

SULIT



First Semester Examination
Academic Session 2018/2019

December 2018/January 2019

**MGM551 – Operation Research
(Penyelidikan Operasi)**

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of TWENTY (20) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi DUA PULUH (20) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **ELEVEN** (11) questions.

Arahan : Jawab **SEBELAS** (11) soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

...2/-

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Question 1

Petromal Oil Company has 15,000 barrels of base oil 1 and 20,000 barrels of base oil 2. The company sells two products: petrol and lubricating oil. Both products are produced by combining base oil 1 and base oil 2. The quality level of each base oil is as follows: oil 1 is grade 10; oil 2 is grade 5. Petrol must have an average quality level of at least 8, and lubricating oil at least 6. Demand for each product must be created by advertising. Each ringgit spent on advertising petrol creates 3 barrels of demand and each ringgit spent on lubricating oil creates 5 barrels of demand. Petrol is sold for RM250 per barrel, lubricating oil for RM400 per barrel. Formulate a Linear program to help Petromal maximizes profit.

[10 marks]

Soalan 1

Syarikat minyak Petromal mempunyai 15,000 tong minyak asas 1 dan 20,000 tong minyak asas 2. Syarikat ini menjual 2 produk: petrol dan minyak pelincir. Setiap produk adalah hasil campuran minyak asas 1 dan minyak asas 2. Petrol mesti mempunyai purata kualiti paras yang sekurang-kurangnya 8, dan minyak pelincir sekurang-kurangnya 6. Permintaan bagi setiap produk mesti didapati melalui iklan. Setiap ringgit dibelanja untuk petrol menghasilkan 3 tong permintaan dan setiap ringgit dibelanja untuk minyak pelincir menghasilkan 5 tong permintaan. Petrol dijual dengan harga RM250 setong, minyak pelincir dijual dengan harga RM400 setong. Rumus suatu masalah Pengaturcaraan Linear untuk membantu Petromal memaksimumkan keuntungan.

[10 markah]

...3/-

Question 2

We have the following transportation tableau:

		Destination				
<u>Source</u>		1	2	3	4	Supply
A		9	10	12	15	300
B		15	12	10	8	250
C		12	14	11	12	400
Demand		150	250	300	350	

(C_{ij} is the unit cost of transportation from source i to destination j and it is located in the upper right hand corner of each cell).

- (i) Get the basic feasible solution using the least cost method.
- (ii) Get the basic feasible solution using the Vogel's approximation method.
- (iii) From either (i) or (ii), show one further iteration.

[10 marks]

...4/-

Soalan 2

Diberikan tablo pengangkutan seperti berikut:

		<i>Destinasi</i>				
<i>Punca</i>		<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>Bekalan</i>
<i>A</i>		9	10	12	15	300
<i>B</i>		15	12	10	8	250
<i>C</i>		12	14	11	12	400
<i>Permintaan</i>		150	250	300	350	

(C_{ij} ialah kos pengangkutan seunit dari punca i ke destinasi j dan ia diletakkan di dalam petak atas kanan setiap sel).

- (i) Dapatkan penyelesaian asas tersaur dengan kaedah kos terkecil.
- (ii) Dapatkan penyelesaian asas tersaur dengan kaedah penghampiran Vogel.
- (iii) Daripada (i) atau (ii), tunjukkan satu lelaran seterusnya.

[10 markah]

Question 3

Show **one** iteration using the two-phase method for the following problem.

$$\text{Minimize } Z = 6x_1 + 3x_2 + 7x_3$$

$$\text{Subject to } x_1 + 4x_2 + 2x_3 = 30$$

$$3x_1 + x_2 + 2x_3 \leq 60$$

$$x_1 + 2x_2 + 4x_3 \geq 40$$

$$\text{where } x_1 \geq 0, x_2 \geq 0 \text{ and } x_3 \geq 0$$

[5 marks]

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Soalan 3

Tunjukkan **satu** lelaran menggunakan teknik dua fasa untuk masalah berikut.

