

SULIT



UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2017/2018 Academic Session

May / June 2018

MAT101 - Calculus
(Kalkulus)

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **ELEVEN (11)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEBELAS (11)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all six (6)** questions.

Arahan: Jawab **semua enam (6)** soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

...2/-

SULIT

Question 1

(a) Suppose $f(x) = \begin{cases} x^2 & , \quad x > 2 \\ a+1 & , \quad x = 2 \\ x+2 & , \quad 0 < x < 2 \\ \frac{|x|}{x} & , \quad x < 0 \end{cases}$.

(i) Find $\lim_{x \rightarrow 2} f(x)$.

(ii) Determine the value of a if f is continuous at 2.

(iii) Find $\lim_{x \rightarrow 0^-} f(x)$.

[35 marks]

(b) Find the following limits.

(i) $\lim_{t \rightarrow 0} \frac{\sin 3t}{\sin 2t}$

(ii) $\lim_{x \rightarrow 0} \frac{3 - \sqrt{9+x}}{x}$

[30 marks]

(c) Show that $\lim_{x \rightarrow 0} \left(x^2 \sin \frac{1}{x} \right) = 0$.

[25 marks]

(d) Show that $\lim_{x \rightarrow a} [f(x)g(x)]$ may exist although $\lim_{x \rightarrow a} f(x)$ or $\lim_{x \rightarrow a} g(x)$ does not exist.

[10 marks]

...3/-

Soalan 1

$$(a) \text{ Andaikan } f(x) = \begin{cases} x^2 & , \quad x > 2 \\ a+1 & , \quad x = 2 \\ x+2 & , \quad 0 < x < 2 \\ \frac{|x|}{x} & , \quad x < 0 \end{cases}$$

(i) Cari $\lim_{x \rightarrow 2} f(x)$.

(ii) Tentukan nilai a jika f adalah selanjar pada 2.

(iii) Cari $\lim_{x \rightarrow 0^-} f(x)$.

[35 markah]

(b) Cari had yang berikut.

(i) $\lim_{t \rightarrow 0} \frac{\sin 3t}{\sin 2t}$

(ii) $\lim_{x \rightarrow 0} \frac{3 - \sqrt{9+x}}{x}$

[30 markah]

(c) Tunjukkan bahawa $\lim_{x \rightarrow 0} \left(x^2 \sin \frac{1}{x} \right) = 0$.

[25 markah]

(d) Tunjukkan bahawa $\lim_{x \rightarrow a} [f(x)g(x)]$ boleh wujud walaupun $\lim_{x \rightarrow a} f(x)$ atau $\lim_{x \rightarrow a} g(x)$ tidak wujud.

[10 markah]

...4/-

SULIT

Question 2

- (a) Prove that $\lim_{x \rightarrow 1} (2x - 1) = 1$ using the ε - δ -definition. [25 marks]
- (b) The limit $\lim_{h \rightarrow 0} \frac{e^h - 1}{h}$ represents the derivative of some function f at some number a . State such function f and the corresponding number a . Hence, deduce the value of $\lim_{h \rightarrow 0} \frac{e^h - 1}{h}$. [15 marks]
- (c) Suppose $f(x) = \begin{cases} x^2 & , x \geq 0 \\ x^3 & , x < 0 \end{cases}$. Using the definition of derivative, determine whether $f'(0)$ exists. [25 marks]
- (d) Find the derivatives of the following functions. **Do not simplify your answer.**
- (i) $y = x^e + e$
- (ii) $y = \frac{\tan x}{\sqrt{x}}$
- (iii) $y = e^{(x^2+1)^5}$
- (iv) $y = xe^x \sin x$
- (v) $y = \cos(x \ln x)$ [35 marks]

Soalan 2

- (a) Buktikan bahawa $\lim_{x \rightarrow 1} (2x - 1) = 1$ dengan menggunakan takrif ε - δ . [25 markah]
- (b) Had $\lim_{h \rightarrow 0} \frac{e^h - 1}{h}$ mewakili terbitan bagi suatu fungsi f pada suatu nombor a . Nyatakan fungsi f tersebut dan nombor a yang sepadan. Justeru itu, deduksikan nilai untuk $\lim_{h \rightarrow 0} \frac{e^h - 1}{h}$. [15 markah]

- (c) Andaikan bahawa $f(x) = \begin{cases} x^2 & , x \geq 0 \\ x^3 & , x < 0 \end{cases}$. Seterusnya, dengan menggunakan takrif terbitan, tentukan sama ada $f'(0)$ wujud.

