First Semester Examination
2017/2018 Academic Session

January 2018

MSG453 - Queueing System and Simulation
[Sistem Giliran dan Simulasi]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **SEVENTEEN (17)** pages of printed material before you begin the examination.

[*Sila pastikan bahawa kertas peperiksaan ini mengandungi* ***TUJUH BELAS (17)*** *muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*]

Instructions: Answer **all three (3)** questions.

*[**Arahan:** Jawab **semua tiga (3)** soalan].*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

Question 1

(a) Consider an M/M/1 queueing system with an arrival rate of λ and a service rate of μ.

(i) Draw a rate diagram of this queueing system.

(ii) Using the birth and death process and under the assumption that the system is stable, show that the probability that the system is in state n is:

$$P_n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n \quad \text{for} \quad n = 0,1,2\cdots$$

(iii) Next, show that the average state of the system is:

$$L = \frac{\lambda}{\mu - \lambda}$$

[40 marks]

(b) A fast-food restaurant has one drive-in window. It is estimated that cars arrive according to a Poisson distribution at the rate of 2 every 5 minutes and that there is enough space to accommodate a line of 10 cars. Other arriving cars can wait outside this space, if necessary. It takes 1.5 minutes on the average to fill an order, but the service time actually varies according to an exponential distribution. Determine the following:

(i) The probability that the facility is idle.

(ii) The expected number of customers waiting but currently not being served.

(iii) The expected waiting time until a customer can place his order at the window.

(iv) The probability that the waiting line will exceed the capacity of the space leading to the drive-in window.

(v) The probability that the waiting time per customer will exceed the average waiting time in the queue.

(vi) To attract more business, the owner of the fast-food restaurant decided to give a free drink to each customer who waits more than 5 minutes for service. Normally, a drink costs 50 cents. How much is the owner expected to pay daily for free drinks? Assume that the restaurant is open for 12 hours daily.

(vii) Assuming that customers who cannot join the line in front of the service window will normally go elsewhere, determine items (i), (ii), (iii) and (iv) as above.

[40 marks]

(c) Suppose that a one person tailor shop is in business of making men suits. Each suit requires four distinct tasks to be performed before it is completed. Assume all four tasks must be completed on each suit before another is started. The time to perform each task has an exponential distribution with a mean of 2 hours. If orders for a suit come at the average rate of 5.5 per week (assume an 8-hour per day and 6 day per week), how long can a customer expect to wait to have a suit made?

[20 marks]

Soalan 1

(a) Pertimbangkan sistem giliran M/M/1 dengan kadar ketibaan λ dan kadar layana μ.

(i) Lukiskan gambar rajah kadar bagi sistem giliran itu.

(ii) Dengan menggunakan proses lahir-mati dan di bawah andaian bahawa sistem berkeadaan mantap, tunjukkan bahawa kebarangkalian sisitem berkeadaan n adalah:

$$P_n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n \quad \textit{untuk} \quad n = 0,1,2\cdots$$

(iii) Seterusnya, tunjukkan bahawa keadaan purata sistem adalah:

$$L = \frac{\lambda}{\mu - \lambda}$$

[40 markah]

(b) Sebuah restoran makanan segera mempunyai satu tingkap perkhidmatan pandu lalu. Kenderaan tiba mengikut agihan Poisson pada kadar 2 setiap 5 minit dan ruang yang disediakan dapat menampung satu barisan 10 kenderaan. Jika perlu, kenderaan lain yang tiba boleh menunggu di luar ruang yang disediakan. Masa layanan setiap pesanan adalah 1.5 minit pada puratanya dengan mengikuti agihan eksponen. Tentukan hal-hal yang berikut:

(i) Kebarangkalian bahawa perkhidmatan yang disediakan adalah bersenang.

(ii) Bilangan jangkaan pelanggan yang sedang menunggu untuk dilayan.

(iii) Masa menunggu jangkaan sehingga seseorang pelanggan dapat membuat pesanan di tingkap.

(iv) Kebarangkalian bahawa saiz barisan menunggu adalah melebihi kapasiti ruang menunggu yang disediakan.

