First Semester Examination
Academic Session 2018/2019

December 2018/January 2019

MAT518 - Numerical Methods For Differential Equations
(Kaedah Berangka Untuk Persamaan Pembezaan)

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX (6) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM (6) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **FOUR (4)** questions.

*[**Arahan :** Jawab **EMPAT (4)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Question 1

(a) Consider the wave equation $W_{tt} = W_{xx}$. Write down the centred time, centred space (CTCS) finite difference scheme for this equation. Investigate the consistency and stability of the scheme.

(b) State the Lax Equivalence Theorem and discuss the convergence of the scheme in 1(a).

[100 marks]

Soalan 1

(a) Pertimbang persamaan gelombang $W_{tt} = W_{xx}$. Tulis skema beza pusat masa dan beza pusat ruang (CTCS) untuk persamaan ini. Kaji kekonsistenaan dan kestabilan skema.

(b) Nyatakan Teorem Kesetaraan Lax dan bincangkan penumpuan skema dalam 1(a).

[100 markah]

Question 2

(a) Consider the two-dimensional heat equation $U_t = U_{xx} + U_{yy}$. Write down the FTCS scheme, the fully implicit scheme and the ADI scheme. What are the advantages of the ADI scheme over the FTCS scheme and the fully implicit scheme?

(b) Consider $C_t + uC_x = 0$ where u is a known constant. Write down the FTCS scheme and conduct a stability analysis.

[100 marks]

Soalan 2

(a) Pertimbang persamaan haba dua dimensi $U_t = U_{xx} + U_{yy}$. Tulis skema FTCS, skema tersirat penuh dan skema ADI. Apakah kelebihan skema ADI berbanding skema FTCS dan skema tersirat penuh?

(b) Pertimbang $C_t + uC_x = 0$ dengan u pemalar yang nilainya diketahui. Tulis skema FTCS dan jalankan analisis kestabilan.

[100 markah]

Question 3

(a) Consider the linear boundary value problem

$$y'' = y$$
$$y(0) = 2, y(1) = 8$$

Write down the finite difference scheme which uses central differences.

Using $N=4$ i.e h=1/5 i.e $\Delta x=1/5$, write down the associated system of linear equations.

(b) Consider the nonlinear boundary value problem

$$y'' = y^2$$
$$y(0) = 2, y(1) = 8$$

Write down the finite difference scheme which uses central differences.

Using $N=4$ i.e h=1/5 i.e $\Delta x=1/5$, write down the associated system of nonlinear equations.

(c) Consider the ordinary differential equation $y' = f(x, y)$ with initial condition $y(x_0) = y_0$.

Name two linear multi-step methods. Define consistency and convergence for linear multi-step methods. What is meant when a linear multi-step method is said to satisfy the root condition?

[100 marks]

Soalan 3

(a) *Pertimbang masalah nilai sempadan linear.*

$$y'' = y$$
$$y(0) = 2, y(1) = 8$$

Tulis skema beza terhingga yang menggunakan beza pusat.
Dengan menggunakan N=4 i.e h=1/5 i.e Δx=1/5 , tulis sistem persamaan linear yang bersekutu.

(b) *Pertimbang masalah nilai sempadan tak linear*

$$y'' = y^2$$
$$y(0) = 2, y(1) = 8$$

Tulis skema beza terhingga yang menggunakan beza pusat.
Dengan menggunakan N=4 i.e h=1/5 i.e Δx=1/5, tulis sistem persamaan tak linear yang bersekutu.

(c) *Pertimbang persamaan pembezaan biasa $y' = f(x, y)$ dengan syarat awal $y(x_0) = y_0$.*

Namakan dua kaedah multi langkah linear. Takrifkan kekonsistenan dan penumpuan untuk kaedah multi langkah linear. Apa yang dimaksudkan apabila suatu kaedah multi-langkah linear dikatakan memenuhi syarat punca?

[100 markah]

Question 4

(a) Consider Laplaces equation

$U_{xx} + U_{yy} = 0$

with $R = \{(x, y) \mid 0 < x < 1, 0 < y < 1\}$

and boundary condition U=0 along the boundaries.

If n=m=3 i.e $\Delta x = \Delta y = 1/3$ i.e h=k=1/3, set up (but do not solve) the associated linear system which when solved will give U at the grid points.

(b) Consider the system

$$4x + y - z = 2$$
$$2x + 5y + z = 8$$
$$-x + y + 3z = 3$$

Will the Gauss Seidel iterative method converge in solving this system? Explain why. Carry out one iteration of the S.O.R method with $\omega = 1.2$ and zero starting vector.

(c) State the Stein-Rosenberg theorem.

(d) Calculate the truncation error for the central difference scheme for Laplaces equation $W_{xx} + W_{yy} = 0$

[100 marks]

Soalan 4

(a) *Pertimbang persamaan Laplace*

$U_{xx} + U_{yy} = 0$

dengan $R = \{(x, y) \mid 0 < x < 1, 0 < y < 1\}$

dan syarat sempadan U=0 sepanjang sempadan.

Jika n=m=3 i.e $\Delta x = \Delta y = 1/3$ i.e h=k=1/3, *bangunkan (tapi jangan selesaikan) sistem persamaan linear bersekutu yang apabila diselesaikan akan memberi nilai U di titik titik grid.*

(b) *Pertimbang sistem*

$$4x + y - z = 2$$
$$2x + 5y + z = 8$$
$$-x + y + 3z = 3$$

Adakah kaedah lelaran Gauss Seidel akan menumpu dalam menyelesaikan sistem ini? Terangkan mengapa. Laksanakan satu lelaran kaedah S.O.R dengan $\omega = 1.2$ *dan vektor permulaan sifar.*

(c) *Nyatakan Teorem Stein-Rosenberg.*

(d) *Kira ralat pangkasan untuk skema beza pusat untuk persamaan Laplace* $W_{xx} + W_{yy} = 0$

[100 markah]

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