First Semester Examination
2017/2018 Academic Session

January 2018

MSG489 - Numerical Methods for Differential Equations
(Kaedah Berangka Untuk Persamaan Pembezaan)

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **EIGHT (8)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN (8)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions : Answer **all four (4)** questions.

*[**Arahan** : Jawab **semua empat (4)** soalan.*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Question 1

(a) Consider the initial value problem $\frac{dy}{dx} = ty + y^2 - 2, y(0) = 1$

(i) Use Euler's method with $h = 0.5$ to estimate $y(1.0)$.

(ii) Compute $\ddot{y}$ in terms of t and y.

(iii) Use the Taylor series of order 2 with $h = 0.5$ to estimate $y(1)$.

[50 marks]

(b) The following initial value problem models the population in Malaysia. Suppose that

$$\frac{dP}{dt} = 2.5 \times 10^{-3} P, P(0) = 58.043$$

where P is the population in millions, t is measured in years and $t=0$ corresponds to the year 1996.

(i) Show that Euler's method applied to this initial value problem leads to

$$P_n = (1 + 2.5 \times 10^{-3} h)^n \times 58.043$$

where P_n is the approximation to $P(nh)$.

(ii) Use a time step of h equal to 6 months to approximate the predicted population for the year 2050.

[50 marks]

Soalan 1

(a) *Pertimbangkan masalah nilai awal* $\frac{dy}{dx} = ty + y^2 - 2, y(0) = 1$

(i) *Gunakan Kaedah Euler dengan* $h = 0.5$ *untuk menganggarkan* $y(1.0)$.

(ii) *Hitungkan* y'' *dalam sebutan* t *dan* y.

(iii) *Gunakan siri Taylor kuasa 2 dengan* $h = 0.5$ *untuk menganggarkan* $y(1)$.

[50 markah]

(b) *Masalah nilai awalan berikut modelkan populasi di Malaysia. Sekiranya*

$$\frac{dP}{dt} = 2.5 \times 10^{-3} P, P(0) = 58.043$$

di mana P adalah populasi dalam billion, t *diukur dalam masa dan* $t=0$ *dikaitkan dengan tahun 1996.*

(i) *Tunjukkan kaedah Euler diaplikasikan kepada masalah nilai awalan ini menyebabkan*

$$P_n = (1 + 2.5 \times 10^{-3} h)^n \times 58.043$$

di mana P_n *adalah anggaran kepada* $P(nh)$.

(ii) *Gunakan langkah masa* h *bersamaan dengan 6 bulan untuk menganggarkan populasi anggaran untuk tahun 2050.*

[50 markah]

Question 2

(a) As part of his job at a restaurant, Jim learned to cook up a big pot of soup late at night, just before restaurant closing time, so that there would be plenty of soup to feed customers the next day. He also found out that, while refrigeration was essential to preserve the soup overnight, but the soup was too hot to be put directly into the fridge when it was ready. The soup had just boiled at $100°C$ and the fridge was not powerful enough to accommodate a big pot of soup if it was any warmer than $20°C$. Jim discovered that by cooling the pot in a sink full of cold water (kept running, so that its temperature was roughly at $5°C$) and stirring occasionally, he could bring the temperature of the soup to $60°C$ in ten minutes. How long before restaurant closing time should the soup be ready so that Jim could put it in the fridge and leave on time?

[50 marks]

(b) A 10 gallon tank initially contains salt at a concentration of 1 gram/gallon. Water with an increasing concentration given by $\mathbf{1 - e^{-t}}$ gram/gallon of salt flows into the tank at a rate of 5 gallon/day and the mixture in the tank flows out at the same rate.

(i) Assuming that the salt distributes itself uniformly, construct a mathematical model of this flow process for the salt content $\mathbf{y(t)}$ of the tank.

(ii) Solve the initial value problem.

(iii) What is the limiting value of the salt content as $t \to \infty$?

[50 marks]

Soalan 2

(a) *Sebagai sebahagian daripada tugasan di restoran, Jim memasak sup dalam periuk yang besar pada sebelah malam, sebelum masa tutup restoran supaya sup sudah pun tersedia untuk pelanggan pada keesokan harinya. Dia juga tahu, walaupun sup perlu disimpan di dalam peti ais supaya tahan, sup itu terlalu panas untuk terus diletakkan dalam peti sejuk apabila siap. Sup itu baru mendidih pada* $100°C$ *dan peti sejuk tidak cukup berkuasa untuk menampung periuk besar sup sekiranya ia panas daripada* $20°C$ *. Jim mendapat tahu bahawa dengan menyejukkan periuk dalam sinki dipenuhi dengan air sejuk (air yang terus mengalir, supaya suhu secara kasarnya pada* $5°C$ *) dan dikacau sekali sekala, dia dapat menurunkan suhu sup kepada* $60°C$ *dalam sepuluh minit. Berapa lamakah sebelum waktu tutup restoran, sup itu perlu siap supaya Jim dapat meletakkannya dalam peti sejuk dan balik tepat pada masanya?*

[50 markah]

(b) *Pada awalnya sebuah tangki 10 gelen mengandungi garam dengan kepekatan 1 gram/gelen. Air dengan peningkatan kepekatan garam diberikan* $1 - e^{-t}$ *gram/gelen yang mengalir masuk ke dalam tangki dengan kadar 5 gelen/sehari dan campuran mengalir keluar dari tangki dengan kadar yang sama.*

(i) *Andaikan garam cair secara sekata, bina satu model matematik untuk proses pengaliran kandungan garam* $y(t)$ *dalam tangki.*

(ii) *Selesaikan masalah nilai awal.*

(iii) *Apakah nilai had untuk kandungan garam tersebut apabila* $t \to \infty$ *?*

[50 markah]

Question 3

(a) Solve the initial value problem $\frac{dy}{dx} = x^2 + y^3, y(1) = 0$ on the interval [1, 1.6] using the predictor-corrector method:

$$\textbf{Predictor}: y_{i+1} = y_i + \frac{h}{2}(3f_i - f_{i-1})$$

$$\textbf{Corrector}: y_{i+1} = y_i + \frac{h}{12}(5f_{i+1} + 8f_i - f_{i-1})$$

with the step length, $h = 0.2$. Perform three corrector iterations per step. Compute the starting value using Taylor series second order method with the same step length, h.