(v) *Kebarangkalian bahawa masa menunggu seorang pelanggan adalah melebihi masa menunggu jangkaan.*

(vi) *Bagi menarik lebih ramai pelanggan, tuan punya restoran telah memutuskan untuk memberi satu minuman percuma kepada setiap pelanggan yang menunggu lebih daripada 5 minit sebelum dilayan. Satu minuman biasanya bernilai 50 sen. Berapa banyakkah tuan punya restoran dijangka membayar setiap hari untuk minuman percuma? Andaikan bahawa restoran dibuka 12 jam sehari.*

(vii) *Dengan mengandaikan bahawa pelanggan yang tidak dapat menunggu di ruang menunggu yang disediakan akan pergi ke tempat lain, tentukan perkara (i), (ii), (iii) dan (iv) seperti yang di atas.*

[40 markah]

(c) *Katakan sebuah kedai menjahit perseorangan berurusan membuat pakaian sut lelaki. Setiap sut perlu diproses menggunakan empat langkah berlainan. Andaikan bahawa keempat-empat langkah itu mesti disiapkan terhadap sesuatu sut terlebih dahulu sebelum proses membuat suit berikutnya boleh dimulakan. Masa untuk melaksanakan setiap langkah itu adalah mengikut agihan eksponen dengan min 2 jam. Jika pesanan untuk membuat sut diterima dengan kadar purata 5.5 seminggu (andaikan 8 jam sehari dan 6 hari seminggu) mengikut agihan Poisson, berapa lamakah seorang pelanggan dijangka menunggu supaya pesanannya dapat disiapkan?*

[20 markah]

Question 2

(a) Consider a gas station located on a highway with 5 pumps. Cars arrive to the gas station according to a Poisson process at a rate 50 cars/hour. Any car that is able to enter the gas station stops by one of the available pumps. If all pumps are occupied, the driver will not enter the gas station. The gas station has three workers to service the cars. Each car takes an exponential amount of time for service with average of 5 minutes. The workers remember the order in which cars arrived so they service the cars on a first come first serviced basis. In the long run:

(i) What is the probability that all workers are idle?

(ii) What is the probability that an arriving car will be able to enter the gas station?

(iii) What is the probability that a car will have to wait for a worker?

(iv) On average, how many cars will find all pumps occupied in one hour?

(v) On average, how many cars will be in the station?

(vi) On average, how many cars waiting for service in the station?

(vii) Assume that a driver is in a hurry, so he will enter the gas station if and only if he will be serviced immediately. What is the probability that he will enter this gas station?

(viii) On average, how long a car will have to wait for service?

[35 marks]

(b) A hair saloon is staffed by two barbers, each attending customers separately. Customers arrival to the saloon follows a Poisson process. As long as the number of customers waiting for service does not exceed one, the arrival rate is 12 customers per hour. If there are two people waiting, the arrival rate will be reduced to 8 customers per hour. If there are three people waiting, the arrival rate will be reduced further to 4 customers per hour. Customers will go elsewhere if, upon arrival, there are four people waiting. The time taken for a hair-cut follows exponential distribution with a mean time of 10 minutes.

(i) Draw a rate-diagram to represent the queueing system.

(ii) Determine the probability that an arriving customer will be serviced immediately upon arrival.

(iii) Determine the expected number of customers waiting in the saloon.

(iv) Determine the expected waiting time.

[35 marks]

(c) ASG is a service company that performs a variety of odd jobs such as yard work, tree pruning and house painting. The company's four employees leave the office with the first assignment of the day. After completing an assignment, the employee would call the office requesting instruction for the next job to be performed. The time to complete an assignment is exponential with a mean of 45 minutes. The travel time between jobs is also exponential, with a mean of 20 minutes.

(i) Determine the average number of employees who are travelling between jobs.

(ii) Compute the probability that no employee is on the road.

[30 marks]

Soalan 2

(a) *Pertimbangkan sebuah stesyen minyak yang mempunyai 5 pam yang terletak di sebuah lebuhraya. Ketibaan kereta ke stesyen itu berlaku mengikut proses Poisson dengan kadar 50 kereta/jam. Mana-mana kereta yang berjaya memasuki stesyen itu akan berhenti di salah satu daripada pam yang ada. Jika kesemua pam dipenuhi, pemandu tidak akan memasuki stesyen itu. Stesyen minyak itu mempunyai tiga pekerja yang ditugaskan melayan kereta yang tiba. Setiap kereta memmerlukan masa layan eksponen dengan purata 5 minit. Para pekerja mengingati susunan ketibaan kereta dan mereka melayani kereta berasaskan konsep yang tiba dahulu didahulukan. Dalam jangka masa yang panjang:*

(i) *Apakah kebarangkalian bahawa kesemua pekerja sedang bersenang?*

(ii) *Apakah kebarangkalian bahawa sebuah kereta yang tiba akan berjaya memasuki stesyen minyak itu?*

