UNIVERSITI SAINS MALAYSIA

First Semester Examination
2017/2018 Academic Year

January 2018

MSS 212 - Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **SIX** (6) questions.

*[**Arahan**: Jawab **ENAM** (6) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. (a) Let

$$A = \begin{bmatrix} 1 & 1 & 1 & k \\ 1 & 1 & k & k \\ 1 & k & k & k \\ k & k & k & k \end{bmatrix}.$$

Using row reduction method, find values of the constant k for which the matrix A is invertible.

(b) Give an example of a 3×3 matrix B with all nonzero entries such that $\det(B) = 14$.

(c) Let $\sigma = (24315)$ and $\tau = (43125)$ be permutations in S_5. Find $\tau \circ \sigma$ and the number of inversions of $\tau \circ \sigma$.

[12 marks]

1. *(a) Biar*

$$A = \begin{bmatrix} 1 & 1 & 1 & k \\ 1 & 1 & k & k \\ 1 & k & k & k \\ k & k & k & k \end{bmatrix}.$$

Menggunakan kaedah penurunan baris, dapatkan nilai-nilai bagi k supaya matriks A tersongsangkan.

(b) Beri satu contoh matriks B bersaiz 3×3 *dengan semua pemasukan bukan sifar sedemikian* $\det(B) = 14$.

(c) Biar $\sigma = (24315)$ *dan* $\tau = (43125)$ *pilihatur dalam S_5. Dapatkan $\tau \circ \sigma$ dan bilangan penyongsangan bagi $\tau \circ \sigma$.*

[12 markah]

2. (a) Determine whether the subset $\mathbb{R}^2$ is a subspace of the vector space $\mathbb{C}^2$ over $\mathbb{C}$.

(b) Find a basis for the vector space

$$M = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{C} \right\}$$

over $\mathbb{R}$ and determine its dimension.

(c) Let $\{v_1, v_2, v_3\}$ be a basis for a vector space V. Show that $\{u_1, u_2, u_3\}$ is also a basis where $u_1 = v_1$, $u_2 = v_1 - v_2$, $u_3 = v_1 + v_2 - v_3$.

[12 marks]

...3/-

2. *(a) Tentukan sama ada subset* $\mathbb{R}^2$ *merupakan subruang kepada ruang vektor* $\mathbb{C}^2$ *atas* $\mathbb{C}$.

(b) Dapatkan asas bagi ruang vektor

$$M = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{C} \right\}$$

atas $\mathbb{R}$ *dan tentukan dimensinya.*

(c) Biar $\{v_1, v_2, v_3\}$ *suatu asas bagi ruang vektor* V. *Tunjukkan bahawa* $\{u_1, u_2, u_3\}$ *dengan* $u_1 = v_1$, $u_2 = v_1 - v_2$, $u_3 = v_1 + v_2 - v_3$ *juga suatu asas.*

[12 markah]

3. Let $\mathbb{P}_3(\mathbb{C})$ be a vector space of polynomials of degree less than or equal to 3 over $\mathbb{C}$.

(a) Construct an isomorphism to show that $\mathbb{P}_3(\mathbb{C}) \cong \mathbb{C}^4$.

(b) Let $T : \mathbb{P}_3(\mathbb{C}) \to \mathbb{C}^4$ be a linear transformation.

(i) What is the matrix representation of T relative to the standard basis?

(ii) Let $\alpha = \{1, x, x^2, x^3\}$ and $\beta = \{(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)\}$ be the ordered basis for $\mathbb{P}_3(\mathbb{C})$ and $\mathbb{C}^4$, respectively. What is the matrix representation of T relative to α, β?

[18 marks]

3. *Biar* $\mathbb{P}_3(\mathbb{C})$ *ruang vektor yang terdiri daripada polinomial dengan darjah kurang atau sama dengan 3 atas* $\mathbb{C}$.

(a) Bina satu isomorfisma untuk menunjukkan $\mathbb{P}_3(\mathbb{C}) \cong \mathbb{C}^4$.

(b) Biar $T : \mathbb{P}_3(\mathbb{C}) \to \mathbb{C}^4$ *suatu penjelmaan linear.*

(i) Apakah matriks perwakilan bagi T *terhadap asas piawai?*

(ii) Biar $\alpha = \{1, x, x^2, x^3\}$ *dan* $\beta = \{(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)\}$ *asas bertertib bagi* $\mathbb{P}_3(\mathbb{C})$ *dan* $\mathbb{C}^4$, *masing-masing. Apakah matriks perwakilan bagi* T *terhadap* α, β*?*

[18 markah]

4. Determine if the following statements are true or false. Justify your answer.

(a) If A is a 3×3 matrix with characteristic polynomial $\lambda(\lambda - 1)(\lambda + 1)$, then A is diagonalisable.

(b) If A and B are invertible $n \times n$ matrices, then $A + B$ is also invertible.

(c) If A is invertible, then A is diagonalisable.

[14 marks]

4. Tentukan sama ada pernyataan berikut adalah benar atau palsu. Justifikasi jawapan anda.

(a) Jika A suatu matriks 3×3 dengan polinomial cirian $\lambda(\lambda - 1)(\lambda + 1)$, maka A terpepenjurukan.

(b) Jika A dan B adalah matriks $n \times n$ tersongsangkan, maka $A + B$ juga tersongsangkan.

(c) Jika A tersongsangkan, maka A terpepenjurukan.

[14 markah]

5. Let

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix}.$$

Find

(a) an orthogonal basis for $\mathbb{R}^2$ consisting of eigenvectors of A.

(b) an orthogonal matrix P and a diagonal matrix D such that $A = PDP^T$.

(c) a symmetric matrix B such that $B = A^2$.

[22 marks]

5. *Biar*

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix}.$$

Dapatkan

(a) *asas ortogonal bagi $\mathbb{R}^2$ yang mengandungi vektor eigen bagi A.*

(b) *matriks ortogonal P dan suatu matrix pepenjuru D sedemikian $A = PDP^T$.*

(c) *matriks simetri B sedemikian $B = A^2$.*

[22 markah]

6. Let $V = \mathbb{R}^3$ and

$$A = \begin{bmatrix} -2 & 2 & 1 \\ -7 & 4 & 2 \\ 5 & 0 & 0 \end{bmatrix}.$$

(a) Find Jordan Canonical Form for A.

(b) Compute A^{20}.

[22 marks]

6. *Biar $V = \mathbb{R}^3$ dan*

$$A = \begin{bmatrix} -2 & 2 & 1 \\ -7 & 4 & 2 \\ 5 & 0 & 0 \end{bmatrix}.$$

(a) *Dapatkan Bentuk Berkanun Jordan bagi A.*

(b) *Hitung A^{20}.*

[22 markah]

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