UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2017/2018

May/June 2018

MAA 101 - Calculus for Science Students I
[Kalkulus untuk Pelajar Sains I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **SEVEN (7)** pages of printed materials before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH (7)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **SIX** (6) questions.

*[**Arahan:** Jawab **ENAM** (6) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

Question 1

(a) Suppose $f(x) = x^2 - 1$ and $g(x) = 2x + 1$. Find the following compositions with their domains:

(i) $f \circ g$
(ii) $g \circ f$
(iii) $f \circ f$
(iv) $g \circ g$

(b) How is the graph $y = 2\sin(x)$ related to the graph of $y = \sin(x)$? Use your answer to sketch the graph of $y = 2\sin(x)$.

(c) Given $f(x) = \frac{4x-1}{2x+3}, x \neq -\frac{3}{2}$. Is f one-to-one (give justification to your answer)? If it is, find $f^{-1}(x)$ and its range.

[30 marks]

Soalan 1

(a) Andaikan $f(x) = x^2 - 1$ dan $g(x) = 2x + 1$. Cari komposisi berikut beserta domainnya:

(i) $f \circ g$
(ii) $g \circ f$
(iii) $f \circ f$
(iv) $g \circ g$

(b) Bagaimanakah graf $y = 2sin(x)$ berkait dengan graf $y = sin(x)$? Gunakan jawapan anda ini untuk melakarkan graf $y = 2sin(x)$.

(c) Diberi $f(x) = \frac{4x-1}{2x+3}, x \neq -\frac{3}{2}$. Adakah f satu-ke-satu (berikan justifikasi kepada jawapan anda)? Jika ya, cari $f^{-1}(x)$ dan julatnya.

[30 markah]

Question 2

(a) Compute the following limits:

(i) $\lim_{t \to 2} \left(\frac{t^2-2}{t^3-3t+5} \right)^2$
(ii) $\lim_{x \to -2} \frac{x+2}{x^3+8}$
(iii) $\lim_{t \to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t}$

(b) Find $\lim_{x \to -6} \frac{2x+12}{|x+6|}$, if it exists. If the limit does not exist, explain why.

(c) Prove that $\lim_{x\to 0^+}\left(\sqrt{x}e^{\sin\left(\frac{\pi}{x}\right)}\right)=0$.
(Hint: Start with $-1\le \sin\left(\frac{\pi}{x}\right)\le 1$)

(d) Determine all horizontal and vertical asymptotes for $y=\frac{1+x^4}{x^2-x^4}$.

(e) Determine the value of constant *c* in the following function such that *f* is continuous on $(-\infty,\infty)$:

$$f(x)=\begin{cases} cx^2+2x, & x<2 \\ x^3-cx, & x\ge 2 \end{cases}$$

[40 marks]

Soalan 2

(a) *Kirakan had-had berikut:*

(i) $\lim_{t\to 2}\left(\frac{t^2-2}{t^3-3t+5}\right)^2$

(ii) $\lim_{x\to -2}\frac{x+2}{x^3+8}$

(iii) $\lim_{t\to 0}\frac{\sqrt{1+t}-\sqrt{1-t}}{t}$

(b) *Cari* $\lim_{x\to -6}\frac{2x+12}{|x+6|}$*, jika wujud. Sekiranya had ini tidak wujud, terangkan sebabnya.*

(c) *Buktikan bahawa* $\lim_{x\to 0^+}\left(\sqrt{x}e^{\sin\left(\frac{\pi}{x}\right)}\right)=0$.
(Petunjuk: Mulakan dengan $-1\le \sin\left(\frac{\pi}{x}\right)\le 1$*)*

(d) *Tentukan kesemua asimptot mengufuk dan menegak bagi* $y=\frac{1+x^4}{x^2-x^4}$.

(e) *Tentukan nilai pemalar c dalam fungsi berikut supaya f adalah selanjar pada* $(-\infty,\infty)$*:*

$$f(x)=\begin{cases} cx^2+2x, & x<2 \\ x^3-cx, & x\ge 2 \end{cases}$$

[40 markah]

Question 3

(a) Find the first derivative of the function $f(t) = 5t - 9t^2$ using the definition of derivative. State the domains of $f(t)$ and $f'(t)$.

(b) Find the first derivatives of the following functions:

(i) $f(x) = (\sqrt{x} + \frac{1}{\sqrt[3]{x}})^2$
(ii) $f(x) = (x^3 - 2x)(x^{-4} + x^{-2})$
(iii) $f(x) = \cos\left(\frac{1-e^{2x}}{1+e^{2x}}\right)$
(iv) $f(x) = \sec(x)\tan(x)$

(c) The curve with equation $y^2 = x^3 + 3x^2$ is called the Tschirnhausen cubic.

(i) Use implicit differentiation to find $\frac{dy}{dx}$.
(ii) Find an equation of the tangent line to this curve at the point $(1, -2)$.
(iii) At which points does this curve have horizontal tangents?

[30 marks]

Soalan 3

(a) Cari terbitan pertama fungsi $f(t) = 5t - 9t^2$ menggunakan definisi terbitan. Nyatakan domain $f(t)$ dan $f'(t)$.

(b) Cari terbitan pertama bagi fungsi-fungsi berikut:

(i) $f(x) = (\sqrt{x} + \frac{1}{\sqrt[3]{x}})^2$
(ii) $f(x) = (x^3 - 2x)(x^{-4} + x^{-2})$
(iii) $f(x) = cos\left(\frac{1-e^{2x}}{1+e^{2x}}\right)$
(iv) $f(x) = sec(x)\,tan(x)$

(c) Suatu lengkung dengan persamaan $y^2 = x^3 + 3x^2$ dipanggil kubik Tschirnhausen.