(iii) *Apakah kebarangkalian bahawa sebuah kereta terpaksa menunggu pekerja?*

(iv) *Pada puratanya, berapakah bilangan kereta dalam masa sejam yang akan mendapati bahawa kesemua pam dipenuhi?*

(v) *Pada puratanya, berapakah bilangan kereta di dalam stesyen?*

(vi) *Pada puratanya, berapakan bilangan kereta yang menanti untuk dilayan di dalam stesyen?*

(vii) *Andaikan bahawa seorang pemandu itu berada dalam keadaan tergesa-gesa, jadi, beliau akan memasuki stesyen minyak itu jika dan hanya jika beliau akan mendapat layanan segera. Apakah kebarangkalian bahawa beliau akan memasuki stesyen minyak itu?*

(viii) *Pada puratanya, berapa lamakah sesebuah kereta perlu menunggu untuk dilayan?*

[35 markah]

(b) *Sebuah kedai gunting mempunyai dua orang tukang gunting yang mengendalikan pelanggan secara berasingan. Ketibaan pelanggan adalah mengikut proses Poisson. Selagi bilangan pelanggan yang sedang menunggu tidak melebihi seorang, kadar ketibaan adalah 12 pelanggan sejam. Jika terdapat dua orang yang sedang menunggu, kadar ketibaan akan menyusut menjadi 8 sejam. Jika terdapat tiga orang yang sedang menunggu, kadar ketibaan adalah 4 sejam. Jika seorang pelanggan tiba dan didapatinya terdapat empat pelanggan lain yang sedang menunggu, dia akan pergi ke kedai yang lain. Masa yang diperlukan untuk mengunting adalah mengikut agihan eksponen dengan min 10 minit.*

(i) *Lukiskan gambar rajah kadar bagi sistem giliran ini.*

(ii) *Tentukan kebarangkalian bahawa seorang pelanggan yang tiba akan terus mendapat layanan.*

(iii) *Tentukan bilangan jangkaan pelanggan yang sedang menunggu.*

(iv) *Tentukan masa jangkaan menunggu.*

[35 markah]

(c) *ASG adalah sebuah syarikat yang menawarkan pelbagai perkhidmatan seperti membersihkan halaman, memotong pokok dan mengecat rumah. Syarikat itu mempunyai empat orang pekerja yang bertugas bersendirian. Setiap pagi pekerja-pekerja itu akan meninggalkan pejabat menuju ke tempat tugasan pertama mereka untuk hari itu. Setelah selesai melaksanakan sesuatu tugas, pekerja-pekerja itu akan menelefon pejabat untuk menerima arahan tentang tugas mereka yang berikutnya. Masa yang diperlukan untuk menyiapkan sesuatu tugas adalah mengikut agihan eksponen dengan min 45 minit. Masa perjalanan dari satu tempat bertugas ke tempat bertugas yang lain pula adalah eksponen dengan min 20 minit.*

(i) *Tentukan bilangan purata pekerja yang sedang dalam perjalanan dari satu tempat bertugas ke tempat bertugas yang lain.*

(ii) *Tentukan kebarangkalian bahawa tidak ada seorang pekerjapun yang sedang dalam perjalanan.*

[30 markah]

Question 3

(a) Consider a small machine shop that consists of three machines: A, B, and C. Although each of the machines performs different operations, it can be safely assumed that the processing-time distributions (including setups) of the three machines are identical.

Processing Time (hours)	Probability
1	0.05
2	0.20
3	0.30
4	0.20
5	0.25

Customer orders for various machined parts arrive at the shop according to the following distributions:

Inter-arrival Time (hours)	Probability
2	0.25
3	0.35
4	0.20
5	0.15
6	0.05

Routing a customer order through the shop depends on the work that has to be done to it. Two major routing exist:

Routing	Percentage of Orders Having the Routing
A-B-C	30
A-C-B	70

The machine shop operates 24 hours a day (three shifts). All orders are processed on a FIFO basis at each machine. Assuming that the shop is empty at the start, simulate the arrival and processing of 10 customers orders. For each order, determine first its arrival time, then its routing. Each time an order is processed on a machine, determine its processing time (you need to determine three different processing times for each order). Make a chart for each machine as follows:

Machine A

Job Number	Arrival Time	Start Service	End Service

Once a job is finished at a machine it goes to the next one on its routing. As soon as the machine is finished with a job, it takes the job with the earliest arrival time of those available (in queue) for processing. When a job is processed by the last machine on its routing, it leaves the system.

