

**SULIT**

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First Semester Examination  
Academic Session 2018/2019

December 2018/January 2019

**MSG488 - Mathematical Algorithms for Computer Graphics  
(Algoritma Matematik untuk Grafik Komputer)**

Duration : 3 hours  
[Masa : 3 jam]

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Please check that this examination paper consists of FIVE (5) pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA (5) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions** : Answer **FOUR** (4) questions.

**Arahan** : Jawab **EMPAT** (4) soalan.]

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].*

...2/-

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**Question 1**

(a) Given

$$y = \begin{cases} 1 + ax^2 - x^3, & x < 1 \\ x^2 + bx + c, & x \geq 1 \end{cases},$$

determine the coefficients  $a$ ,  $b$  and  $c$  such that  $y$  is a spline function of degree 3.

(b) Consider a quadratic Bézier curve with three control points  $(1, 1)$ ,  $(2, 3)$  and  $(4, 2)$ . Suppose the curve is divided into two Bézier curve segments at  $x = 3$ , find the control points of each curve segment.

[ 100 marks ]

**Soalan 1**

(a) Diberi

$$y = \begin{cases} 1 + ax^2 - x^3, & x < 1 \\ x^2 + bx + c, & x \geq 1 \end{cases},$$

tentukan koefisien  $a$ ,  $b$  dan  $c$  supaya  $y$  adalah satu fungsi splin berdarjah 3.

(b) Pertimbangkan satu lengkung Bézier kuadratik dengan titik kawalan  $(1, 1)$ ,  $(2, 3)$  dan  $(4, 2)$ . Andaikan lengkung dibahagikan kepada dua segmen lengkung Bézier pada  $x = 3$ , cari titik-titik kawalan setiap segmen lengkung.

[ 100 markah ]

**Question 2**(a) Let the Bernstein polynomials of degree  $n$  be denoted by

$$B_i^n(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}, \quad 0 \leq t \leq 1, \quad \text{for } i = 0, 1, \dots, n.$$

Given a rational cubic Bézier curve

$$r(t) = \frac{B_0^3(t) - 2B_1^3(t) - 3B_2^3(t) + B_3^3(t)}{B_0^3(t) + 2B_1^3(t) + 2B_2^3(t) + B_3^3(t)}, \quad t \in [0, 1].$$

Use the de Casteljau Algorithm to find the curve value  $r$  at  $t = 0.75$ .

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- (b) (i) Use Chaikin Subdivision Algorithm to refine the polyline shown in Figure 1. Obtain all new vertices and sketch the polyline through the new vertices.
- (ii) What do we find if the Chaikin Algorithm is applied infinitely to the polyline in Figure 1? Sketch the result.

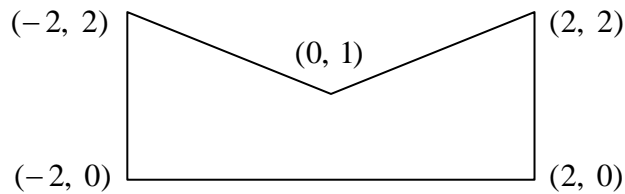


Figure 1

[ 100 marks ]

**Soalan 2**

- (a) Katakan polinomial Bernstein berdarjah  $n$  ditakrif sebagai

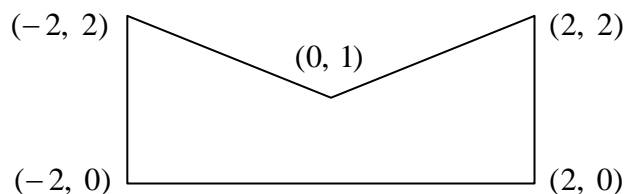
$$B_i^n(t) = \frac{n!}{i!(n-i)!} t^i (1-t)^{n-i}, \quad 0 \leq t \leq 1, \quad \text{untuk } i = 0, 1, \dots, n.$$

Diberi satu lengkung Bézier kubik nisbah

$$r(t) = \frac{B_0^3(t) - 2B_1^3(t) - 3B_2^3(t) + B_3^3(t)}{B_0^3(t) + 2B_1^3(t) + 2B_2^3(t) + B_3^3(t)}, \quad t \in [0, 1].$$

Gunakan Algoritma de Casteljau untuk mencari nilai lengkung  $r$  pada  $t = 0.75$ .