$$\text{Minimumkan} \quad Z = 6x_1 + 3x_2 + 7x_3$$

$$\text{Terhadap} \quad x_1 + 4x_2 + 2x_3 = 30$$

$$3x_1 + x_2 + 2x_3 \leq 60$$

$$x_1 + 2x_2 + 4x_3 \geq 40$$

$$\text{dengan} \quad x_1 \geq 0, \quad x_2 \geq 0 \text{ and} \quad x_3 \geq 0$$

[5 markah]

Question 4

(a) Standardize the following Linear Programming (LP) formulation:

$$\text{Maximize} \quad Z = 20x_1 + 12x_2 - 18x_3$$

$$\text{Subject to} \quad 6x_1 + 2x_2 - 5x_3 = 25$$

$$5x_1 - 4x_2 + x_3 \geq -35$$

$$2x_1 - 3x_2 + 4x_3 \leq 30$$

$$\text{where} \quad x_1 \geq 0, \quad x_2 \text{ unrestricted and} \quad -5 \leq x_3 \leq 15$$

(b) From (a), if the LP remains the same and only the condition on variable x_3 is changed to $x_3 \geq 0$, show its dual formulation.

[10 marks]

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Soalan 4

(a) Piawaikan model Pengaturcaraan Linear (PL) berikut:

$$\text{Maksimumkan } Z = 20x_1 + 12x_2 - 18x_3$$

$$\text{Terhadap } 6x_1 + 2x_2 - 5x_3 = 25$$

$$5x_1 - 4x_2 + x_3 \geq -35$$

$$2x_1 - 3x_2 + 4x_3 \leq 30$$

$$\text{Dengan } x_1 \geq 0, x_2 \text{ tak tersekat dan } -5 \leq x_3 \leq 15$$

(b) Daripada (a,) jika PL tidak berubah kecuali syarat bagi pembolehubah x_3 ,
iaitu $x_3 \geq 0$, tunjukkan rumus dualnya.

[10 markah]

Question 5

Given the following linear programming problem:

$$\text{Minimize } Z = 8x_1 + 6x_2 + 16x_3$$

$$\text{Subject to } 4x_1 + 2x_2 - 8x_3 \geq 16$$

$$2x_1 - 4x_2 + 2x_3 \geq 12$$

$$\text{where } x_i \geq 0 \quad \forall i.$$

Show the simplex tableau after **one** iteration using the dual simplex method.

[5 marks]

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Soalan 5

Diberikan rumus pengaturcaraan linear berikut:

$$\text{Minimumkan } Z = 8x_1 + 6x_2 + 16x_3$$

$$\text{Terhadap } 4x_1 + 2x_2 - 8x_3 \geq 16$$

$$2x_1 - 4x_2 + 2x_3 \geq 12$$

$$\text{dengan } x_i \geq 0 \quad \forall i.$$

Tunjukkan tablo simpleks setelah satu lelaran menggunakan kaedah simpleks dual.

[5 markah]

Question 6

Snacko manufactures three types of snacks in a bag. Each snack consists of mixed nuts and chocolate. The compositions of each type of 1 kg snack bag and the profits earned are shown in the table below.

Snack Type	Amount of Mixed Nuts (kg)	Amount of Chocolate (kg)	Profit per kg (bag) (RM)
1	2	5	5
2	1	4	8
3	3	2	6

240 kg of mixed nuts and 360 kg of chocolates are available. After defining x_i to be the number of Type i snack manufactured (for $i = 1, 2, 3$), the Linear Programming formulation is:

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Maximize $Z = 5x_1 + 8x_2 + 6x_3$

Subject to $2x_1 + x_2 + 3x_3 \leq 240$
 $5x_1 + 4x_2 + 2x_3 \leq 360$

where $x_1, x_2, x_3 \geq 0$.

