First Semester Examination
2017/2018 Academic Session

January 2018

MSG488 - Mathematical Algorithms for Computer Graphics
[Algoritma Matematik untuk Grafik Komputer]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **SIX (6)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **ENAM (6)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all four (4)** questions.

*[**Arahan:** Jawab **semua empat (4)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

Question 1

(a) Consider a cubic Bézier curve $\boldsymbol{P}(t)$ with four control points $(1,\ 0)$, $(0,\ 0)$, $(0,\ 1)$ and $(1,\ 1)$. Use the de Casteljau Algorithm to evaluate the tangent to the curve at $t = 1/3$.

(b) Suppose the cubic Bézier curve given in (a) is divided into three Bézier curve segments with equal length of intervals, find the control points of each curve segment.

[100 marks]

Soalan 1

(a) Pertimbangkan satu lengkung Bézier kubik $\boldsymbol{P}(t)$ *dengan empat titik kawalan* $(1,\ 0)$, $(0,\ 0)$, $(0,\ 1)$ *dan* $(1,\ 1)$. *Gunakan Algoritma de Casteljau untuk menilai tangen lengkung pada* $t = 1/3$.

(b) Andaikan lengkung Bézier kubik yang diberikan dalam (a) dibahagikan kepada tiga segmen lengkung Bézier dengan panjang selang sama, cari titik-titik kawalan setiap segmen lengkung.

[100 markah]

Question 2

Let $\boldsymbol{u} = (u_0,\ u_1,\ \ldots,\ u_{n+k})$ be a non-decreasing knot vector where n and k are positive integers with $n \geq k-1$. The normalized B-spline basis functions of order k are defined recursively by

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \quad \text{for} \quad k > 1,$$

and

$$N_i^1(u) = \begin{cases} 1, & u_i \leq u < u_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

where $i = 0,\ 1,\ \ldots,\ n$.

(a) Given a B-spline curve of order 3

$$\boldsymbol{P}(u) = \begin{pmatrix}1\\1\end{pmatrix} N_0^3(u) + \begin{pmatrix}1\\2\end{pmatrix} N_1^3(u) + \begin{pmatrix}2\\2\end{pmatrix} N_2^3(u) + \begin{pmatrix}2\\1\end{pmatrix} N_3^3(u),$$

with $\boldsymbol{u} = (0, 1, 2, 3, 4, 5, 6)$. Suppose a knot value $u = 3.5$ is inserted twice into $\boldsymbol{u}$ without changing the shape of $\boldsymbol{P}$. Find the new set of the de Boor points for $\boldsymbol{P}$.

(b) Suppose $\boldsymbol{u} = (-4, -3, -1, 0, 1, 4, 6, 8)$, find a cubic Bézier curve which is identical to

$$\boldsymbol{P}(u) = \begin{pmatrix}1\\1\end{pmatrix} N_0^4(u) + \begin{pmatrix}1\\2\end{pmatrix} N_1^4(u) + \begin{pmatrix}2\\2\end{pmatrix} N_2^4(u) + \begin{pmatrix}2\\1\end{pmatrix} N_3^4(u).$$

[100 marks]

Soalan 2

Katakan $\boldsymbol{u} = (u_0, u_1, \ldots, u_{n+k})$ *ialah suatu vektor simpulan tak menyusut di mana* n *dan* k *adalah integer positif dengan* $n \geq k-1$. *Fungsi asas splin-B ternormal berperingkat* k *ditakrif secara rekursif oleh*

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \text{ untuk } \quad k > 1,$$

dan

$$N_i^1(u) = \begin{cases} 1, & u_i \leq u < u_{i+1} \\ 0, & \text{sebaliknya} \end{cases}$$

di mana $i = 0, 1, \ldots, n$.

(a) *Diberi suatu lengkung splin-B berperingkat* 3

$$\boldsymbol{P}(u) = \begin{pmatrix}1\\1\end{pmatrix} N_0^3(u) + \begin{pmatrix}1\\2\end{pmatrix} N_1^3(u) + \begin{pmatrix}2\\2\end{pmatrix} N_2^3(u) + \begin{pmatrix}2\\1\end{pmatrix} N_3^3(u),$$

dengan $\boldsymbol{u} = (0, 1, 2, 3, 4, 5, 6)$. *Andaikan satu nilai simpulan* $u = 3.5$ *dimasukkan dua kali ke dalam* $\boldsymbol{u}$ *tanpa mengubah bentuk* $\boldsymbol{P}$. *Cari set titik de Boor baru untuk* $\boldsymbol{P}$.

(b) *Andaikan* $\boldsymbol{u} = (-4, -3, -1, 0, 1, 4, 6, 8)$, *cari lengkung Bézier kubik yang serupa dengan*

$$\boldsymbol{P}(u) = \begin{pmatrix}1\\1\end{pmatrix} N_0^4(u) + \begin{pmatrix}1\\2\end{pmatrix} N_1^4(u) + \begin{pmatrix}2\\2\end{pmatrix} N_2^4(u) + \begin{pmatrix}2\\1\end{pmatrix} N_3^4(u).$$

[100 markah]

Question 3

Let the Bernstein polynomials of degree n be denoted by

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!}\, t^i \left(1-t\right)^{n-i}, \qquad 0 \le t \le 1, \qquad \text{for } \; i = 0,\, 1,\, \ldots,\, n.$$

(a) Figure 1 shows a rational quadratic curve

$$\boldsymbol{R}(t) = \frac{\boldsymbol{C}_0 B_0^2(t) \;+\; 2\boldsymbol{C}_1 B_1^2(t) \;+\; \boldsymbol{C}_2 B_2^2(t)}{B_0^2(t) \;+\; 2B_1^2(t) \;+\; B_2^2(t)},\; t \in [0,\; 1]$$

and its control polygon. The line segment $\boldsymbol{C}_1\boldsymbol{M}$ intersects the curve at a point $\boldsymbol{P}$ where $\boldsymbol{M}$ divides the line segment $\boldsymbol{C}_0\boldsymbol{C}_2$ with ratio $3:2$, find the point $\boldsymbol{P}$.

