

**SULIT**

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First Semester Examination  
Academic Session 2018/2019

December 2018/January 2019

**MAA111 - Algebra for Science Students  
(Algebra untuk Pelajar Sains)**

Duration : 3 hours  
[Masa : 3 jam]

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Please check that this examination paper consists of FIVE (5) pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA (5) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions:** Answer **THREE** (3) questions.

**[Arahan:** Jawab **TIGA** (3) soalan.]

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].*

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**SULIT**

**Question 1**

- (a) Determine the values of  $a$  for which the system has no solutions, exactly one solution, or infinitely many solutions

$$\begin{aligned}x + 2y - 3z &= 3, \\3x - y + 5z &= 2, \\4x + y + (a^2 - 2)z &= a + 4.\end{aligned}$$

[ 10 marks ]

- (b) Use determinant to find the area of the triangle with vertices  $(3,1)$ ,  $(7,1)$  and  $(7,9)$ .

[ 8 marks ]

- (c) Let  $T_1$  and  $T_2$  be linear transformation from  $\mathbb{R}^3$  to  $\mathbb{R}^3$  such that  $T_1(x, y, z) = (2x + y, 0, x + z)$  and  $T_2(x, y, z) = (x - y, z, y)$ . Find the standard matrices for the compositions  $T = T_2 \circ T_1$  and  $T' = T_1 \circ T_2$  and determine if  $T$  and  $T'$  are invertible. If it is, find its inverse.

[ 10 marks ]

**Soalan 1**

- (a) Tentukan nilai-nilai  $a$  di mana sistem ini mempunyai tiada penyelesaian, satu penyelesaian, atau penyelesaian tak terhingga

$$\begin{aligned}x + 2y - 3z &= 3, \\3x - y + 5z &= 2, \\4x + y + (a^2 - 2)z &= a + 4.\end{aligned}$$

[ 10 markah ]

- (b) Gunakan penentu untuk dapatkan luas segitiga dengan bucu-bucu  $(3,1)$ ,  $(7,1)$  dan  $(7,9)$ .

[ 8 markah ]

- (c) Biar  $T_1$  and  $T_2$  sebagai transformasi linear dari  $\mathbb{R}^3$  ke  $\mathbb{R}^3$  sedemikian hingga  $T_1(x, y, z) = (2x + y, 0, x + z)$  dan  $T_2(x, y, z) = (x - y, z, y)$ . Dapatkan matriks piawai bagi gubahan  $T = T_2 \circ T_1$  dan  $T' = T_1 \circ T_2$  dan tentukan sama ada jika  $T$  dan  $T'$  adalah tersongsangkan. Jika ya, dapatkan songsangannya.

[ 10 markah ]

...3/-

**Question 2**

- (a) Let  $T_1(1,0) = (2,0)$  and  $T_2(0,1) = (0,1)$ . Give a geometric description for transformation  $T_1$  and  $T_2$ .

[ 5 marks ]

- (b) (i) Find matrix  $A$  given that  $(4A)^{-1} = \begin{pmatrix} 2 & 4 \\ -3 & 2 \end{pmatrix}$ .  
 (ii) Assuming that all matrices are  $n \times n$  and invertible, solve the following equation for  $D$ .

$$C^T B^{-1} A^2 B A C^{-1} D A^{-2} B^T C^{-2} = C^T.$$

[ 15 marks ]

- (c) Find eigenvalues of matrix  $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{pmatrix}$  and a basis for each of the corresponding eigenspaces. Determine whether the matrix is diagonalizable; justify your answers.

[ 18 marks ]

**Soalan 2**

- (a) Biar  $T_1(1,0) = (2,0)$  dan  $T_2(0,1) = (0,1)$ . Terangkan secara geometri transformasi bagi  $T_1$  dan  $T_2$ .

[ 5 markah ]

- (b) (i) Dapatkan matriks  $A$  jika diberikan  $(4A)^{-1} = \begin{pmatrix} 2 & 4 \\ -3 & 2 \end{pmatrix}$ .  
 (ii) Andaikan semua matriks adalah bersaiz  $n \times n$  dan tersongsangkan, selesaikan persamaan berikut bagi  $D$ .

$$C^T B^{-1} A^2 B A C^{-1} D A^{-2} B^T C^{-2} = C^T.$$

[ 15 markah ]

...4/-

- (c) Dapatkan nilai-nilai eigen bagi matriks  $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{pmatrix}$  dan asas yang sepadan dengan setiap ruang eigen. Tentukan sama ada matriks tersebut terpepenjurukan; berikan alasan untuk jawapan anda.

[ 18 markah ]

**Question 3**

- (a) Find a basis for the solution space of the following homogeneous linear system, and the dimension of that space.

$$\begin{aligned} x + y + z &= 0, \\ 3x + 2y - 2z &= 0, \\ 4x + 3y - z &= 0, \\ 6x + 5y + z &= 0. \end{aligned}$$

[ 10 marks ]

- (b) Find a subset of the vectors that forms a basis for the space spanned by the following vectors. Then express each vectors that is not in the basis as a linear combination of the basis vectors.

$$\begin{aligned} \vec{v}_1 &= (1, -2, 0, 3), \vec{v}_2 = (2, -4, 0, 6), \\ \vec{v}_3 &= (-1, 1, 2, 0), \vec{v}_4 = (0, -1, 2, 3). \end{aligned}$$

[ 10 marks ]

- (c) Find the orthogonal projection of  $\vec{u}$  on the subspace of  $\mathbb{R}^4$  spanned by vectors  $\vec{v}_1, \vec{v}_2$  and  $\vec{v}_3$ ;

$$\begin{aligned} \vec{u} &= (-2, 0, 2, 4); \vec{v}_1 = (1, 1, 3, 0), \\ \vec{v}_2 &= (-2, -1, -2, 1), \vec{v}_3 = (-3, -1, 1, 3). \end{aligned}$$

[ 14 marks ]

...5/-

**Soalan 3**

- (a) Dapatkan asas bagi ruang penyelesaian sistem linear homogen berikut, dan dimensi bagi ruang tersebut.

$$\begin{aligned}x + y + z &= 0, \\3x + 2y - 2z &= 0, \\4x + 3y - z &= 0, \\6x + 5y + z &= 0.\end{aligned}$$

[ 10 markah ]

- (b) Dapatkan subset vektor yang membentuk asas bagi ruang yang direntang oleh vektor-vektor berikut. Seterusnya tulis setiap vektor yang bukan asas sebagai gabungan linear vektor-vektor asas.

$$\begin{aligned}\vec{v}_1 &= (1, -2, 0, 3), \vec{v}_2 = (2, -4, 0, 6), \\ \vec{v}_3 &= (-1, 1, 2, 0), \vec{v}_4 = (0, -1, 2, 3).\end{aligned}$$

[ 10 markah ]

- (d) Dapatkan unjuran berortogon bagi  $\vec{u}$  pada subruang  $\mathbb{R}^4$  direntang oleh vektor-vektor  $\vec{v}_1, \vec{v}_2$  dan  $\vec{v}_3$ .

$$\begin{aligned}\vec{u} &= (-2, 0, 2, 4); \vec{v}_1 = (1, 1, 3, 0), \\ \vec{v}_2 &= (-2, -1, -2, 1), \vec{v}_3 = (-3, -1, 1, 3).\end{aligned}$$

[ 14 markah ]

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