After adding slack variables s_1 and s_2 , the optimal solution found by the simplex method is:

BFS	x_1	x_2	x_3	s_1	s_2	Solution
Z	$53/5$	0	0	$4/5$	$9/5$	840
x_3	$3/10$	0	1	$2/5$	$-1/10$	60
x_2	$11/10$	1	0	$-1/5$	$3/10$	60

Answer the following questions:

- (i) For what range of values of profit for the Type 2 snack would the current basis remain optimal? If the profit for a Type 2 snack bag is RM13.00, what would be the new optimal solution?
- (ii) For what range of values values of profit for the Type 1 snack would the current basis remain optimal? If the profit for a Type 1 snack bag is RM12.00, what would be the new optimal solution?
- (iii) For what range of the amount of mixed nuts would the current basis remain optimal?
- (iv) Suppose that an additional 100 kg of mixed nuts is available. What would be the new Sweetco profit and what is the amount of each snack that should be produced?
- (v) Snacko is considering making a Type 4 snack. Type 4 snack earns RM17 profit per kg and it requires 3 kg of mixed nuts and 4 kg of chocolate. Should they manufacture any of this Type 4 snack?

[10 marks]

..9/-

Soalan 6

Snacko menghasilkan tiga jenis snek yang dibungkus dalam beg. Setiap jenis snek hanya mengandungi campuran kacang dan coklat. Kandungan bagi setiap beg 1 kg snek dan keuntungannya diberikan di dalam jadual berikut:

Jenis Snek	Amaun Campuran Kacang (kg)	Amaun Coklat (kg)	Keuntungan per kg Beg (RM)
1	2	5	5
2	1	4	8
3	3	2	6

240 kg campuran kacang dan 360 kg coklat sedia ada. Biarkan x_i mewakili amaun snek Jenis i yang dihasilkan (bagi $i = 1, 2, 3$), maka rumus Pengaturcaraan Linearnya ialah:

$$\begin{aligned} \text{Maksimumkan } Z &= 5x_1 + 8x_2 + 6x_3 \\ \text{Terhadap} \quad &2x_1 + x_2 + 3x_3 \leq 240 \\ &5x_1 + 4x_2 + 2x_3 \leq 360 \\ \text{dengan} \quad &x_1, x_2, x_3 \geq 0. \end{aligned}$$

Setelah memasukkan pembolehubah lalai s_1 dan s_2 , penyelesaian optimum yang didapati menggunakan kaedah simpleks adalah:

Asas	x_1	x_2	x_3	s_1	s_2	Penyelesaian
Z	53/5	0	0	4/5	9/5	840
x_3	3/10	0	1	2/5	-1/10	60
x_2	11/10	1	0	-1/5	3/10	60

Jawab soalan berikut:

- (i) Berapakah julat bagi keuntungan snek Jenis 2 supaya penyelesaian asas yang didapati kekal optimum? Jika keuntungan per beg snek Jenis 2 ialah RM13.00, apakah penyelesaian optimum yang baru?

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- (ii) Berapakah julat bagi keuntungan snek Jenis 1 supaya penyelesaian asas yang didapati kekal optimum? Jika keuntungan per beg snek Jenis 1 ialah RM12.00, apakah penyelesaian optimum yang baru?
- (iii) Berapakah julat bagi amaun campuran kacang supaya penyelesaian asas kekal optimum?
- (iv) Andaikan tambahan 100 kg. campuran kacang boleh didapati, berapakah keuntungan Snacko dan amaun pengeluaran setiap jenis snek?
- (v) Snacko bercadang membuat snek Jenis 4. Keuntungan sekilogram snek Jenis 4 ialah RM17 dan ia memerlukan 3 kg gula dan 4 kg coklat. Patutkah mereka membuat snek jenis 4 ini?

[10 markah]

Question 7

Five workers have to work on four tasks, if unable, it is marked as (-). The cost taken for a worker to complete a task is as follows:

Worker	Cost (RM)			
	Task 1	Task 2	Task 3	Task 4
1	33	30	35	25
2	28	37	-	32
3	38	35	36	40
4	26	32	28	24
5	31	-	35	38

- Determine the assignment of worker to task that minimizes the total cost.
- What is the minimum total cost?
- Which worker is not assigned to any task?
- Assume that the numbers in the table are revenues, determine the maximum revenue and its assignment.

