Second Semester Examination
2017/2018 Academic Session

May/June 2018

MAT203 - Vector Calculus
(Kalkulus Vektor)

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **SEVEN (7)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH (7)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all TEN (10)** questions.

*[**Arahan:** Jawab **semua SEPULUH (10)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

Question 1

Let $\mathbf{a} = \langle 2, 1 \rangle$ and $\mathbf{b} = \langle 8, 12 \rangle$.

(a) Draw $\mathbf{a}$ and $\mathbf{b}$ on the plane of two dimensional Cartesian coordinate system.

(b) Let $\mathbf{u} = proj_{\mathbf{a}}\,\mathbf{b}$ and find $\mathbf{u}$.

(c) Find $\mathbf{v}$ where $\mathbf{v} = \mathbf{b} - \mathbf{u}$ and add both $\mathbf{u}$ and $\mathbf{v}$ on the drawing in part (a).

(d) Show that $\mathbf{u}$ is orthogonal to $\mathbf{v}$.

[10 marks]

Soalan 1

Biar $\mathbf{a} = \langle 2, 1 \rangle$ *dan* $\mathbf{b} = \langle 8, 12 \rangle$.

(a) Lukiskan $\mathbf{a}$ *dan* $\mathbf{b}$ *pada satah dalam sistem koordinat Cartesan dua matra.*

(b) Biar $\mathbf{u} = proj_{\mathbf{a}}\,\mathbf{b}$ *dan dapatkan* $\mathbf{u}$.

(c) Dapatkan $\mathbf{v}$ *yang mana* $\mathbf{v} = \mathbf{b} - \mathbf{u}$ *dan tambahkan kedua-dua* $\mathbf{u}$ *dan* $\mathbf{v}$ *pada lukisan dalam bahagian (a).*

(d) Tunjukkan bahawa $\mathbf{u}$ *adalah berserenjang dengan* $\mathbf{v}$.

[10 markah]

Question 2

(a) Find the point at which the line $x = 1 - t, y = t, z = 1 + 3t$ intersects with the plane $4x - 2y - z = 6$.

(b) Find cylindrical coordinates and spherical coordinates for the point whose Cartesian coordinates are $(x, y, z) = (-4, 8, -2)$.

[12 marks]

Soalan 2

(a) Dapatkan titik yang mana garis lurus $x = 1 - t, y = t, z = 1 + 3t$ *bersilang dengan satah* $4x - 2y - z = 6$.

(b) Dapatkan koordinat silinder dan koordinat sfera untuk titik yang koordinat Cartesannya ialah $(x, y, z) = (-4, 8, -2)$.

[12 markah]

Question 3

The position function of a moving particle is $\mathbf{r}(t) = t^2\mathbf{i} + 5t\mathbf{j} + (t^2 - 16t)\mathbf{k}$. Find the time, t, at which the speed of the particle is at its minimum.

[7 marks]

Soalan 3

Fungsi kedudukan bagi suatu zarah yang bergerak ialah $\mathbf{r}(t) = t^2\mathbf{i} + 5t\mathbf{j} + (t^2 - 16t)\mathbf{k}$. *Dapatkan masa, t, yang mana kelajuan zarah tersebut adalah minimum.*

[7 markah]

Question 4

Given a curve $\mathbf{x}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}$.

(a) Find the point, the unit tangent vector, $\mathbf{T}$, the principal unit normal vector, $\mathbf{N}$ and the binormal vector, $\mathbf{B}$ at $t = 0$.

(b) Find the equation for the osculating plane at $t = 0$.

[10 marks]

Soalan 4

Diberi suatu lengkung $\mathbf{x}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}$.

(a) Dapatkan titik, vektor tangen unit, $\mathbf{T}$, *vektor normal unit utama,* $\mathbf{N}$ *dan vektor binormal,* $\mathbf{B}$ *pada* $t = 0$.

(b) Dapatkan persamaan untuk satah oskulasi pada $t = 0$.

[10 markah]

Question 5

Let $\mathfrak{R}$ be the region in $\mathbb{R}^2$ described by the parametric equations:

$$x = u(3t^2 - u^2) \text{ and } y = t(3u^2 - t^2)$$

for $0 \leq t \leq 1$ and $0 \leq u \leq 1$.

(a) Find the Jacobian $J = \begin{vmatrix} \dfrac{\partial x}{\partial t} & \dfrac{\partial x}{\partial u} \\ \dfrac{\partial y}{\partial t} & \dfrac{\partial y}{\partial u} \end{vmatrix}$.

(b) Use the formula $dA = \left| J \right| dt\, du$ to compute the area of the region $\mathfrak{R}$.

