UNIVERSITI SAINS MALAYSIA

First Semester Examination
2017/2018 Academic Session

January 2018

MSG352 - Linear and Integer Programming
[Pengaturcaraan Linear dan Integer]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of TEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEPULUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

Question 1

(a) Determine whether the following statements are true (**T**) or false (**F**). If the statement is false, explain why. State all theorems used in your explanation.

(i) The Revised Simplex Method is less efficient than the Simplex Method.

(ii) It is possible to convert a primal Linear Program (LP) into multiple dual LPs.

(iii) Given any feasible solution to the primal LP, we can use the complementary slackness Theorem to obtain the optimal solution to the dual LP.

(iv) The Branch-and-Bound Method can be used to solve any pure Integer Program (IP).

(v) Any LP can be solved using the Dual Simplex Method.

(vi) A Mixed Integer Program has real variables and integer variables.

(vii) The Revised Simplex Method can be used to solve the LPs at each node of the Branch-and-Bound tree.

(viii) When solving Goal Programming problems, we will not always obtain a solution that is optimal for all goals.

(ix) The Cutting Plane Method adds linear inequalities to the LP relaxation of an IP to reduce the LP's feasible region without cutting off any integer feasible solutions.

(x) The optimal objective value obtained by solving an IP is always better than the optimal objective value obtained by solving its LP relaxation.

(b) Consider an IP with 2 variables, x_1 and x_2. **Sketch** a 2D graph for each of the following situations:

(i) The optimal objective value for both the IP and its LP relaxation are the same.

(ii) The optimal objective value for both the IP and its LP relaxation are different.

Your graph must clearly indicate:

- The feasible region of the IP.
- The feasible region of the LP relaxation.
- The optimal solution for the IP.
- The optimal solution for the LP relaxation.
- The objective function line and its direction.
- All necessary labels.

[25 marks]

Soalan 1

(a) *Tentukan sama ada pernyataan-pernyataan berikut adalah benar (**T**) atau tidak benar (**F**). Bagi pernyataan-pernyataan yang tidak benar, terangkan sebabnya. Nyatakan semua teorem yang anda gunakan dalam penerangan anda.*

(i) *Kaedah Simpleks Tertilik Semula adalah kurang efisien berbanding Kaedah Simpleks.*

(ii) *Kita boleh mengubah suatu model Pengaturcaraan Linear (PL) primal kepada banyak model (PL) dual.*

(iii) *Diberi suatu penyelesaian tersaur bagi model PL primal, kita boleh menggunakan Teorem Kelalaian Lengkap untuk mendapatkan penyelesaian optimum bagi model PL dual.*

(iv) *Kaedah Cabang-dan-Batas boleh digunakan untuk menyelesaikan sebarang model Pengaturcaraan Integer (PI) tulen.*

(v) *Sebarang model PL boleh diselesaikan menggunakan Kaedah Simpleks Dual.*

(vi) *Model PI bercampur mempunyai pembolehubah nyata dan pembolehubah integer.*

(vii) *Kaedah Simpleks Tertilik Semula boleh digunakan untuk meyelesaikan model PL di setiap nod dalam gambarajah pohon bagi kaedah Cabang dan Batas.*

(viii) *Apabila menyelesaikan masalah Pengaturcaraan Gol, kita tidak akan sentiasa dapat penyelesaian yang optimum bagi setiap gol.*

(ix) *Kaedah Satah Potongan menambahkan ketaksamaan linear pada PL tak tegang bagi suatu PI untuk mengurangkan ruang penyelesaian PL tanpa mengecualikan mana-mana penyelesaian tersaur integer.*

(x) *Nilai matlamat optimum yang didapati apabila menyelesaikan suatu PI sentiasa lebih baik daripada nilai matlamat optimum yang didapati apabila menyelesaikan PL tak tegang.*

(b) *Pertimbangkan suatu PI yang mempunyai 2 pembolehubah x_1 dan x_2.* ***Lakarkan*** *graf 2D bagi setiap situasi yang berikut:*

(i) *Nilai matlamat optimum bagi PI dan PL tak tegang adalah sama.*

(ii) *Nilai matlamat optimum bagi PI dan PL tak tegang adalah berlainan.*

Graf anda mesti menunjukkan dengan jelas:

- *Ruang penyelesaian bagi PI.*
- *Ruang penyelesaian bagi PL tak tegang.*
- *Penyelesaian optimum bagi PI.*
- *Penyelesaian optimum bagi PL tak tegang.*
- *Garis fungsi matlamat dan arah tujunya.*
- *Semua label yang patut.*

[25 markah]

Question 2

(a) Consider the following Integer Program (IP):

$$\begin{aligned} &\text{maximise} && z = 5x_1 + 2x_2 \\ &\text{subject to} \\ &&& 3x_1 + x_2 \leq 12 \\ &&& x_1 + x_2 \leq 5 \\ &&& x_1, \quad x_2 \in \mathbb{Z}_{\geq 0}. \end{aligned}$$

Given that the set of optimal basic variables for the Linear Programming relaxation to the IP is $\{x_1, x_2\}$, solve the IP using the Cutting Plane Method.

(b) Consider the following Linear Program (LP):

$$\begin{aligned} &\text{maximise} && z = 9x_1 - 4x_2 + 6x_3 + 8x_4 \\ &\text{subject to} \\ &&& 3x_1 + 3x_2 - 2x_3 + 5x_4 \leq 6 \\ &&& x_1 - 2x_2 + 2x_3 + 3x_4 \leq 6 \\ &&& x_1, \quad x_2, \quad x_3, \quad x_4 \geq 0. \end{aligned}$$

The tableau for the LP at iteration k is as follows:

Basic	x_1	x_2	x_3	x_4	s_1	s_2	Solution
z	0	13	-12	7	3	0	18
x_1	1	1	$-2/3$	$5/3$	$1/3$	0	2
s_2	0	-3	$8/3$	$4/3$	$-1/3$	1	4

Continue solving the LP using the **Revised Simplex Method**.
Do not start from the first iteration.