[10 marks]

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Soalan 7

Lima pekerja boleh melaksanakan empat tugas, jika tidak, ia ditandakan dengan (-). Kos bagi setiap pekerja melaksanakan setiap tugas adalah seperti berikut:

<i>Pekerja</i>	<i>Kos (RM)</i>			
	<i>Tugas 1</i>	<i>Tugas 2</i>	<i>Tugas 3</i>	<i>Tugas 4</i>
<i>1</i>	<i>33</i>	<i>30</i>	<i>35</i>	<i>25</i>
<i>2</i>	<i>28</i>	<i>37</i>	<i>-</i>	<i>32</i>
<i>3</i>	<i>38</i>	<i>35</i>	<i>36</i>	<i>40</i>
<i>4</i>	<i>26</i>	<i>32</i>	<i>28</i>	<i>24</i>
<i>5</i>	<i>31</i>	<i>-</i>	<i>35</i>	<i>38</i>

- (i) *Tentukan pemberian pekerja kepada tugas supaya jumlah kos diminimumkan.*
- (ii) *Berapakah jumlah kos keseluruhan yang minimum?*
- (iii) *Pekerja manakah yang tidak diberi tugas?*
- (iv) *Andaikan bahawa angka di dalam jadual adalah pendapatan, tentukan pendapatan maksimum dan umpukannya.*

[10 markah]

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Question 8

After solving a transportation problem, we obtain the following optimal tableau

		Destination				Supply
		1	2	3	4	
Source	A	50	40	30	25	240
				80	160	
B	40	30	25	35	180	
		140	40			
C	30	25	20	30	130	
	130	0				
Demand		130	140	120	160	

(C_{ij} is the unit cost of transportation from source i to destination j and it is located in the upper right hand corner of each cell).

- (i) What the optimal transportation cost?
- (ii) Get the new solution if both the supply from Source B and demand at Destination 2 increased by 20 units.
- (iii) Get the new solution if both the supply from Source A and demand at Destination 1 increased by 30 units.
- (iv) Find the range for C_{13} so that the solution remains optimal.
- (v) Find the range for C_{21} so that the solution remains optimal.
- (vi) Get the alternative optimal tableau if it exists. Show your work.

[10 marks]

...13/-

Soalan 8

Setelah menyelesaikan suatu masalah pengangkutan, kita dapat tablo optimum berikut,

		Destinasi				Bekalan
		1	2	3	4	
Punca	A	50	40	30	25	240
				80	160	
B	40	30	25	35	180	
		140	40			
C	30	25	20	30	130	
	130	0				
Permintaan		130	140	120	160	

(C_{ij} ialah kos pengangkutan seunit dari punca i ke destinasi j dan ia terletak di penjuru kanan atas setiap sel).

- Berapakah jumlah kos pengangkutan optimum?
- Dapatkan penyelesaian baru jika bekalan dari Punca B dan permintaan di Destinasi 2 meningkat kedua-duanya sebanyak 20 unit.
- Dapatkan penyelesaian baru jika bekalan dari Punca A dan permintaan di Destinasi 1 meningkat kedua-duanya sebanyak 30 unit.
- Dapatkan julat bagi C_{13} supaya penyelesaian didapati kekal optimum.
- Dapatkan julat bagi C_{21} supaya penyelesaian didapati kekal optimum
- Dapatkan tablo optimum alternatif jika wujud. Tunjukkan jalan kerja anda.

[10 markah]

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Question 9

Consider the list of activities and their precedence for a project.

Activity	Precedence Activities	Duration (days)
A	-	5
B	-	4
C	-	7
D	A	6
E	A	8
F	B, D	7
G	C	6
H	D, E	5
I	F, G	4

- (i) Draw the project network diagram.
- (ii) Show the critical path for the project.
- (iii) Give the shortest time to complete the project.