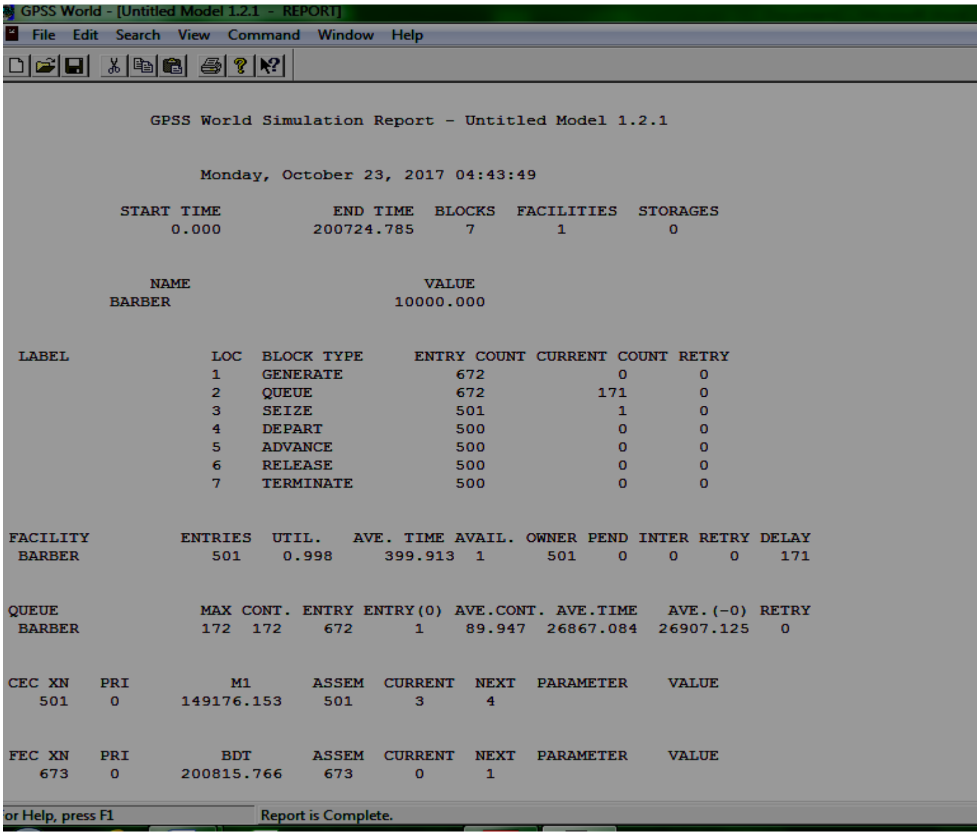
Compute the average waiting time per job, the total idle time of the machines, and the maximum queue length at each machine.

Perform a hand simulation. Use the enclosed two digit random number table with the first column for the *inter-arrival time*, the second column for the *routing* and the third column for the *processing time*. The simulation clock starts at time 0.

[50 marks]

(b) A hospital has 100 patients arriving every day. Daytime is defined as 08:00 to 16:00 (8 hours). 70% of the arrivals are Planned and arrive at 08:00. The rest are Emergency (no appointment) and arrive randomly at a daytime rate that is 5 times as high as the rest of the day. After arrival, a patient has to be examined by a Doctor that has a Poisson processing rate of 2 per hour. Emergency patients have precedence (priority). 7% of Emergency patients have to be examined twice (never three or more times). Draw the simulation model for this problem as if you were using the simulation software Arena. Provide explanation wherever appropriate.

(c) Referring to the GPSS World program output below, interpret and describe all model-specific parameters necessary to actually run the queuing example application. Also justify and explain the type of server model that was used based on the given output.



GPSS World Simulation Report - Untitled Model 1.2.1

Monday, October 23, 2017 04:43:49

START TIME	END TIME	BLOCKS	FACILITIES	STORAGES
0.000	200724.785	7	1	0

NAME	VALUE
BARBER	10000.000

LABEL	LOC	BLOCK TYPE	ENTRY COUNT	CURRENT COUNT	RETRY
	1	GENERATE	672	0	0
	2	QUEUE	672	171	0
	3	SEIZE	501	1	0
	4	DEPART	500	0	0
	5	ADVANCE	500	0	0
	6	RELEASE	500	0	0
	7	TERMINATE	500	0	0

FACILITY	ENTRIES	UTIL.	AVE. TIME	AVAIL.	OWNER	PEND	INTER	RETRY	DELAY
BARBER	501	0.998	399.913	1	501	0	0	0	171

QUEUE	MAX	CONT.	ENTRY	ENTRY(0)	AVE.CONT.	AVE.TIME	AVE.(-0)	RETRY
BARBER	172	172	672	1	89.947	26867.084	26907.125	0

CEC XN	PRI	M1	ASSEM	CURRENT	NEXT	PARAMETER	VALUE
501	0	149176.153	501	3	4		

FEC XN	PRI	BDT	ASSEM	CURRENT	NEXT	PARAMETER	VALUE
673	0	200815.766	673	0	1		

[50 marks]