[50 marks]

(b) Find the approximate value of $y(0.5)$ for the initial value problem $\frac{dy}{dx} = x + y, y(0) = 1$ using the multistep method $y_{i+1} = y_{i-1} + \frac{h}{3}(f_{i+1} + 4f_i + f_{i-1})$ with $h = 0.1$. Compute the starting value using classical Runge-Kutta fourth order method with the same step length.

[50 marks]

Soalan 3

(a) Selesaikan masalah nilai awal $\frac{dy}{dx} = x^2 + y^3, y(1) = 0$ *dalam selang [1, 1.6] dengan menggunakan Kaedah Penganggaran-Pembaikian:*

$$\textbf{\textit{Penganggaran}}: y_{i+1} = y_i + \frac{h}{2}(3f_i - f_{i-1})$$

$$\textbf{\textit{Pembaikian}}: y_{i+1} = y_i + \frac{h}{12}(5f_{i+1} + 8f_i - f_{i-1})$$

dengan saiz langkah, $h = 0.2$*. Lakukan tiga lelaran pembaikian bagi setiap langkah. Kirakan nilai awal menggunakan kaedah siri Taylor peringkat dua dengan saiz langkah yang sama.*

[50 markah]

(b) Cari nilai anggaran untuk $y(0.5)$ *untuk masalah nilai awal* $\frac{dy}{dx} = x + y, y(0) = 1$ *menggunakan kaedah pelbagai langkah* $y_{i+1} = y_{i-1} + \frac{h}{3}(f_{i+1} + 4f_i + f_{i-1})$ *dengan* $h = 0.1$ *. Kirakan nilai permulaan menggunakan kaedah Runge-Kutta klasik tertib keempat dengan saiz langkah yang sama.*

[50 markah]

Question 4

(a) Find the solution of $\nabla^2 u = 12x, 0 \leq x \leq 5, 0 \leq y \leq 4,$ subject to the boundary conditions $u(x,y) = x^3 + 3xy^2$ on $x = 1, y = 0, y = 1$ and $2u + \frac{\partial u}{\partial x} = 3y^2$ on $x = 0$ by using the five point formula. Use central difference approximation in the boundary conditions. Assume the step length is $h = 2$ along the axes.

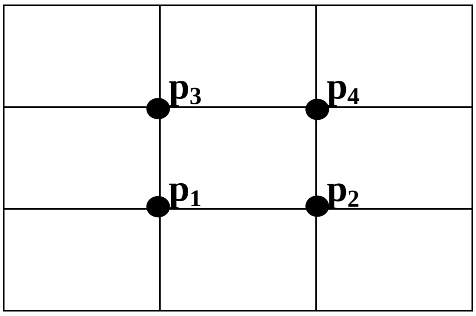
[50 marks]

(b) Determine the system of four unknown, p_1, p_2, p_3 and p_4 for computing approximation for the harmonic function $u(x,y)$ in the rectangle $R = \{(x,y) : 0 \leq x \leq 3, 0 \leq y \leq 3\}$ shown in the figure under the conditions:

$u(x,0) = 10, u(x,3) = 90$ for $0 < x < 3$,

$u(0,y) = 70, u(3,y) = 0$ for $0 < y < 3$.

Hence find p_1, p_2, p_3 and p_4.



[50 marks]

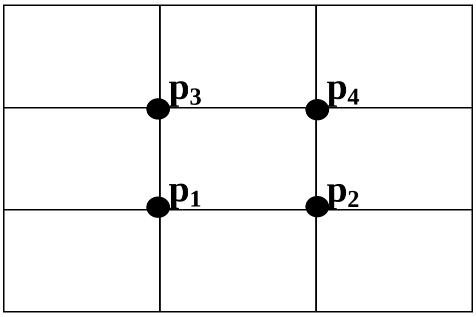
Soalan 4

(a) *Dapatkan penyelesaian untuk* $\nabla^2 u = 12x, 0 \le x \le 5, 0 \le y \le 4,$ *berdasarkan syarat sempadan* $u(x,y) = x^3 + 3xy^2$ *pada* $x = 1, y = 0, y = 1$ *dan* $2u + \frac{\partial u}{\partial x} = 3y^2$ *pada* $x = 0$ *dengan menggunakan formula lima titik. Gunakan penganggaran pembezaan tengah di dalam syarat-syarat sempadan. Andaikan saiz langkah adalah* $h = 2$ *sepanjang paksi-paksi.*

[50 markah]

(b) *Tentukan sistem untuk empat nilai yang tidak diketahui,* p_1, p_2, p_3 *dan* p_4 *untuk mengira penganggaran untuk sistem harmonik* $u(x,y)$ *di dalam segiempat tepat* $R = \{(x,y) : 0 \le x \le 3, 0 \le y \le 3\}$ *ditunjukkan dalam rajah di bawah syarat-syarat:* $u(x,0) = 10, u(x,3) = 90$ *untuk* $0 < x < 3$ *,*
$u(0,y) = 70, u(3,y) = 0$ *untuk* $0 < y < 3$.

Dengan itu dapatkan p_1, p_2, p_3 *dan* p_4.



[50 markah]

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