[10 marks]

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Soalan 9

Pertimbangkan senarai kegiatan dan kegiatan pendahulu bagi suatu projek.

<i>Kegiatan</i>	<i>Kegiatan Pendahulu</i>	<i>Jangkamasa (hari)</i>
<i>A</i>	-	5
<i>B</i>	-	4
<i>C</i>	-	7
<i>D</i>	<i>A</i>	6
<i>E</i>	<i>A</i>	8
<i>F</i>	<i>B, D</i>	7
<i>G</i>	<i>C</i>	6
<i>H</i>	<i>D, E</i>	5
<i>I</i>	<i>F, G</i>	4

- (i) *Lakarkan gambarajah aliran projek ini.*
- (ii) *Tunjukkan lintasan genting projek ini.*
- (iii) *Berikan masa terpendek menyiapkan projek ini.*

[10 markah]

...16/-

Question 10

(a) Consider the following two-person game:

		Player B		
		B1	B2	B3
Player A	A1	6	7	9
	A2	8	6	8
	A3	6	4	6

- (i) Why pure strategies cannot be used to solve this game? Explain your answer.
- (ii) Apply the dominance method to give the reduced matrix.
- (iii) Based on the answer obtained in part (ii), calculate the game value and probabilities of strategies for Players A and B using the odds method.
- (b). Find each player's optimal strategy and the value of the two-person zero sum game in the table below:

4	5	3	8
6	7	5	9
5	7	5	4
6	6	5	5

[10 marks]

...17/-

Soalan 10

(a) Pertimbangkan permainan dua orang berikut:

		Pemain B		
		B1	B2	B3
Pemain A	A1	6	7	9
	A2	8	6	8
	A3	6	4	6

- (i) Kenapa strategi tulen tidak boleh digunakan untuk menyelesaikan permainan ini? Beri penjelasan anda.
- (ii) Gunakan kaedah dominan untuk mendapatkan matriks terkurangkan.
- (iii) Berdasarkan jawapan yang didapati di bahagian (ii), kira nilai permainan dan kebarangkalian strategi untuk pemain A dan B menggunakan kaedah odds.
- (b) Cari nilai dan strategi optimum bagi masalah permainan jumlah sifar dua-orang berikut.

4	5	3	8
6	7	5	9
5	7	5	4
6	6	5	5

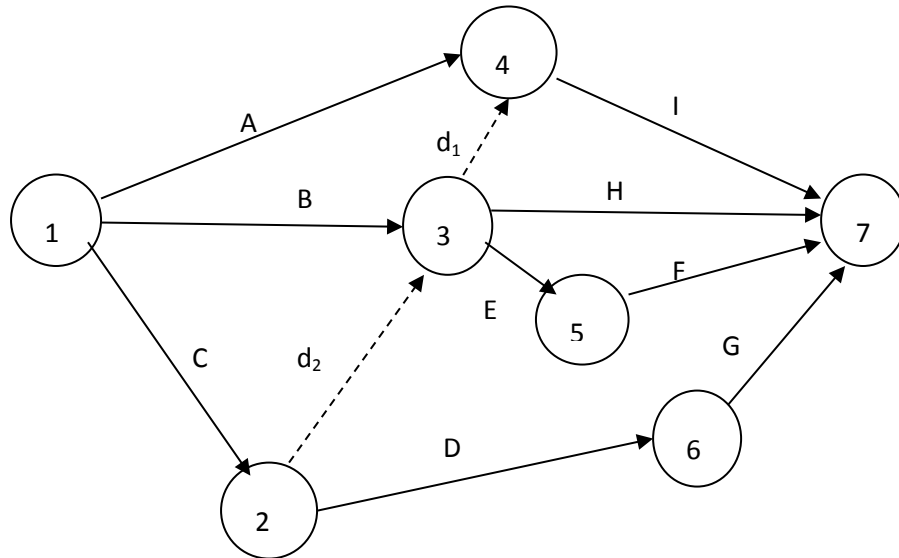
[10 markah]

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Question 11

The network diagram below represents a project.