Soalan 3

(a) *Sebuah bengkel mempunyai tiga buah mesin: A, B dan C. Walaupun ketiga-tiga buah mesin itu menjalankan operasi yang berlainan, masa pemprosesannya masih boleh dianggap sama (termasuk masa penyediaannya).*

Masa Pemprosesan (jam)	*Kebarangkalian*
1	*0.05*
2	*0.20*
3	*0.30*
4	*0.20*
5	*0.25*

Pesanan pelanggan untuk pelbagai bahagian mesin tiba di bengkel itu mengikut agihan berikut:

Lat ketibaan (jam)	*Kebarangkalian*
2	*0.25*
3	*0.35*
4	*0.20*
5	*0.15*
6	*0.05*

Pergerakan pesanan pelanggan melalui mesin-mesin di dalam bengkel adalah bergantung kepada jenis kerja yang perlu dilakukan. Dua jenis pergerakan utama adalah:

Pergerakan	*Peratusan Pesanan yang Melalui Pergerakan Ini*
A-B-C	*30*
A-C-B	*70*

Bengkel itu dibuka 24 jam sehari (tiga syif). Semua pesanan diproses mengikut FIFO pada setiap mesin. Dengan menggunakan andaian bahawa bengkel itu bersenang pada permulaannya, simulasikan ketibaan dan pemprosesan 10 pesanan pelanggan. Untuk setiap pesanan, tentukan waktu ketibaan dan jenis pergerakannya. Setiap kali pesanan diproses oleh mesin, tentukan masa pemprosesannya (anda perlu menentukan tiga masa pemprosesan berlainan untuk setiap pesanan). Bentukkan jadual seperti berikut untuk setiap mesin:

Mesin A

No. Pesanan	*Waktu Ketibaan*	*Mula Layanan*	*Selesai Layanan*

Selepas sesuatu pesanan selesai diproses di sebuah mesin, ia akan pergi ke mesin berikutnya mengikut jadual pergerakan. Sebaik sahaja sebuah mesin selesai dengan sesuatu pesanan, ia akan menerima pesanan yang mempunyai waktu ketibaan terawal dari antara yang sedia ada (dalam giliran) untuk diproses. Apabila sesuatu pesanan telah diproses oleh mesin yang terakhir dalam jadualnya, ia akan meninggalkan sistem

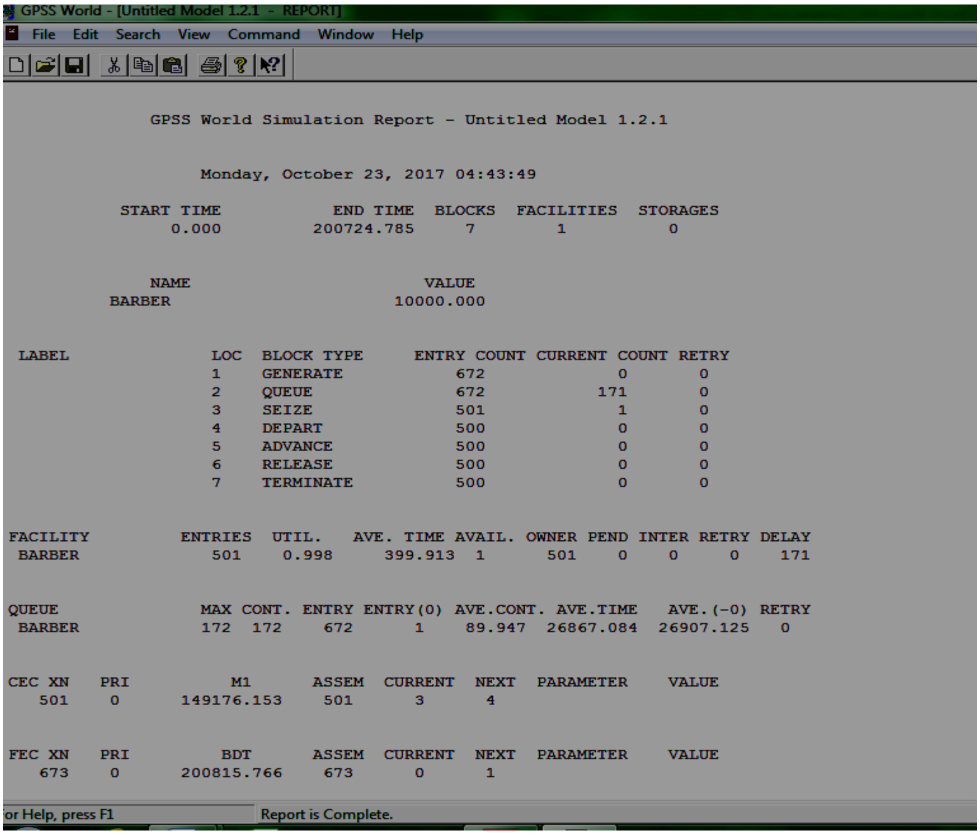
Hitung purata masa menunggu setiap pesanan, jumlah masa mesin bersenang dan panjang maksimum barisan menunggu untuk setiap mesin.

