Second Semester Examination
2017/2018 Academic Session

May / June 2018

MAA111 - Algebra for Science Students
(Algebra untuk Pelajar Sains)

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **SEVEN (7)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH (7)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all four (4)** questions.

*[**Arahan:** Jawab **semua empat (4)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Question 1

(a) Given

$$\begin{array}{ccccccccc} x & + & y & + & z & = & 2, \\ x & + & 4y & - & z & = & k, \\ 2x & - & y & + & 4z & = & k^2, \end{array}$$

Find the value(s) of k, if any, such that the system have :

(i) no solution,

(ii) a unique solution, and

(iii) infinitely many solutions.

[10 marks]

(b) Solve the matrix equation $ABXA^{-1}B^{-1} = I + A$ for X (assuming that the matrices involved are invertible). Simplify your answer.

[10 marks]

(c) Determine whether each statement is true or false.

(i) Addition of matrices is not commutative.

(ii) If matrices $n \times n$ A and B are nonsingular matrices, then $A + B$ is a nonsingular matrix.

(iii) If the determinant of an $n \times n$ matrix is nonzero, then $A\vec{x} = \vec{0}$ has only the trivial solution.

(iv) A set of vectors $S = \{\overrightarrow{v_1}, \overrightarrow{v_2}, \ldots, \overrightarrow{v_k}\}$ in a vector space is called linearly dependent if the vector equation $c\overrightarrow{v_1} + c\overrightarrow{v_2} + \ldots + c\overrightarrow{v_k} = \vec{0}$ has only the trivial solution.

(v) The null space of A is the solution space of the homogeneous system $A\vec{x} = \vec{0}$.

[5 marks]

Soalan 1

(a) *Diberi*

$$\begin{array}{rcrcrcl} x & + & y & + & z & = & 2, \\ x & + & 4y & - & z & = & k, \\ 2x & - & y & + & 4z & = & k^2, \end{array}$$

Dapatkan nilai-nilai k *,jika wujud, supaya sistem ini*

(i) *tiada penyelesaian,*

(ii) *mempunyai penyelesaian unik, dan*

(iii) *penyelesaian tak terhingga.*

[10 markah]

(b) *Selesaikan persamaan matriks* $ABXA^{-1}B^{-1} = I + A$ *bagi* X *(dengan mengandaikan bahawa matriks yang terlibat adalah tersongsangkan). Ringkaskan jawapan anda.*

[10 markah]

(c) *Tentukan sama ada setiap pernyataan adalah benar atau palsu*

(i) *Penambahan matriks bukan komutatif.*

(ii) *Jika matriks* $n \times n$ A *dan* B *bukan matriks tak singular, maka matriks* $A + B$ *adalah tak singular.*

(iii) *Jika penentu bagi matriks* $A, n \times n$ *adalah tak sifar, maka* $A\vec{x} = \vec{0}$ *hanya mempunyai penyelesian remeh.*

(iv) *Suatu set vektor* $S = \left\{\overrightarrow{v_1}, \overrightarrow{v_2}, \ldots, \overrightarrow{v_k}\right\}$ *dalam ruang vektor bersandar linear sekiranya persamaan* $c\overrightarrow{v_1} + c\overrightarrow{v_2} + \ldots + c\overrightarrow{v_k} = \vec{0}$ *ini hanya mempunyai penyelesaian remeh.*

(v) *Ruang nol bagi* A *adalah ruang penyelesaian bagi sistem homogen* $A\vec{x} = \vec{0}$ *.*

[5 markah]

Question 2

(a) Given $A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 2 & 5 & 1 & 1 & 0 \\ 3 & 7 & 2 & 2 & -2 \\ 4 & 9 & 3 & -1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 3 & 0 & -4 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Based on the fact that matrices A and B are row-equivalent,

(i) find the rank and nullity of A,

(ii) find a basis for the nullspace of A,

(iii) find a basis for the row space of A,

(iv) find a basis for the column space of A,

(v) determine whether A is linearly dependent and, if so, find expressions of some of its elements as linear combinations of the others.

[19 marks]

(b) Let A be an $m \times n$ matrix (where $m < n$) whose rank is r.

(i) What is the largest value r can be?

(ii) How many vectors are in a basis for the row space of A?

(iii) How many vectors are in a basis for the column space of A?

