

SULIT



First Semester Examination
Academic Session 2018/2019

December 2018/January 2019

**MAT382 - Introductory Numerical Methods
(*Pengenalan Kaedah Berangka*)**

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of FIVE (5) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi FIVE (5) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **SIX** (6) questions.

[Arahan: Jawab **SIX** (6) soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

...2/-

SULIT

Question 1

Find the roots of the equation $x^2 + 3x - 8^{-14}$ with three-digit accuracy.

[5 marks]

Soalan 1

Carikan punca-punca persamaan $x^2 + 3x - 8^{-14}$ dengan ketepatan tiga digit.

[5 markah]

Question 2

(a) The equation $f(x) = x^2 - 2e^x = 0$ has a solution in the interval $[-1, 1]$.

(i) Calculate x_2 using the Secant method with $x_0 = -1$ and $x_1 = 1$.

(ii) Calculate x_3 using Newton's method with x_2 from part (i).

[10 marks]

(b) The equation $f(x) = 2 - x^2 \sin x = 0$ has a solution in the interval $[-1, 2]$.

(i) Verify that the Bisection method can be applied to the function $f(x)$ on $[-1, 2]$.

(ii) Find the number of iterations needed for accuracy 0.000001 using the error formula for the Bisection method.

(iii) Compute x_3 for the Bisection method.

[15 marks]

Soalan 2

(a) Persamaan $f(x) = x^2 - 2e^x = 0$ mempunyai penyelesaian dalam selang $[-1, 1]$.

(i) Hitungkan x_2 menggunakan kaedah Sekan dengan $x_0 = -1$ dan $x_1 = 1$.

(ii) Hitungkan x_3 menggunakan kaedah Newton dengan x_2 daripada bahagian (i).

[10 markah]

(b) Persamaan $f(x) = 2 - x^2 \sin x = 0$ mempunyai penyelesaian dalam selang $[-1, 2]$.

(i) Tentusahkan kaedah Pembahagian Dua Sama dapat digunapakai terhadap fungsi $f(x)$ pada $[-1, 2]$.

(ii) Carikan bilangan lelaran yang diperlukan untuk ketepatan 0.000001 menggunakan formula ralat bagi kaedah Pembahagian Dua Sama.

(iii) Kirakan x_3 bagi kaedah Pembahagian Dua Sama berkenaan.

[15 markah]

...3/-

Question 3

(a) Let $A = LU$ where $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 4 & 1 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$.

(i) Solve the linear system $Ax = b$ where $b = (0, -2, 2)$ using the given factorization.

(ii) Using the given factorization, what is the determinant of A ?

[10 marks]

(b) Solve the linear system

$$\begin{cases} 0.211x_1 + 0.811x_2 = 1.52 \\ 1.71x_1 - 1.06x_2 = 0.512 \end{cases}$$

using three-digit rounding arithmetic and Gaussian elimination with partial pivoting.

[10 marks]

Soalan 3

(a) Andaikan $A = LU$ yang mana $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix}$ dan $U = \begin{bmatrix} 4 & 1 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$.

(i) Selesaikan sistem linear $Ax = b$ yang mana $b = (0, -2, 2)$ menggunakan pemfaktoran yang diberi.

(ii) Dengan menggunakan pemfaktoran yang diberi, apakah determinan A ?

[10 markah]

(b) Selesaikan sistem linear

$$\begin{cases} 0.211x_1 + 0.811x_2 = 1.52 \\ 1.71x_1 - 1.06x_2 = 0.512 \end{cases}$$

menggunakan aritmetik pembundaran tiga digit dan kaedah penghapusan Gaussian secara pemangsaan separa.

[10 markah]

Question 4

Let $x_0 = 0$, $x_1 = 0.5$, and $x_2 = 1.0$. Given $f(x) = -2e^{-x} + \frac{1}{4}x^4 - \frac{1}{120}x^5 + 2x$ and its derivatives

$$f'(x) = 2e^{-x} + x^3 - \frac{1}{24}x^4 + 2, \quad f^{(4)}(x) = -2e^{-x} + 6 - x,$$

$$f''(x) = -2e^{-x} + 3x^2 - \frac{1}{6}x^3, \quad f^{(5)}(x) = 2e^{-x} - 1,$$

$$f'''(x) = 2e^{-x} + 6x - \frac{1}{2}x^2, \quad f^{(6)}(x) = -2e^{-x}.$$

- (a) Find the Lagrange Interpolating Polynomial, $P_2(x)$, of degree at most 2 for $f(x)$ using x_0 , x_1 and x_2 . [5 marks]
- (b) Give the general error formula for $f(x) - P_2(x)$. [5 marks]
- (c) Use the formula from (b) to find a bound for the absolute error at $x = 0.65$ assuming $f'''(x)$ has no relevant critical points. [5 marks]

Soalan 4

Andaikan $x_0 = 0$, $x_1 = 0.5$, and $x_2 = 1.0$. Diberi $f(x) = -2e^{-x} + \frac{1}{4}x^4 - \frac{1}{120}x^5 + 2x$ dan terbitannya

$$f'(x) = 2e^{-x} + x^3 - \frac{1}{24}x^4 + 2, \quad f^{(4)}(x) = -2e^{-x} + 6 - x,$$

$$f''(x) = -2e^{-x} + 3x^2 - \frac{1}{6}x^3, \quad f^{(5)}(x) = 2e^{-x} - 1,$$

$$f'''(x) = 2e^{-x} + 6x - \frac{1}{2}x^2, \quad f^{(6)}(x) = -2e^{-x}.$$

- (a) Carikan Polinomial Interpolasi Lagrange, $P_2(x)$, berdarjah 2 paling tinggi bagi $f(x)$ menggunakan x_0 , x_1 dan x_2 . [5 markah]
- (b) Berikan formula ralat umum bagi $f(x) - P_2(x)$. [5 markah]
- (c) Gunakan formula daripada (b) untuk mencari batas kepada ralat mutlak pada $x = 0.65$ dengan anggapan bahawa $f'''(x)$ tiada titik kritikal yang relevan. [5 markah]

Question 5

Let $f(x) = x \ln x + x^4$.

- (a) Approximate $I = \int_1^3 f(x)dx$ using Simpson's $\frac{3}{8}$ Rule. [5 marks]
- (b) Find the least upper bound for the absolute error of the approximation in part (a) using the error formula. [5 marks]

Soalan 5

Andaikan $f(x) = x \ln x + x^4$.

- (a) Anggarkan $I = \int_1^3 f(x)dx$ menggunakan Kaedah Simpson $\frac{3}{8}$. [5 markah]
- (b) Carikan batas atas terkecil bagi ralat mutlak anggaran di bahagian (a) menggunakan formula ralat yang berkaitan. [5 markah]

Question 6

Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$. Perform two iterations of the Power Method on A.

[10 marks]

Soalan 6

Andaikan $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$. Jalankan dua lelaran Kaedah Kuasa terhadap A.

[10 markah]