First Semester Examination
Academic Session 2018/2019

December 2018/January 2019

MAA101 - Calculus for Science Students I
(Kalkulus untuk Pelajar Sains I)

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of EIGHT (8) pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN (8) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **FOUR (4)** questions.

*[**Arahan :** Jawab **EMPAT (4)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Question 1

(a) Suppose $f(x) = \sin x$ and $k(x) = \sqrt{x}$ where $x \geq 0$. Find the following composition with their domain and range.

(i) $f \circ k$

(ii) $k \circ f$

(iii) $k \circ k$

[30 marks]

(b) The functions f and g are defined as follows

$$f(x) = \ln(x+1), \quad x > -1,$$
$$g(x) = x^2 + 2x \ , \quad x > -1.$$

(i) Sketch the graph of f and g separately.

(ii) State the range of f and g.

(iii) Determine whether f and g are **one-to-one** function. For each one-to-one function, find its inverse.

(iv) Find the composite function $f \circ g$.

(v) Sketch the graph of $f \circ g$.

(vi) State the range of $f \circ g$.

(vii) Find the values of x if $f^{-1} \circ g = e^3 - 1$.

[70 marks]

Soalan 1

(a) Andaikan $f(x) = \sin x$ dan $k(x) = \sqrt{x}$ yang mana $x \geq 0$. Cari komposisi berikut berserta domain dan julatnya.

(i) $f \circ k$

(ii) $k \circ f$

(iii) $k \circ k$

[30 markah]

(b) *Fungsi f dan g ditakrifkan seperti berikut*

$$f(x) = \ln(x+1), \quad x > -1,$$
$$g(x) = x^2 + 2x \ , \quad x > -1.$$

(i) *Lakarkan graf f dan g secara berasingan.*

(ii) *Nyatakan julat bagi f dan g.*

(iii) *Tentukan sama ada f dan g ialah fungsi* ***satu-ke-satu****. Bagi setiap fungsi satu-ke-satu, cari fungsi songsangnya .*

(iv) *Cari fungsi gubahan bagi $f \circ g$.*

(v) *Lakarkan graf $f \circ g$.*

(vi) *Nyatakan julat bagi $f \circ g$.*

(vii) *Cari nilai x sekiranya $f^{-1} \circ g = e^3 - 1$.*

[70 markah]

Question 2

(a) f is defined as follows

$$f(x) = \begin{cases} 6x-3 & , x < 1 \\ 3x & , 1 \le x < 2 \\ 3x^2 - 6 & , x \ge 2 \end{cases}.$$

(i) Find $\lim_{x \to 1} f(x)$ and $\lim_{x \to 2} f(x)$.

(ii) Is f continuous at $x = 1$ and $x = 2$?

(iii) Is f differentiable at $x = 1$ and $x = 2$?

[30 marks]

(b) Find the first derivative of $f(x) = 1 - 3x^2$ by using the definition of derivatives. Hence find an equation of the tangent line at point $x = 1$.

[20 marks]

(c) Find the first derivatives of the following functions:

(i) $y = x^e + \pi$

(ii) $y = \dfrac{\sin x}{\sqrt{x}}$

(iii) $y = e^{\left(x^2+1\right)^5}$

(iv) $y = \sin^2 3x + \cos^5 2x^3$

(vi) $x^2 + 3y^3 = xy^2$

[30 marks]

(d) Describe three situations in which a function can fail to be differentiable. Illustrate with sketches.

[10 marks]

(e) Find the number c guaranteed by the Mean Value Theorem for $f(x) = (x+1)^3$ in $[-1,1]$.

[10 marks]

Soalan 2

(a) f ditakrifkan seperti berikut

$$f(x) = \begin{cases} 6x-3 & , x<1 \\ 3x & ,1 \le x < 2 \\ 3x^2-6 & , x \ge 2 \end{cases}.$$

(i) Cari $\lim\limits_{x\to 1} f(x)$ *dan* $\lim\limits_{x\to 2} f(x)$.

(ii) Adakah f selanjar pada $x=1$ *dan* $x=2$ *?*

(iii) Adakah f terbezakan pada $x=1$ *dan* $x=2$ *?*

[30 markah]

(b) Cari terbitan pertama bagi $f(x) = 1-3x^2$ *dengan menggunakan takrifan terbitan. Oleh itu, cari persamaan garisan tangen pada titik* $x=1$.

[20 markah]

(c) *Cari terbitan pertama bagi fungsi berikut:*

(i) $y = x^e + \pi$

(ii) $y = \dfrac{\sin x}{\sqrt{x}}$

(iii) $y = e^{\left(x^2+1\right)^5}$

(iv) $y = \sin^2 3x + \cos^5 2x^3$

(vi) $x^2 + 3y^3 = xy^2$

[30 markah]

(d) *Terangkan tiga keadaan yang menyebabkan suatu fungsi gagal untuk dibezakan. Ilustrasikan dengan menggunakan lakaran.*

[10 markah]

(e) *Cari nilai c yang dijamin oleh Teorem Nilai Min untuk* $f\left(x\right) = \left(x+1\right)^3$ *dalam* $\left[-1,1\right]$.