[6 marks]

Soalan 2

(a) *Diberi* $A=\begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 2 & 5 & 1 & 1 & 0 \\ 3 & 7 & 2 & 2 & -2 \\ 4 & 9 & 3 & -1 & 4 \end{bmatrix}$ dan $B=\begin{bmatrix} 1 & 0 & 3 & 0 & -4 \\ 0 & 1 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Berdasarkan kepada fakta bahawa A *dan* B *ialah matriks baris bersamaan,*

(i) *cari pangkat dan kenolan* A,

(ii) *cari asas untuk ruang nol* A,

(iii) *dapatkan asas bagi ruang baris* A,

(iv) *dapatkan asas bagi ruang lajur* A,

(v) *tentukan sama ada bergantung secara linear dan, jika ya, cari ungkapan beberapa elemennya sebagai kombinasi linear yang lain.*

[19 markah]

(b) *Biar* A *suatu matriks* $m \times n$ *(dengan* $m < n$*) dan mempunyai pangkat* r.

(i) *Apakah nilai terbesar yang boleh dicapai oleh* r*?*

(ii) *Berapakah bilangan vektor dalam asas bagi ruang baris* A*?*

(iii) *Berapakah bilangan vektor dalam asas bagi ruang lajur* A*?*

[6 markah]

Question 3

(a) Find a least squares solution to the inconsistent system $A\vec{x}=\vec{b},$ where

$$A=\begin{bmatrix} 1 & 5 \\ 2 & -2 \\ -1 & 1 \end{bmatrix} \text{ and } \vec{b}=\begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}.$$

[6 marks]

(b) Let W be the subspace of $\mathbb{R}^3$ spanned by

$$\overrightarrow{W_1} = \begin{bmatrix} 1 \\ -3 \\ 5 \\ 0 \\ 5 \end{bmatrix}, \overrightarrow{W_2} = \begin{bmatrix} -1 \\ 1 \\ 2 \\ -2 \\ 3 \end{bmatrix}, \overrightarrow{W_3} = \begin{bmatrix} 0 \\ -1 \\ 4 \\ -1 \\ 5 \end{bmatrix}.$$

Find a basis for $W^{\perp}$.

[11 marks]

(c) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that rotates each point 90° counterclockwise about the origin. Show that T is a linear transformation.

[8 marks]

Soalan 3

(a) Cari *penyelesaian kuasa dua terkecil bagi sistem tak konsisten* $A\vec{x} = \vec{b}$, *yang mana*

$$A = \begin{bmatrix} 1 & 5 \\ 2 & -2 \\ -1 & 1 \end{bmatrix} \text{ dan } \vec{b} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}.$$

[6 markah]

(b) *Biar* W *adalah suatu subruang bagi* $\mathbb{R}^3$ *direntang oleh*

$$\overrightarrow{W_1} = \begin{bmatrix} 1 \\ -3 \\ 5 \\ 0 \\ 5 \end{bmatrix}, \overrightarrow{W_2} = \begin{bmatrix} -1 \\ 1 \\ 2 \\ -2 \\ 3 \end{bmatrix}, \overrightarrow{W_3} = \begin{bmatrix} 0 \\ -1 \\ 4 \\ -1 \\ 5 \end{bmatrix}.$$

Dapatkan asas bagi $W^{\perp}$ *.*

[11 markah]

(c) *Biar* $T : \mathbb{R}^2 \to \mathbb{R}^2$ *suatu transformasi yang memutar setiap titik* 90° *berlawanan dengan asalnya. Tunjukkan bahawa* T *ialah suatu transformasi linear.*

[8 markah]

Question 4

(a) Find the eigenvalues and the corresponding eigenspaces of

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 0 & -3 \\ 1 & 0 & -1 \end{bmatrix}.$$

If possible, find a matrix P that diagonalizes matrix A.

[17 marks]

(b) Assume A and B are $n \times n$ matrices with $\det(A) = 3$ and $\det(B) = -2.$ Using the given information, find

(i) $\det(3B^T)$,

(ii) $\det(B^{-1}A)$.

[8 marks]

Soalan 4

(a) Dapatkan nilai eigen serta ruang eigen yang sepadan bagi

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 0 & -3 \\ 1 & 0 & -1 \end{bmatrix}.$$

Dapatkan matriks P yang memenjurukan matriks A, jika boleh.

[17 markah]

(b) Andaikan A dan B suatu matriks $n \times n$ yang mempunyai penentu $(A) = 3$ dan penentu $(B) = -2.$ Menggunakan maklumat diberi, dapatkan

(i) Penentu $(3B^T)$,

(ii) Penentu $(B^{-1}A)$

[8 markah]

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