[25 markah]

- (d) Cari terbitan bagi fungsi yang berikut. **Jangan permudahkan jawapan anda.**

(i) $y = x^e + e$

(ii) $y = \frac{\tan x}{\sqrt{x}}$

(iii) $y = e^{(x^2+1)^5}$

(iv) $y = xe^x \sin x$

(v) $y = \cos(x \ln x)$

[35 markah]

Question 3

- (a) Suppose $\int_1^7 f(x) dx = 10$, $\int_4^7 f(x) dx = 1$ and $\int_1^4 g(x) dx = 4$. Compute $\int_1^4 f(x) + 3g(x) dx$.

[25 marks]

- (b) (i) Find $F'(x)$, where $F(x) = \int_{x^2}^x \sin(t^2) dt$.
- (ii) Find $F'(x)$, where $F(x) = \left(\int_0^x t \cos t dt \right)^5$.

[30 marks]

- (c) Show that $f(x) = \frac{x^3}{1+x^2}$ is an odd function. Hence, deduce the value of $\int_{-3}^3 \frac{x^3}{1+x^2}$.

[25 marks]

...6/-

- (d) Using the fact that the right Riemann sum R_n for $f(x) = x^2$ from 0 to 1 is $\sum_{i=1}^n \frac{i^2}{n^3}$, deduce the value of $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^2}{n^3}$.

[20 marks]

Soalan 3

- (a) Andaikan $\int_1^7 f(x) dx = 10$, $\int_4^7 f(x) dx = 1$ dan $\int_1^4 g(x) dx = 4$. Hitungkan $\int_1^4 f(x) + 3g(x) dx$.

[25 markah]

- (b) (i) Cari $F'(x)$ dengan $F(x) = \int_{x^2}^x \sin(t^2) dt$.
- (ii) Cari $F'(x)$ dengan $F(x) = \left(\int_0^x t \cos t dt \right)^5$.

[30 markah]

- (c) Tunjukkan bahawa $f(x) = \frac{x^3}{1+x^2}$ ialah fungsi ganjil. Justeru itu, deduksikan nilai $\int_{-3}^3 \frac{x^3}{1+x^2}$.

[25 markah]

- (d) Berdasarkan fakta bahawa hasil tambah Riemann kanan R_n bagi $f(x) = x^2$ dari 0 ke 1 ialah $\sum_{i=1}^n \frac{i^2}{n^3}$, deduksikan nilai $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^2}{n^3}$.

[20 markah]

...7/-

Question 4

- (a) The region bounded by the graph of $y = x^2$, the horizontal line $y = 4$ and the y -axis in the first quadrant is rotated about the vertical line $x = 2$. Express the volume of the solid generated using
- the shell method;
 - the washer method.

Do not evaluate your definite integrals.

[40 marks]

- (b) Find the following integral.

(i) $\int \sec^2 3x \sqrt{1 + \tan 3x} dx$

(ii) $\int x^3 \ln x dx$

(iii) $\int \frac{x-4}{x^2-5x+6} dx$

[60 marks]

Soalan 4

- (a) Rantau yang dibendung oleh graf $y = x^2$, garis mengufuk $y = 4$ dan paksi y dalam sukuan pertama diputarkan sekitar garis tegak $x = 2$. Ungkapkan isipadu pejal yang djanakan dengan menggunakan
- kaedah cangkerang;
 - kaedah cakera.

Jangan nilaikan kamiran tentu anda.

[40 markah]

...8/-

SULIT

(b) Cari kamiran yang berikut.

(i) $\int \sec^2 3x \sqrt{1 + \tan 3x} dx$

(ii) $\int x^3 \ln x dx$

(iii) $\int \frac{x-4}{x^2-5x+6} dx$

[60 markah]

Question 5

(a) (i) State the Intermediate Value Theorem.

[10 marks]

(ii) Show that the equation $2x + \cos x = 0$ has a real root between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

[30 marks]

(b) Suppose $a, b \in \mathbb{R}$ with $a < b$. Prove that $|\sin b - \sin a| < |b - a|$.

[35 marks]

(c) Prove that the equation $x^3 + x = 1$ cannot have more than one real roots.

[25 marks]

Soalan 5

(a) (i) Nyatakan Teorem Nilai Pertengahan.

[10 markah]

(ii) Tunjukkan bahawa persamaan $2x + \cos x = 0$ mempunyai punca nyata di antara $-\frac{\pi}{2}$ dan $\frac{\pi}{2}$.