[10 markah]

Question 3

(a) Find the following limit:

(i) $\lim\limits_{x\to 1} \dfrac{x^2 + 2x - 3}{x - 1}$

(ii) $\lim\limits_{x\to 0} \dfrac{x}{4 - \sqrt{x+16}}$

(iii) $\lim\limits_{x\to \infty} \sqrt[3]{\dfrac{8x^{19} - 6x^5}{7x^{19} + 1}}$

(iv) $\lim\limits_{x\to 0} \left(x^2 \sec^2 x + \dfrac{\tan x}{x} \right)$, if $0 \le x^2 \sec^2 x \le x^4 + x^2$

(v) $\lim\limits_{x\to \infty} (e^x - x)^{\frac{1}{x}}$

[30 marks]

(b) Let $g(x)$ be the content of oxygen in a pond x days after sewage is dumped into the pond. Given that

$$g(x) = 1 - \frac{10}{x+10} + \frac{100}{(x+10)^2}, \qquad x \geq 0 .$$

(i) Find the lowest and highest content of oxygen, if exists.

(ii) On which day does the oxygen content increase the fastest?

[50 marks]

(c) Find the area of the largest rectangle where the perimeter of the rectangle is 10 cm.

[20 marks]

Soalan 3

(a) Cari had berikut:

(i) $\lim\limits_{x \to 1} \dfrac{x^2 + 2x - 3}{x - 1}$

(ii) $\lim\limits_{x \to 0} \dfrac{x}{4 - \sqrt{x+16}}$

(iii) $\lim\limits_{x \to \infty} \sqrt[3]{\dfrac{8x^{19} - 6x^5}{7x^{19} + 1}}$

(iv) $\lim\limits_{x \to 0} \left(x^2 \sec^2 x + \dfrac{\tan x}{x} \right)$, *jika* $0 \leq x^2 \sec^2 x \leq x^4 + x^2$

(v) $\lim\limits_{x \to \infty} (e^x - x)^{\frac{1}{x}}$

[30 markah]

(b) Andaikan $g(x)$ merupakan kandungan oksigen dalam suatu kolam x hari selepas kumbahan dibuang ke dalam kolam. Diberikan

$$g(x) = 1 - \frac{10}{x+10} + \frac{100}{(x+10)^2}, \qquad x \geq 0 .$$

(i) Cari kandungan oksigen yang paling rendah dan paling tinggi, jika wujud.

(ii) Pada hari manakah kandungan oksigen meningkat dengan paling cepat?

[50 markah]

(c) Cari luas segi empat terbesar di mana perimeter segi empat ini ialah 10 cm.

[20 markah]

Question 4

(a) Evaluate the following integral:

(i) $\int \frac{1}{\sqrt[3]{(1-2x)^2}}\,dx$

(ii) $\int x^2 e^x\,dx$

(iii) $\int_1^2 \frac{1}{u^3+u^2}\,du$

(iv) $\int_2^{2\sqrt{3}} \frac{1}{v\sqrt{v^2-3}}\,dv$

[60 marks]

(b) Given $f(x)=\sin x$, $0 \le x \le \pi/2$.

(i) Sketch the graph of the inverse function f^{-1}.

(ii) Find the area under the graph of $f^{-1}(x)$ from $x=0$ to $x=1$.

[20 marks]

(c) State the integral, but do not evaluate, for the volume of a pyramid with height h and base an equilateral triangle with side s.

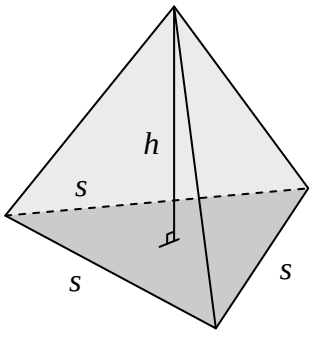


Figure 1

[20 marks]

Soalan 4

(a) *Nilaikan kamiran berikut:*

(i) $\int \frac{1}{\sqrt[3]{(1-2x)^2}}\,dx$

(ii) $\int x^2 e^x\,dx$

(iii) $\int_1^2 \frac{1}{u^3+u^2}\,du$

(iv) $\int_2^{2\sqrt{3}} \frac{1}{v\sqrt{v^2-3}}\,dv$

[60 markah]

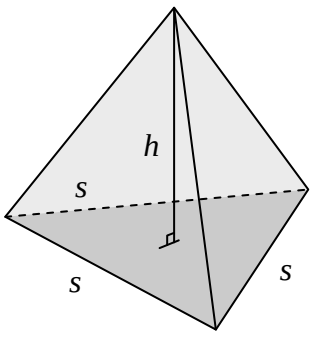
(b) *Diberi* $f(x)=\sin x,\ 0\le x\le \pi/2$.

(i) *Lakarkan graf fungsi songsang* f^{-1}.

(ii) *Cari luas di bawah graf* $f^{-1}(x)$ *dari* $x=0$ *ke* $x=1$.

[20 markah]

(c) *Nyatakan kamiran, tetapi tidak menilai, bagi isipadu piramid dengan tinggi* h *dan tapak segi tiga sama sisi dengan sisi* s.



Rajah 1

[20 markah]

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