[10 marks]

Soalan 5

Biar $\mathfrak{R}$ suatu kawasan dalam $\mathbb{R}^2$ diterangkan oleh persamaan parametrik:

$$x = u(3t^2 - u^2) \;\; dan \;\; y = t(3u^2 - t^2)$$

untuk $0 \leq t \leq 1$ *dan* $0 \leq u \leq 1$.

(a) Dapatkan Jacobian $J = \begin{vmatrix} \dfrac{\partial x}{\partial t} & \dfrac{\partial x}{\partial u} \\ \dfrac{\partial y}{\partial t} & \dfrac{\partial y}{\partial u} \end{vmatrix}$.

(b) Gunakan rumus $dA = \left| J \right| dt\, du$ *untuk mengira luas rantau* $\mathfrak{R}$.

[10 markah]

Question 6

Let W be the region in $\mathbb{R}^3$ inside the sphere of radius 2 centered at the origin and outside the cylinder of radius 1 centered about the $z-$axis. Use cylindrical coordinates to evaluate

$$\iiint_W (x^2 + y^2)\, dV.$$

(Hint: Integrate with respect to r first.)

[10 marks]

Soalan 6

Biar W suatu kawasan di $\mathbb{R}^3$ dalam bahagian sfera yang berjejari 2 berpusat di asalan dan di luar silinder yang berjejari 1 berpusat sekitar paksi$-z$. Gunakan koordinat silinder untuk menilai

$$\iiint_W (x^2+y^2)\,dV.$$

(Petunjuk: Kamirkan dengan r terdahulu.)

[10 markah]

Question 7

Let $\mathbf{F}$ be the vector field $\mathbf{F}(x,y) = (2x-3y)\,\mathbf{i} - 3x\,\mathbf{j}$ and let C be the part of the unit circle (oriented in the counterclockwise direction) that lies in the first quadrant.

(a) Write down a parametrization for C.

(b) Use the parametrization in part (a) to evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$.

(c) Show that this vector field is conservative and find a function, f, such that $\mathbf{F} = \nabla f$.

(d) Now use the Fundamental Theorem of Line Integrals to evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$.

[13 marks]

Soalan 7

Biar $\mathbf{F}$ suatu medan vektor $\mathbf{F}(x,y) = (2x-3y)\,\mathbf{i} - 3x\,\mathbf{j}$ dan biar C sebahagian daripada bulatan unit (berorientasi ke arah lawan jam) yang terletak dalam kuadran pertama.

(a) Tuliskan suatu parameteran bagi C.

(b) Gunakan parameteran dalam bahagian (a) untuk menilai kamiran garis $\int_C \mathbf{F}\cdot d\mathbf{r}$.

(c) Tunjukkan bahawa medan vektor ini adalah medan abadi dan dapatkan satu fungsi, f, iaitu $\mathbf{F} = \nabla f$.

(d) Sekarang gunakan Teorem Asas bagi Kamiran Garis untuk menilai kamiran garisan $\int_C \mathbf{F}\cdot d\mathbf{r}$.

[13 markah]

Question 8

Use Green's Theorem to evaluate the line integral $\int_C x^2y^2\,dx + 4xy^3\,dy$ along the positively oriented triangle, C, with vertices at $(0,0)$, $(1,3)$, and $(0,3)$.

[8 marks]

Soalan 8

Gunakan Teorem Green untuk menilai kamiran garis $\int_C x^2y^2\,dx + 4xy^3\,dy$ *di sepanjang sisi segitiga yang berorientasi positif, C, dengan bucu-bucu* $(0,0)$, $(1,3)$, *dan* $(0,3)$.

[8 markah]

Question 9

Evaluate the surface integral $\iint_S y\,dS$ where S is the surface $z = \frac{2}{3}(x^{3/2} + y^{3/2})$ in $0 \le x, y \le 1$.

[8 marks]

Soalan 9

Nilaikan kamiran permukaan $\iint_S y\,dS$ *yang mana S ialah permukaan* $z = \frac{2}{3}(x^{3/2} + y^{3/2})$ *dalam* $0 \le x, y \le 1$.

[8 markah]

Question 10

(a) Define and briefly explain the Stokes Theorem.

(b) Use Stokes Theorem to evaluate

$$\iint_S curl(\mathbf{F}) \cdot d\mathbf{S}$$

where $\mathbf{F}(x,y,z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$ and S is the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$ and is oriented upward.

[12 marks]

Soalan 10

(a) Nyatakan dan terangkan secara ringkas Teorem Stokes.

(b) Gunakan Teorem Stokes untuk menilai

$$\iint_S curl(\mathbf{F}) \cdot d\mathbf{S}$$

yang mana $\mathbf{F}(x,y,z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$ *dan S ialah bahagian paraboloid* $z = 9 - x^2 - y^2$ *yang terletak di atas satah* $z = 5$ *dan berorientasi ke atas.*

[12 markah]