Second Semester Examination
2017/2018 Academic Session

May / June 2018

MSG265 - Design And Analysis Of Experiments
(Rekabentuk dan Analisis Ujikaji)

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **TWELVE (12)** pages of printed materials before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA BELAS (12)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions : Answer **all four (4)** questions.

*[**Arahan** : Jawab **semua empat (4)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

Question 1

(a) Consider the three-factor model

$$y_{ijk} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\beta\gamma)_{jk} + \varepsilon_{ijk}$$

Assuming that all the factors are random

(i) Complete the analysis of variance table below.

(ii) Propose appropriate test statistics for all effects.

Source	DF	E(MS)
A		$\sigma^2 + c\sigma_{\tau\beta}^2 + bc\sigma_{\tau}^2$
	b-1	$\sigma^2 + c\sigma_{\tau\beta}^2 + a\sigma_{\beta\gamma}^2 + ac\sigma_{\beta}^2$
C		$\sigma^2 + a\sigma_{\beta\gamma}^2 + ab\sigma_{\gamma}^2$
	(a-1)(b-1)	$\sigma^2 + c\sigma_{\tau\beta}^2$
BC		$\sigma^2 + a\sigma_{\beta\gamma}^2$
	b(a-1)(c-1)	σ^2
Total		

[40 marks]

(b) Consider the following incomplete ANOVA table:

Source	SS	DF	MS	F
A		1	50.00	
B	80.00		40.00	
AB	30.00	2		
Error				
Total	172.00	17		

In addition to the ANOVA table, the experiment has been replicated three times and that the totals of the three replicates are 10, 12 and 14 respectively. The original experiment was run as a completely randomized design. Assuming both factors are fixed. Answer the following questions:

(i)   Complete the ANOVA table above.

(ii)   Suppose that the experiment had been run in a randomized complete block design, construct the ANOVA table and give your conclusion.

(iii)   Is blocking important in this experiment?

[60 marks]

Soalan 1

(a)   Pertimbangkan model tiga-faktor

$$y_{ijk} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\beta\gamma)_{jk} + \varepsilon_{ijk}$$

Andaikan semua faktor adalah rawak:

(i)   Lengkapkan jadual analisis varians di bawah.

(ii)   Cadangkan ujian statistik yang bersesuaian bagi semua kesan.

Sumber	DF	E(MS)
A		$\sigma^2 + c\sigma_{\tau\beta}^2 + bc\sigma_{\tau}^2$
	b-1	$\sigma^2 + c\sigma_{\tau\beta}^2 + a\sigma_{\beta\gamma}^2 + ac\sigma_{\beta}^2$
C		$\sigma^2 + a\sigma_{\beta\gamma}^2 + ab\sigma_{\gamma}^2$
	(a-1)(b-1)	$\sigma^2 + c\sigma_{\tau\beta}^2$
BC		$\sigma^2 + a\sigma_{\beta\gamma}^2$
	b(a-1)(c-1)	σ^2
Total		

[40 markah]

(b)   *Pertimbangkan jadual ANOVA tidak lengkap di bawah:*

Sumber	SS	DF	MS	F
A		1	50.00	
B	80.00		40.00	
AB	30.00	2		
Ralat				
Jumlah	172.00	17		

Sebagai tambahan kepada jadual ANOVA, eksperimen ini direplika sebanyak tiga kali dan jumlah bagi setiap replika adalah 10, 12 dan 14 masing-masing. Eksperimen asal dilakukan sebagai rekabentuk rawak lengkap. Andaikan kedua-dua faktor adalah tetap. Jawab soalan-soalan berikut:

(i)   *Lengkapkan jadual ANOVA di atas.*

(ii)   *Sekiranya eksperimen ini dilakukan dalam rekabentuk blok rawak lengkap, bina jadual ANOVA dan berikan kesimpulan anda.*

(iii)   *Adakah pemblokan penting dalam eksperimen ini?*

                           [60 markah]

Question 2

(a)   State the three basic principles of experimental design and two important properties of replication.

                           [10 marks]

(b)   A tire company with 4 factories has 3 teams of workers at each factory (the teams at Factory A are not the same as those at Factory B, and so on). The company is interested in comparing output among factories, as well as among teams within factories. The experiment is conducted to measure the output of each team on 3 occasions.

    (i)   Write the appropriate statistical model for the experiment, stating all elements (parameters and random variables).

    (ii)   Give the general analysis of variance for the situation described in this experiment. Your ANOVA table should consist of source of variation, degrees of freedom, expected mean squares and F_o.

                           [40 marks]

(c)   A study was conducted to measure Irrigation and Nitrogen effects on Sweet Corn Row Numbers. Plots were 115 by 63-feet, arranged in a split-plot design, with four irrigation treatments as whole plots and 15 nitrogen applications randomly allocated on 23 by 21-foot subplots in each irrigation treatment. There were 3 replications of the experiment (blocks).

         (i)   State the design being used here?

         (ii)   State the linear model which you would use to analyze these data. Define all the parameters involved in this model.

         (iii)   Complete the following ANOVA table and state the conclusion.