The normal and crash durations with the normal and crash costs are given as follows:

Activity	Duration (days)		Direct Cost (RM)	
	Normal	Crash	Normal	Crash
A	7	5	120	140
B	5	4	90	130
C	7	5	160	190
D	4	3	180	210
E	6	4	250	310
F	7	5	180	240
G	9	6	120	200
H	8	7	110	160
I	7	5	200	280

...19/-

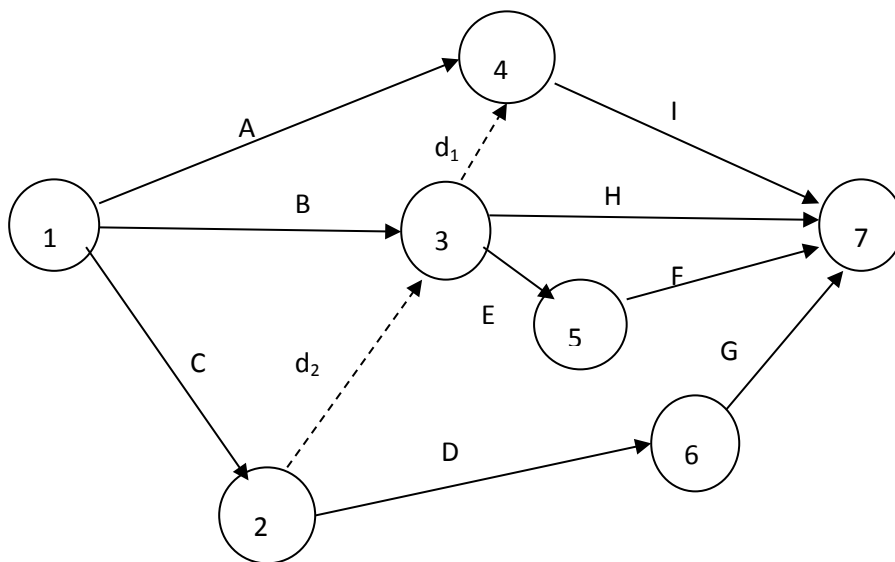
The indirect cost per day is RM160. The contract agreement states that there is a penalty cost of RM120 per day if the project is completed later than 19 days and there is a bonus of RM140 per day if it is completed earlier than 19 days.

- (i) Determine the critical path.
- (ii) List all of the critical activities.
- (iii) Determine the total cost for the project.
- (iv) Perform **two** iterations to crash the project. Show the total cost for each iteration.

[10 marks]

Soalan 11

Gambarajah rangkaian berikut mewakili suatu projek:



Jangkamasa biasa dan nahas serta kos biasa dan nahas bagi setiap kegiatan diberikan seperti berikut:

...20/-

<i>Kegiatan</i>	<i>Jangkamasa (hari)</i>		<i>Kos Langsung (RM)</i>	
	<i>Biasa</i>	<i>Nahas</i>	<i>Biasa</i>	<i>Nahas</i>
<i>A</i>	7	5	120	140
<i>B</i>	5	4	90	130
<i>C</i>	7	5	160	190
<i>D</i>	4	3	180	210
<i>E</i>	6	4	250	310
<i>F</i>	7	5	180	240
<i>G</i>	9	6	120	200
<i>H</i>	8	7	110	160
<i>I</i>	7	5	200	280

Kos tak langsung projek ialah RM160 sehari. Tercatat di dalam perjanjian kontrak bahawa denda sebanyak RM120 sehari dikenakan jika projek ini siap lewat daripada 19 hari dan ditawarkan bonus sebanyak RM140 sehari jika ia siap awal daripada 19 hari.

- (i) Tentukan lintasan genting projek ini.*
- (ii) Senaraikan kesemua kegiatan genting.*
- (iii) Tentukan jumlah kos projek ini.*
- (iv) Lakukan **dua** lelaran pemampatan bagi projek ini. Tunjukkan jumlah kos projek bagi setiap lelaran.*

[10 markah]