Lakukan simulasi dengan tangan. Guna jadual nombor rawak dua digit yang disertakan dengan lajur pertama untuk lat ketibaan, lajur kedua untuk pergerakan dan lajur ketiga untuk masa pemprosesan. Jam simulasi bermula pada waktu 0.

[50 markah]

(b) *Seramai 100 pesakit datang ke sebuah hospital 100 pesakit pada setiap hari. Waktu siang ditakrifkan bermula pada jam 08:00 pagi hingga 16:00 petang (8 jam). 70% daripada ketibaan pesakit adalah secara temujanji dan tiba di hospital pada jam 08:00 pagi. Selebihnya merupakan kes kecemasan (tanpa temujanji) dan tiba secara rawak pada kadar siang hari iaitu 5 kali lebih tinggi untuk sepanjang hari. Selepas ketibaan, seorang pesakit perlu diperiksa oleh Doktor yang mempunyai kadar pemprosesan Poisson sebanyak 2 per jam. Pesakit kecemasan mempunyai keutamaan. 7% pesakit kecemasan perlu diperiksa dua kali (tidak pernah tiga kali atau lebih). Lukiskan model simulasi untuk masalah ini seolah-olah anda menggunakan perisian simulasi Arena. Berikan penjelasan di mana sesuai..*

(c) *Merujuk kepada output program GPSS di bawah, tafsir dan bincangkan semua parameter khusus model yang diperlukan untuk menjalankan contoh aplikas sistem giliran. Juga terangkan jenis model kaunter yang digunapakai berdasarkan output yang diberikan.*

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              GPSS World Simulation Report - Untitled Model 1.2.1


                   Monday, October 23, 2017 04:43:49

           START TIME           END TIME  BLOCKS  FACILITIES  STORAGES
                0.000         200724.785     7         1          0


              NAME                       VALUE
          BARBER                      10000.000


 LABEL              LOC  BLOCK TYPE     ENTRY COUNT CURRENT COUNT RETRY
                    1    GENERATE           672             0       0
                    2    QUEUE              672           171       0
                    3    SEIZE              501             1       0
                    4    DEPART             500             0       0
                    5    ADVANCE            500             0       0
                    6    RELEASE            500             0       0
                    7    TERMINATE          500             0       0


FACILITY         ENTRIES  UTIL.   AVE. TIME AVAIL. OWNER PEND INTER RETRY DELAY
 BARBER             501    0.998     399.913  1      501    0    0     0    171


QUEUE              MAX CONT. ENTRY ENTRY(0) AVE.CONT. AVE.TIME   AVE.(-0) RETRY
 BARBER            172  172    672      1    89.947  26867.084  26907.125   0


CEC XN   PRI          M1      ASSEM  CURRENT  NEXT  PARAMETER    VALUE
   501    0      149176.153    501      3       4


FEC XN   PRI         BDT      ASSEM  CURRENT  NEXT  PARAMETER    VALUE
   673    0      200815.766    673      0       1
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[50 markah]

APPENDIX 1 / *LAMPIRAN 1*

Formulas for Queueing Theory:

1. *M/M/1*:

$$\rho = \lambda / \mu$$

$$P_n = (1-\rho)\rho^n \quad \text{for } n = 0, 1, 2, \ldots$$

$$L = \frac{\lambda}{\mu - \lambda}$$

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$W = \frac{1}{\mu - \lambda} \quad , \quad W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$P[w > t] = e^{-t/w}$$

$$P[w_q > t] = \rho e^{-t/w}$$

$$\rho = \frac{\lambda}{s\mu}$$

$$P_0 = \left[\frac{(\lambda/\mu)^s}{s!} \frac{1}{(1-\rho)} + \sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} \right]^{-1}$$

$$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} P_0 & , \quad \text{if } 0 \le n \le s \\ \text{if } n > s & , \quad \text{if } n \le s \end{cases}$$

$$L_q = \frac{(\lambda/\mu)^s \rho}{s!(1-\rho)^2} P_0$$

$$W_q = \frac{L_q}{\lambda} \quad , \quad W = W_q + 1/\mu$$

$$L = L_q + \lambda/\mu$$

$$P[w_q > t] = e^{-\mu t} \left[1 + \frac{P_0 \left(\dfrac{\lambda}{\mu} \right)^s}{s!(1-\rho)} \left(\frac{1 - e^{\mu t(s-1-\lambda/\mu)}}{s - 1 - \lambda/\mu} \right) \right]$$

2. *M/M/s*:

$$P[w_q > t] = [1 - P\{w_q = 0\}] e^{-s\mu(1-\rho)t}$$

$$\text{where} \quad P\{w_q = 0\} = \sum_{n=0}^{s-1} P_n$$

APPENDIX 2 / *LAMPIRAN 2*

3. *M/M/s*: finite population of size *M*.

$$P_0 = \left[\sum_{n=0}^{s-1} \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{M} \binom{M}{n} \frac{n!}{s^{n-s} s!} \left(\frac{\lambda}{\mu}\right)^n \right]^{-1}$$

$$P_n = \begin{cases} P_0 \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n & , \quad \text{if } 0 \le n \le s \\ P_0 \binom{M}{n} \left(\frac{n!}{s^{n-s} s!}\right) \left(\frac{\lambda}{\mu}\right)^n & , \quad \text{if } s < n \le M \\ 0 & , \quad \text{if } n > M \end{cases}$$

$$L = P_0 \left[\sum_{n=0}^{s-1} n \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{M} n \binom{M}{n} \frac{n!}{s^{n-s} s!} \left(\frac{\lambda}{\mu}\right)^n \right]$$

$$L_q = L - s + P_0 \sum_{n=0}^{s-1} (s-n) \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n$$

$$W = \frac{L}{\lambda(M-L)} \quad , \quad W_q = \frac{L_q}{\lambda(M-L)}$$

4. *M/G/1*:

$$P_0 = 1 - \rho$$

$$L_q = \frac{\lambda^2 \sigma^2 + \rho^2}{2(1-\rho)}$$

$$L = \rho + L_q$$

$$W_q = \frac{L_q}{\lambda} \quad , \quad W = w_q + \frac{1}{\mu}$$

5. $M / E_k / 1$:

$$L_q = \frac{1+k}{2k} \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$W_q = \frac{1+k}{2k} \frac{\lambda}{\mu(\mu-\lambda)}$$

$$W = W_q + 1/\mu$$

$$L = \lambda W$$

APPENDIX 3 / *LAMPIRAN 3*

6. *M/M/1/k*:

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{k+1}} & (\rho \neq 1) \\ \dfrac{1}{k+1} & (\rho = 1) \end{cases}$$

For $\rho \neq 1$

$$L = \frac{\rho\left[1-(k+1)\rho^k + k\rho^{k+1}\right]}{\left(1-\rho^{k+1}\right)(1-\rho)}$$

$$L_q = L - (1-P_0) = L - \frac{\rho\left(1-\rho^k\right)}{1-\rho^{k+1}}$$

$$W = L/\lambda' \quad , \quad \lambda' = \mu\left(L - L_q\right)$$

$$W_q = W - 1/\mu = L_q/\lambda'$$

For $\rho = 1$

$$L = \frac{k}{2}$$

7. *M/M/s/k*:

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & (0 \le n < s) \\ \dfrac{1}{s^{n-s}s!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & (s \le n \le k) \end{cases}$$

$$P_0 = \begin{cases} \left[\displaystyle\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{(\lambda/\mu)^s}{s!}\,\frac{1-\left(\dfrac{\lambda}{s\mu}\right)^{k-s+1}}{1-\dfrac{\lambda}{s\mu}}\right]^{-1} & \text{for} \quad \left(\dfrac{\lambda}{s\mu} \neq 1\right) \\ \left[\displaystyle\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{(\lambda/\mu)^s}{s!}(k-s+1)\right]^{-1} & \text{for} \quad \left(\dfrac{\lambda}{s\mu} = 1\right) \end{cases}$$

$$L_q = \frac{P_0 (s\rho)^S \rho}{s!(1-\rho)^2}\left[1-\rho^{k-s+1} - (1-\rho)(k-s+1)\rho^{k-s}\right]$$