(i) Gunakan pembezaan tersirat untuk mencari $\frac{dy}{dx}$,

(ii) Cari persamaan garis tangen untuk lengkung ini pada titik $(1, -2)$.

(iii) Pada titik manakah lengkung ini mempunyai tangen mengufuk?

[30 markah]

Question 4

(a) Given $f(x) = \frac{x}{x^2-9}$. Find

(i) the domain of *f*,
(ii) all the x- and y-intercepts and the asymptotes,
(iii) the intervals on which f is increasing or decreasing,
(iv) the local maximum and minimum points, if any, and
(v) the intervals of concavity and the inflection points, if exist.
Then, sketch the graph of f .

(b) The rate at which photosynthesis takes place for a species of phytoplankton is modelled by the function

$$P(I) = \frac{100I}{I^2 + I + 4} ,$$

where *I* is the light intensity. For what light intensity is *P* a maximum?

[40 marks]

Soalan 4

(a) Diberi $f(x) = \frac{x}{x^2-9}$. *Cari*

(i) domain bagi f,
(ii) kesemua pintasan-x dan -y dan asimptot,
(iii) selang bagi f menokok atau menyusut,
(iv) titik maksimum dan titik minimum tempatan, jika ada, dan
(v) selang kecekungan dan titik lengkuk balas, jika wujud.
Seterusnya, lakarkan graf f.

(b) Kadar fotosintesis berlaku untuk suatu spesies fitoplankton boleh dimodelkan dengan fungsi

$$P(I) = \frac{100I}{I^2 + I + 4} ,$$

yang mana I ialah intensiti cahaya. Pada intensiti cahaya apakah kadar P menjadi maksimum?

[40 markah]

Question 5

Evaluate the following integrals:

(a) $\int (x+1)\sqrt{2x+x^2}\,dx$

(b) $\int (x^2+1)e^{-x}\,dx$ (Use integration by parts)

(c) $\int_0^{\frac{\pi}{2}} \sin^7(x)\cos^5(x)dx$ (Use trigonometric substitution)

(d) $\int_0^1 \frac{x-4}{x^2-5x+6}dx$ (Use partial fractions)

[30 marks]

Soalan 5

Nilaikan kamiran berikut:

(a) $\int (x+1)\sqrt{2x+x^2}\,dx$

(b) $\int (x^2+1)e^{-x}\,dx$ *(Gunakan pengamiran bahagian demi bahagian)*

(c) $\int_0^{\frac{\pi}{2}} sin^7(x)\,cos^5(x)dx$ *(Gunakan penggantian trigonometrik)*

(d) $\int_0^1 \frac{x-4}{x^2-5x+6}dx$ *(Gunakan pecahan separa)*

[30 markah]

Question 6

Let *R* be the region in the first quadrant bounded by the curve $y = x^3$ and $y = 2x - x^2$. Calculate the following quantities

(a) the area of *R*,
(b) the volume obtained by rotating *R* about the x-axis (hint: use washer method), and
(c) the volume obtained by rotating *R* about the y-axis (hint: use cylindrical shell method).

[30 marks]

Soalan 6

Biar R mewakili rantau dalam kuadran pertama yang dibatasi oleh lengkung $y = x^3$ *dan* $y = 2x - x^2$*. Kirakan kuantiti berikut*

(a) luas rantau R,
(b) isipadu pepejal yang terhasil apabila rantau R diputarkan sekitar paksi-x (petunjuk: gunakan kaedah cakera berlubang), dan
(c) isipadu pepejal yang terhasil apabila rantau R diputarkan sekitar paksi-y (petunjuk: gunakan kaedah petala silinder).

[30 markah]

- oooOOooo -

List of Formulas

[*Senarai Formula*]

1. $\tan x = \frac{\sin x}{\cos x}$
2. $\sec x = \frac{1}{\cos x}$
3. $\operatorname{cosec} x = \frac{1}{\sin x}$
4. $\cot x = \frac{1}{\tan x}$
5. $sin^2x + cos^2x = 1$
6. $1 + tan^2x = sec^2x$
7. $1 + cot^2x = cosec^2x$
8. $\frac{d}{dx}(x^n) = nx^{n-1}$
9. $\frac{d}{dx}(e^x) = e^x$
10. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
11. $\frac{d}{dx}(\sin x) = \cos x$
12. $\frac{d}{dx}(\cos x) = -\sin x$
13. $\frac{d}{dx}(\tan x) = \sec^2 x$
14. $\frac{d}{dx}(\sec x) = \sec x \tan x$
15. $\frac{d}{dx}(fg) = fg' + gf'$
16. $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{gf' - fg'}{g^2}$
17. $\frac{d}{dx}\Big(f\big(g(x)\big)\Big) = f'\big(g(x)\big)g'(x)$
18. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$
19. $\int \frac{1}{x} dx = \ln|x| + C$
20. $\int e^x \, dx = e^x + C$
21. $\int \ln x \, dx = x \ln x - x + C$
22. $\int \sin x \, dx = -\cos x + C$
23. $\int \cos x \, dx = \sin x + C$
24. $\int \tan x \, dx = \ln|\sec x| + C$
25. $\int \sec x \, dx = \ln|\sec x + \tan x| + C$
26. $\int \sec^2 x \, dx = \tan x + C$
27. $\int \sec x \tan x \, dx = \sec x + C$

- oooOOooo -