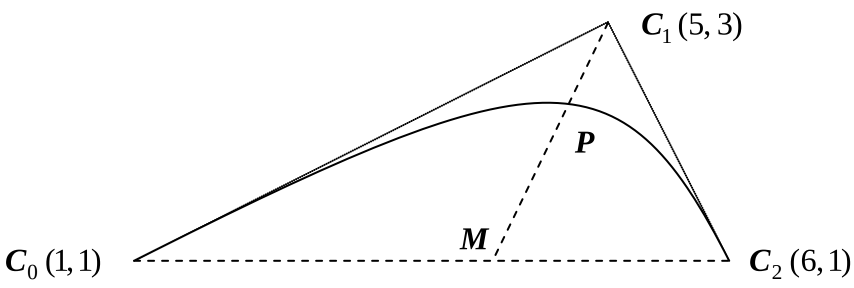


Figure 1

(b) Rewrite the function

$$r(t) = \frac{1}{t^2 + 1},\; t \in [0,\; 1],$$

in the form of non-parametric rational cubic Bézier.

[100 marks]

Soalan 3

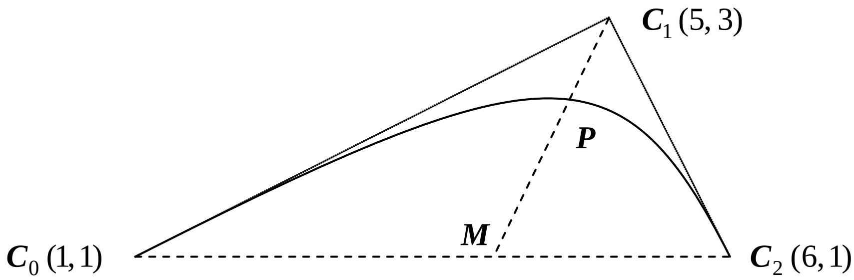
Katakan polinomial Bernstein berdarjah n *ditakrif sebagai*

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!}\, t^i\,(1-t)^{n-i}, \qquad 0 \le t \le 1, \qquad \textit{untuk}\ \ i = 0,\ 1,\ \ldots,\ n.$$

(a) *Rajah 1 menunjukkan satu lengkung kuadratik nisbah*

$$\boldsymbol{R}(t) = \frac{\boldsymbol{C}_0 B_0^2(t)\ +\ 2\boldsymbol{C}_1 B_1^2(t)\ +\ \boldsymbol{C}_2 B_2^2(t)}{B_0^2(t)\ +\ 2B_1^2(t)\ +\ B_2^2(t)},\ t \in [0,\ 1]$$

dan poligon kawalannya. Segmen garis $\boldsymbol{C}_1\boldsymbol{M}$ *bersilang lengkung pada titik* $\boldsymbol{P}$ *di mana* $\boldsymbol{M}$ *membahagikan segmen garis* $\boldsymbol{C}_0\boldsymbol{C}_2$ *dengan nisbah* $3:2$, *cari titik* $\boldsymbol{P}$.



Rajah 1

(b) *Tulis semula fungsi*

$$r(t) = \frac{1}{t^2+1},\ t \in [0,\ 1],$$

dalam bentuk Bézier kubik nisbah tak berparameter.

[100 markah]

Question 4

Let $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ denote 2D points $(1,\ 0)$, $(2,\ 3)$ and $(5,\ 1)$ respectively. A Bézier patch is defined on the triangle $\boldsymbol{ABC}$ by

$$S(u,\ v,\ w) = u^3 + 3v^3 - 2u^2v - uv^2 - 3uvw - 2u^2w - v^2w,$$

where $(u,\ v,\ w)$ are the barycentric coordinates of any point in the triangle with u, v, $w \in [0,\ 1]$ and $u+v+w=1$. Given that the values of S at $\boldsymbol{A}$ and $\boldsymbol{B}$ are 1 and 3 respectively.

(a) Find the value of S at point $(3,\ 2)$.

(b) If the Bézier patch is rewritten in vector-valued form, find all the control points of the patch.

(c) Find the tangent plane to patch S at vertex $\boldsymbol{A}$.

[100 marks]

Soalan 4

Katakan $\boldsymbol{A}$, $\boldsymbol{B}$ *dan* $\boldsymbol{C}$ *menandakan titik 2D* $(1, 0)$, $(2, 3)$ *dan* $(5, 1)$ *masing-masing. Suatu tampalan Bézier ditakrif pada segi tiga* $\boldsymbol{ABC}$ *sebagai*

$$S(u, v, w) = u^3 + 3v^3 - 2u^2v - uv^2 - 3uvw - 2u^2w - v^2w,$$

di mana (u, v, w) *adalah koordinat baripusat bagi sebarang titik dalam segi tiga dengan* u, v, $w \in [0, 1]$ *dan* $u + v + w = 1$. *Diberikan nilai* S *pada* $\boldsymbol{A}$ *dan* $\boldsymbol{B}$ *adalah* 1 *dan* 3 *masing-masing.*

(a) *Cari nilai* S *pada titik* $(3, 2)$.

(b) *Jika tampalan Bézier ditulis semula dalam bentuk vektor, cari semua titik kawalan tampalan.*

(c) *Cari satah tangen ke tampalan* S *pada bucu* $\boldsymbol{A}$.

[100 markah]

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