Source	df	SS	MS	F_o	F_c
Rep		5.94			
Irrigation		2.49			
Rep x Irrigation		3.54			
Nitrogen		80.92			
Irrigation x Nitrogen		14.28			
Error		29.12			
Total		136.29			

[50 marks]

Soalan 2

(a)   Nyatakan tiga prinsip asas rekabentuk ujikaji dan dua kepentingan pengreplikaan.

[10 markah]

(b)   Sebuah syarikat tayar dengan 4 kilang mempunyai 3 pasukan pekerja di setiap kilang (pasukan pekerja di kilang A adalah berbeza dengan pasukan pekerja di kilang B, dan seterusnya). Syarikat tersebut berminat untuk membandingkan output yang dihasilkan antara kilang dan juga antara pasukan pekerja dalam kilang masing-masing. Ujikaji dilakukan untuk mengukur output setiap pasukan sebanyak 3 kali.

(i)   *Tuliskan model statistik yang bersesuaian, nyatakan semua elemen yang terlibat (parameter dan pemboleh ubah rawak).*

(ii)   *Berikan analisis varians umum bagi situasi yang diterangkan dalam ujikaji ini. Jadual ANOVA tersebut perlu mengandungi sumber variasi, darjah kebebasan, jangkaan min kuasa dua dan F_o.*

[40 markah]

(c)   *Satu kajian dilakukan bagi mengukur kesan saliran dan Nitrogen ke atas bilangan baris jagung manis. Plot adalah 115 x 63-kaki, disusun mengikut rekabentuk split-plot, dengan empat rawatan saliran sebagai 'whole plots' dan 15 aplikasi nitrogen diumpukkan secara rawak ke atas 23 x 21-kaki subplot dalam setiap rawatan saliran. Ujikaji diulang sebanyak 3 kali (blok).*

(i)   *Nyatakan rekabentuk yang digunakan?*

(ii)   *Nyatakan model linear yang digunakan untuk menganalisis data ini. Takrifkan semua parameter yang terlibat dalam model ini.*

(iii)   *Lengkapkan jadual ANOVA berikut dan nyatakan kesimpulannya.*

Sumber	dk	SS	MS	F_o	F_c
Rep		5.94			
Saliran		2.49			
Rep x Saliran		3.54			
Nitrogen		80.92			
Saliran x Nitrogen		14.28			
Ralat		29.12			
Jumlah		136.29			

[50 markah]

Question 3

Consider a 2^4 factorial design to identify the effect of pH, concentration, time and pressure on the yield in an industrial process. The experiments were replicated twice. The data are given below:

Treatment	pH	Concentration	Time	Pressure	Yield (Rep I)	Yield (Rep II)
1	3	10	0.2	0	28.64	31.06
2	3	10	0.07	0	27.93	28.71
3	10	10	0.2	0	8.86	10.844
4	10	10	0.07	0	17.99	19.05
5	3	100	0.2	0	3.86	5.02
6	3	100	0.07	0	24.91	23.67
7	10	100	0.2	0	11.31	10.17
8	10	100	0.07	0	12.32	12.3
9	3	10	0.07	3000	36.69	36.03
10	3	10	0.2	3000	23.96	21.76
11	10	10	0.07	3000	23.38	24
12	10	10	0.2	3000	13.71	12.31
13	3	100	0.2	3000	8.68	10.9
14	3	100	0.07	3000	20.09	22.33
15	10	100	0.07	3000	17.45	17.29
16	10	100	0.2	3000	10.43	10.95

Given that:

$$contrast_A = -121.876 \qquad contrast_{AB} = 87.396$$
$$contrast_B = -143.244 \qquad contrast_{AC} = 31.284$$
$$contrast_C = -141.676 \qquad contrast_{AD} = 20.036$$
$$contrast_D = 33.316 \qquad contrast_{BC} = -16.404$$
$$contrast_{BD} = -4.196 \qquad contrast_{CD} = -27.444$$
$$contrast_{ABC} = 60.796 \qquad contrast_{ABD} = -9.076$$
$$contrast_{ACD} = -1.044 \qquad contrast_{BCD} = 40.724$$
$$contrast_{ABCD} = -53.116$$

(a) Analyse the full factorial design using analysis of variance and give your conclusion.

[40 marks]

(b) Due to some restrictions, it is impossible to perform a complete replicate of a factorial design in one block. To overcome this problem, the experimenter decided to run the experiment in two blocks for each replicate.

    (i) Give the outline of the design. What is the name of the design?

    (ii) Analyze the data and draw conclusion. Is there any difference with the findings in part (a)?

