2)^2}$	$\frac{1}{2a^3}(\sin at - at \cos at)$
11	$\frac{s}{(s^2+a^2)^2}$	$\frac{1}{2a}(t \sin at)$
12	$\frac{a}{(s-b)^2+a^2}$	$e^{bt} \sin at$
13	$\frac{s-b}{(s-b)^2+a^2}$	$e^{bt} \cos at$
14	$\frac{1}{s^2+2\zeta\omega s+\omega^2}$	$\frac{1}{\omega_d} e^{-\zeta\omega t} \sin \omega_d t,$
15	$\frac{s+2\zeta\omega}{s^2+2\zeta\omega s+\omega^2}$	$e^{-\zeta\omega t} \left[\cos \omega_d t + \frac{\zeta}{(1-\zeta^2)^{1/2}} \sin \omega_d t \right]$ where $\omega_d = \omega(1-\zeta^2)^{1/2}$

Vibration-related Formulas

1. $\zeta = \frac{c}{2\omega_n m}$
2. $x(t) = e^{-\zeta\omega_n t} (A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t))$
3. $\omega_d = \sqrt{1 - \zeta^2} \omega_n$
4. $x_p = X \sin(\omega t - \phi)$, $X = \frac{F_0 / k}{\left((1 - r^2)^2 + (2\zeta r)^2 \right)^{1/2}}$, $\phi = \tan^{-1} \frac{2\zeta r}{1 - r^2}$
5. $\frac{X}{Y} = \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2}$, $\phi = \tan^{-1} \left[\frac{2\zeta r^3}{1 + (4\zeta^2 - 1)r^2} \right]$
6. $\frac{F_T}{kY} = r^2 \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2}$
7. $TR = \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2}$
8. $\frac{mX}{m_0 e} = \frac{r^2}{\left[(1 - r^2)^2 + (2\zeta r)^2 \right]^{1/2}}$
9. $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
10. $\det(A) = ad - bc$

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