APPENDIX 4 / ***LAMPIRAN 4***

$$L = L_q + s - P_0 \sum_{n=0}^{s-1} \frac{(s-n)(\rho s)^n}{n!}$$

$$W = \frac{L}{\lambda'} \quad , \quad \lambda' = \lambda(1 - P_k)$$

$$W_q = W - \frac{1}{\mu}$$

$$W_q = \frac{L_q}{\lambda'}$$

8. *M/M/s/s*:

$$P_n = \frac{(\lambda/\mu)^n / n!}{\sum_{i=0}^{s} \left(\frac{\lambda}{\mu}\right)^i / i!} \quad \text{for} \quad (0 \le n \le s)$$

$$P_s = \frac{(s\rho)^s / s!}{\sum_{i=0}^{s} (s\rho)^i / i!} \quad \text{where} \quad \left(\rho = \frac{\lambda}{s\mu}\right).$$

$$L = \frac{\lambda}{\mu}(1 - P_s) \quad , \quad W = \frac{L}{\lambda'} \text{ where } \lambda' = \lambda(1 - P_s)$$

9. $M/M/\infty$:

$$P_n = \frac{(\lambda/\mu)^n e^{-\lambda/\mu}}{n!} \quad \text{for } n = 0, 1, 2, \ldots$$

$$L = \lambda/\mu$$

$$W = \frac{1}{\mu}$$

APPENDIX 5 / *LAMPIRAN 5*

10. *M/M/1*: state-dependent service

$$\mu_n = \begin{cases} \mu_1 & (1 \le n \le k) \\ \mu & (n \ge k) \end{cases}$$

$$P_0 = \left[\frac{1-\rho_1^k}{1-\rho_1} + \frac{\rho\rho_1^{k-1}}{1-\rho} \right]^{-1} \quad (\rho_1 = \lambda / \mu_1, \rho = \lambda / \mu < 1)$$

$$L = P_0 \left[\frac{\rho_1 \left[1 + (k-1)\rho_1^k - k\rho_1^{k-1} \right]}{(1-\rho_1)^2} + \frac{\rho\rho_1^{k-1} \left[k - (k-1)\rho \right]}{(1-\rho)^2} \right]$$

$$L_q = L - (1 - P_0)$$

$$W = \frac{L}{\lambda} \qquad W_q = \frac{L_q}{\lambda}$$

$$W = W_q + \frac{1-P_0}{\lambda}$$

$$P_n = \begin{cases} \left(\dfrac{\lambda}{\mu_1} \right)^n P_0 & (0 \le n < k) \\ \dfrac{\lambda^n}{\mu_1^{k-1} \mu^{n-k+1}} P_0 & (n \ge k) \end{cases}$$

11. *M/M/1*: finite population of size *M*.

$$P_0 = \left[\sum_{n=0}^{M} \frac{M!}{(M-n)!} \left(\frac{\lambda}{\mu} \right)^n \right]^{-1}$$

$$P_n = \frac{M!}{(M-n)!} \left(\frac{\lambda}{\mu} \right)^n P_0 \quad \text{for } n = 1, 2, \ldots, M$$

$$L = M - \frac{\mu}{\lambda} [1 - P_0]$$

$$L_q = M - \frac{\lambda + \mu}{\lambda} (1 - P_0)$$

$$W = \frac{L}{\lambda'} \quad , \quad W_q = \frac{L_q}{\lambda'} \quad \text{where } \lambda' = \lambda (M - L)$$

APPENDIX 6 / *LAMPIRAN 6*

TWO-DIGIT RANDOM NUMBER TABLE

03	26	48	92	38	96	41	04	35	84
71	44	81	46	44	47	07	20	58	04
33	75	06	41	87	72	63	88	59	54
53	71	27	13	37	45	89	61	30	26
41	15	43	91	46	81	57	39	34	86
16	18	75	11	26	80	93	97	29	33
88	50	00	56	70	19	90	00	93	95
13	10	08	15	29	33	75	70	43	05
15	72	73	69	27	75	72	95	99	56
64	10	99	02	18	26	78	69	19	12
98	66	53	86	34	71	09	88	56	08
43	05	06	19	91	78	03	65	08	16
69	82	02	61	98	50	74	84	60	41
06	40	10	24	68	42	39	97	25	55
34	86	83	41	33	83	85	92	32	29
46	05	92	36	82	04	67	05	18	69
28	73	59	56	43	88	61	17	07	48
35	53	49	39	98	14	16	76	69	10
90	90	18	27	75	08	75	17	55	68
62	32	97	16	33	66	02	34	62	26

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