[60 marks]

Soalan 3

Pertimbangkan reka bentuk faktoran 2^4 bagi mengenalpasti kesan pH, kepekatan, masa dan tekanan ke atas hasil dalam suatu proses industri. Eksperimen diulang sebanyak dua kali. Data bagi eksperimen diberikan di bawah:

Rawatan	pH	Kepekatan	Masa	Tekanan	Hasil (Rep I)	Hasil (Rep II)
1	3	10	0.2	0	28.64	31.06
2	3	10	0.07	0	27.93	28.71
3	10	10	0.2	0	8.86	10.844
4	10	10	0.07	0	17.99	19.05

5	3	100	0.2	0	3.86	5.02
6	3	100	0.07	0	24.91	23.67
7	10	100	0.2	0	11.31	10.17
8	10	100	0.07	0	12.32	12.3
9	3	10	0.07	3000	36.69	36.03
10	3	10	0.2	3000	23.96	21.76
11	10	10	0.07	3000	23.38	24
12	10	10	0.2	3000	13.71	12.31
13	3	100	0.2	3000	8.68	10.9
14	3	100	0.07	3000	20.09	22.33
15	10	100	0.07	3000	17.45	17.29
16	10	100	0.2	3000	10.43	10.95

Diberikan:

$$kontrast_A = -121.876 \qquad kontrast_{AB} = 87.396$$
$$kontrast_B = -143.244 \qquad kontrast_{AC} = 31.284$$
$$kontrast_C = -141.676 \qquad kontrast_{AD} = 20.036$$
$$kontrast_D = 33.316 \qquad kontrast_{BC} = -16.404$$
$$kontrast_{BD} = -4.196 \qquad kontrast_{CD} = -27.444$$
$$kontrast_{ABC} = 60.796 \qquad kontrast_{ABD} = -9.076$$
$$kontrast_{ACD} = -1.044 \qquad kontrast_{BCD} = 40.724$$
$$kontrast_{ABCD} = -53.116$$

(a) *Analisa reka bentuk faktoran lengkap menggunakan analisis varians dan berikan kesimpulan anda.*

[40 marks]

(b)   *Disebabkan beberapa kekangan, adalah mustahil bagi menjalankan reka bentuk ujikaji faktoran dengan replika penuh dalam satu blok. Bagi mengatasi masalah tersebut, pengujikaji memutuskan untuk menjalankan ujikaji dalam dua blok bagi setiap replika.*

        (i)   *Beri gambaran bagi reka bentuk ujikaji ini. Apakah nama reka bentuk ini?*

        (ii)   *Analisa data dan berikan kesimpulan. Adakah terdapat perbezaan dengan dapatan dalam bahagian (a)?*

                                                     [60 markah]

Question 4

Consider the following experiment:

Run	Treatment Combination
1	d
2	ae
3	b
4	abde
5	cde
6	ac
7	bce
8	abcd

Answer the following questions about this experiment:

(a)   How many factors were investigated in this experiment?

(b)   How many factors are in the basic design?

(c)   Assume that the factors in the experiment are represented by the initial letters of the alphabet (A, B,etc), what are the design generators for the factors beyond the basic design?

(d)   Is this design a principal fraction?

(e)   What is the complete defining relation?

(f)   List all the alias structure for this design.

(g)   Estimate all the factor effects.

                                                     [100 marks]

Soalan 4

Pertimbangkan eksperimen berikut:

Larian	Kombinasi rawatan
1	d
2	ae
3	b
4	abde
5	cde
6	ac
7	bce
8	abcd

Jawab soalan berikut berkaitan eksperimen ini:

(a) Berapa banyak faktor yang diselidik dalam eksperimen ini?

(b) Berapa banyak faktor yang terdapat dalam reka bentuk asas?

(c) Diandaikan bahawa faktor dalam eksperimen ini diwakili dengan abjad (A, B,dan sebagainya), apakah generator reka bentuk bagi faktor yang melebihi reka bentuk asas?

(d) Adakah reka bentuk ini adalah pecahan prinsipal?

(e) Apakah hubungan pentakrif lengkap?

(f) Senaraikan semua struktur alias bagi reka bentuk ini.

(g) Anggarkan semua kesan faktor.

[100 markah]

FORMULA

Expected Mean Square: Split-Plot Design

$$\tau_i : \sigma^2 + ab\sigma_\tau^2$$

$$\beta_j : \sigma^2 + b\sigma_{\tau\beta}^2 + \frac{rb\sum \beta_j^2}{a-1}$$

$$\gamma_k : \sigma^2 + \frac{ra\sum \gamma_k^2}{ab-1}$$

$$(\tau\beta)_{ij} : \sigma^2 + b\sigma_{\tau\beta}^2$$

$$(\beta\gamma)_{jk} : \sigma^2 + \frac{r\sum\sum (\beta\gamma)_{jk}^2}{(a-1)(b-1)}$$

$$\varepsilon_{ijk} : \sigma^2$$

Two-stage Nested Design

$$SS_T = \sum_i \sum_j \sum_k y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \sum_{i=1}^{a} \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$SS_{B(A)} = \sum_{i=1}^{a} \sum_{j=1}^{b} \frac{y_{ij.}^2}{n} - \sum_{i=1}^{a} \frac{y_{i..}^2}{bn}$$

$$SS_E = SS_T - SS_A - SS_{B(A)}$$

Expected Mean Squares: Two-stage Nested Design

$$E[MS_A] = \sigma^2 + \frac{bn\sum \tau_i^2}{a-1}$$

$$E[MS_{B(A)}] = \sigma^2 + \frac{n\sum\sum \beta_{j(i)}^2}{a(b-1)}$$

$$E[MS_E] = \sigma^2$$

- oooOOooo -