- (b) (i) Gunakan Algoritma Subbahagi Chaikin untuk memperhaluskan poligaris yang ditunjukkan dalam Rajah 1. Dapatkan semua titik baru dan lakarkan poligaris yang melalui titik baru.
- (ii) Apa yang kita dapati jika Algoritma Chaikin digunakan tanpa batas pada poligaris dalam Rajah 1? Lakarkan hasilnya



Rajah 1

[ 100 markah ]

**Question 3**

Let  $\mathbf{u} = (u_0, u_1, \dots, u_{n+k})$  be a non-decreasing knot vector where  $n$  and  $k$  are positive integers with  $n \geq k - 1$ . The normalized B-spline basis functions of order  $k$  are defined recursively by

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \quad \text{for } k > 1,$$

and

$$N_i^1(u) = \begin{cases} 1, & u_i \leq u < u_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

where  $i = 0, 1, \dots, n$ .

- (a) Suppose a B-spline curve of order 4

$$\mathbf{P}(u) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} N_0^4(u) + \begin{pmatrix} 1 \\ 2 \end{pmatrix} N_1^4(u) + \begin{pmatrix} 2 \\ 2 \end{pmatrix} N_2^4(u) + \begin{pmatrix} 2 \\ 1 \end{pmatrix} N_3^4(u),$$

is defined with  $\mathbf{u} = (0, 1, 3, 4.75, 5.25, 6, 7, 8)$ . Use the de Boor Algorithm to evaluate the curve point  $\mathbf{P}$  at  $u = 5$ .

- (b) Find a uniform cubic B-spline curve which is identical to the uniform B-spline given below

$$\mathbf{P}(u) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} N_0^3(u) + \begin{pmatrix} 2 \\ 3 \end{pmatrix} N_1^3(u) + \begin{pmatrix} 4 \\ 2 \end{pmatrix} N_2^3(u).$$

[ 100 marks ]

**Soalan 3**

Katakan  $\mathbf{u} = (u_0, u_1, \dots, u_{n+k})$  ialah suatu vektor simpulan tak menyusut di mana  $n$  dan  $k$  adalah integer positif dengan  $n \geq k - 1$ . Fungsi asas splin-B ternormal berperingkat  $k$  ditakrif secara rekursif oleh

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \quad \text{untuk } k > 1,$$

dan

$$N_i^1(u) = \begin{cases} 1, & u_i \leq u < u_{i+1} \\ 0, & \text{sebaliknya} \end{cases}$$

di mana  $i = 0, 1, \dots, n$ .

...5/-

- (a) *Andaikan satu lengkung splin-B berperingkat 4*

$$P(u) = \binom{1}{1} N_0^4(u) + \binom{1}{2} N_1^4(u) + \binom{2}{2} N_2^4(u) + \binom{2}{1} N_3^4(u),$$

*ditakrif dengan  $u = (0, 1, 3, 4.75, 5.25, 6, 7, 8)$ . Gunakan Algoritma de Boor untuk menilai titik lengkung  $P$  pada  $u = 5$ .*

- (b) *Cari satu lengkung splin-B kubik seragam yang sama dengan splin-B seragam yang diberikan di bawah*

$$P(u) = \binom{1}{1} N_0^3(u) + \binom{2}{3} N_1^3(u) + \binom{4}{2} N_2^3(u).$$

[ 100 markah ]

**Question 4**

Given a non-parametric quadratic patch

$$S(u, v, w) = u^2 + 2v^2 + 3w^2 - w - v - u, \quad 0 \leq u, v, w \leq 1,$$

where  $(u, v, w)$  are the barycentric coordinates of any point in a triangular domain.

- (a) Find the value of  $S$  if  $3u = 4v = 6w$ .
- (b) Show that  $S$  is a quadratic curve along the line  $u = v$ . Express the curve in Bézier form by determining the associated control ordinates.

[ 100 marks ]

**Soalan 4**

*Diberi satu tampalan kuadratik tak berparameter*

$$S(u, v, w) = u^2 + 2v^2 + 3w^2 - w - v - u, \quad 0 \leq u, v, w \leq 1,$$

*di mana  $(u, v, w)$  adalah koordinat baripusat bagi sebarang titik dalam satu domain segi tiga.*

- (a) *Cari nilai  $S$  jika  $3u = 4v = 6w$ .*
- (b) *Tunjukkan bahawa  $S$  adalah satu lengkung kuadratik sepanjang garis  $u = v$ . Ungkapkan lengkung dalam bentuk Bézier dengan menentukan ordinat kawalan yang berkaitan.*

[ 100 markah ]

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