[30 markah]

...9/-

SULIT

(b) Andaikan $a, b \in \mathbb{R}$ dengan $a < b$. Buktikan bahawa $|\sin b - \sin a| < |b - a|$.

[35 markah]

(c) Buktikan bahawa persamaan $x^3 + x = 1$ tidak boleh mempunyai lebih daripada satu punca nyata.

[25 markah]

Question 6

(a) Suppose $y = x^{\cos x}$. Find $\frac{dy}{dx}$ using logarithmic differentiation.

[25 marks]

(b) Suppose $f(x) = 2x + \cos x$. Assuming f is one-to-one, find the derivative of f^{-1} at 1.

[25 marks]

(c) Is the following statement true or false? State the correct answer, that is, either “TRUE” or “FALSE”.

(i) Suppose a function f is defined everywhere. If $f(-2) = f(2)$, then f is an even function.

(ii) The limit $\lim_{x \rightarrow \pi} \sin\left(\frac{1}{x}\right)$ does not exist.

(iii) If $\lim_{x \rightarrow a} [f(x) + g(x)]$ does not exist, then it cannot be the case that both $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist.

(iv) If a function f is differentiable at a , then $\lim_{x \rightarrow a} f(x)$ exists.

(v) The natural logarithmic function is defined by $\ln x = \int_1^x \frac{1}{t} dt$, $x > 0$.

(vi) The continuity of a function f on $[a, b]$ is sufficient to guarantee the Riemann integrability of f on $[a, b]$.

(vii) If $\lim_{x \rightarrow a} f(x) = \infty$, then $\lim_{x \rightarrow a} f(x)$ does not exist.

...10/-

SULIT

- (viii) If $f(a) = f(b)$ whenever $a = b$, then f is one-to-one.
- (ix) Suppose a function f is continuous on an open interval (a, b) . The Extreme Value Theorem tells us that the absolute maximum of f exists.
- (x) The Riemann integral $\int_a^b f(x) dx$ is defined to be $\lim_{n \rightarrow \infty} R_n$, where R_n is the corresponding right Riemann sum.

[50 marks]

Soalan 6

- (a) Andaikan $y = x^{\cos x}$. Cari $\frac{dy}{dx}$ dengan menggunakan pembezaan logaritma.
[25 markah]
- (b) Andaikan $f(x) = 2x + \cos x$. Dengan anggapan f adalah satu-ke-satu, cari terbitan f^{-1} pada 1.
[25 markah]
- (c) Adakah kenyataan berikut benar atau palsu? Nyatakan jawapan yang betul, iaitu sama ada "BENAR" atau "PALSU".
- (i) Andaikan fungsi f tertakrif di mana-mana. Jika $f(-2) = f(2)$, maka f ialah fungsi genap.
- (ii) Had $\lim_{x \rightarrow \pi} \sin\left(\frac{1}{x}\right)$ tidak wujud.
- (iii) Jika $\lim_{x \rightarrow a} [f(x) + g(x)]$ tidak wujud, maka tidak mungkin berlakunya kedua-dua $\lim_{x \rightarrow a} f(x)$ dan $\lim_{x \rightarrow a} g(x)$ wujud.
- (iv) Jika fungsi f terbezakan pada a , maka $\lim_{x \rightarrow a} f(x)$ wujud.
- (v) Fungsi logaritma asli ditakrifkan oleh $\ln x = \int_1^x \frac{1}{t} dt$, $x > 0$.
- (vi) Keseluruhan fungsi f pada $[a, b]$ adalah mencukupi untuk memastikan kebolehkamiran Riemann f pada $[a, b]$.
- (vii) Jika $\lim_{x \rightarrow a} f(x) = \infty$, maka $\lim_{x \rightarrow a} f(x)$ tidak wujud.

...11/-

- (viii) *Jika $f(a) = f(b)$ apabila $a = b$, maka f adalah satu-ke-satu.*
- (ix) *Andaikan fungsi f adalah selanjur pada selang terbuka (a, b) . Teorem Nilai Ekstrim memberitahu kita bahawa maksimum mutlak untuk f wujud.*
- (x) *Kamiran Riemann $\int_a^b f(x) dx$ ditakrifkan sebagai $\lim_{n \rightarrow \infty} R_n$, dengan R_n ialah hasil tambah Riemann kanan yang sepadan.*

[50 markah]

-ooo000ooo-