[30 marks]

Soalan 2

(a) *Pertimbangkan model Pengaturcaraan Integer (PI) berikut:*

$$\begin{aligned} &\textit{maksimumkan} && z = 5x_1 + 2x_2 \\ &\textit{terhadap} \\ & && 3x_1 + x_2 \leq 12 \\ & && x_1 + x_2 \leq 5 \\ & && x_1, \quad x_2 \in \mathbb{Z}_{\geq 0}. \end{aligned}$$

Diberi set pembolehubah asas optimum bagi model Pengaturcaraan Linear (PL) tak tegang adalah $\{x_1, x_2\}$, selesaikan PI tersebut menggunakan Kaedah Satah Potongan.

(b) *Pertimbangkan model PL berikut:*

$$\begin{aligned} &\textit{maksimumkan} && z = 9x_1 - 4x_2 + 6x_3 + 8x_4 \\ &\textit{terhadap} \\ & && 3x_1 + 3x_2 - 2x_3 + 5x_4 \leq 6 \\ & && x_1 - 2x_2 + 2x_3 + 3x_4 \leq 6 \\ & && x_1, \quad x_2, \quad x_3, \quad x_4 \geq 0. \end{aligned}$$

Tablo bagi PL pada lelaran k adalah seperti berikut:

Asas	x_1	x_2	x_3	x_4	s_1	s_2	*Penyelesaian*
z	*0*	*13*	*−12*	*7*	*3*	*0*	*18*
x_1	*1*	*1*	$-2/3$	$5/3$	$1/3$	*0*	*2*
s_2	*0*	*−3*	$8/3$	$4/3$	$-1/3$	*1*	*4*

*Teruskan menyelesaikan model PL dengan menggunakan **Kaedah Simpleks Tertilik Semula**. **Jangan** mula dari lelaran pertama.*

[30 markah]

Question 3

Consider the following **uncorrected initial tableau** for a preemptive Goal Programming problem:

Basic	x_1	x_2	x_3	s_1^-	s_1^+	s_2^-	s_2^+	s_3^-	s_3^+	Solution
z_1	0	0	0	$-P_1$	$-P_1$	0	0	0	0	0
z_2	0	0	0	0	0	0	$-P_2$	0	0	0
z_3	0	0	0	0	0	0	0	$-P_3$	0	0
s_1^-	5	3	4	1	-1	0	0	0	0	40
s_2^-	5	7	8	0	0	1	-1	0	0	55
s_3^-	12	9	15	0	0	0	0	1	-1	125

(a) Give the following:

- The Goal Programming formulation for the problem.
- The original inequality or equation (not in standard form) for each goal. Show your working.

(b) Continue solving the problem and report the following:

- The values of x_1, x_2 and x_3.
- The "status" of each goal (met/not met and by how much). [20 marks]

Soalan 3

*Pertimbangkan **tablo permulaan tidak dibetulkan** berikut bagi suatu masalah Pengaturcaraan Gol "preemptive":*

Asas	x_1	x_2	x_3	s_1^-	s_1^+	s_2^-	s_2^+	s_3^-	s_3^+	*Penyelesaian*
z_1	*0*	*0*	*0*	$-P_1$	$-P_1$	*0*	*0*	*0*	*0*	*0*
z_2	*0*	*0*	*0*	*0*	*0*	*0*	$-P_2$	*0*	*0*	*0*
z_3	*0*	*0*	*0*	*0*	*0*	*0*	*0*	$-P_3$	*0*	*0*
s_1^-	*5*	*3*	*4*	*1*	*-1*	*0*	*0*	*0*	*0*	*40*
s_2^-	*5*	*7*	*8*	*0*	*0*	*1*	*-1*	*0*	*0*	*55*
s_3^-	*12*	*9*	*15*	*0*	*0*	*0*	*0*	*1*	*-1*	*125*

(a) *Berikan:*

- *Model Pengaturcaraan Gol bagi masalah tersebut.*
- *Ketaksamaan atau persamaan asal (bukan dalam bentuk piawai) bagi setiap gol. Tunjukkan jalan kerja anda.*

(b) *Teruskan menyelesaikan masalah tersebut dan laporkan perkara-perkara berikut:*

- *Nilai-nilai bagi x_1, x_2 dan x_3.*
- *"Status" bagi setiap gol (dipenuhi/tidak dipenuhi dan sebanyak mana kurang atau lebih).*

[20 markah]

Question 4

(a) HealthyPeople Sdn. Bhd. makes 2 types of protein bars (Chocoloco and Nuttynuts) using 3 ingredients (chocolate chips, nuts and raisins). They have formulated a Linear Program (LP) which maximises revenue (in RM), and obtained the z-row of the optimal tableau, both given below:

$$\text{maximise} \quad z = x_1 + 4x_2$$

subject to

$$\begin{aligned} 2x_1 + x_2 &= 6 && \text{(choc. chips in kg)} \\ -x_1 + x_2 &\geq 3 && \text{(nuts in kg)} \\ x_1 + 2x_2 &\leq 10 && \text{(raisins in kg)} \\ x_1, \quad x_2 &\geq 0. \end{aligned}$$

Basic	x_1	x_2	s_3	e_2	a_1	a_2	Solution
z	0	0	$^7/_3$	0	$M - {}^2/_3$	M	$^{58}/_3$

Find and interpret the shadow prices of each constraint. State all theorem and methods used.

(b) Solve the following LP using the Dual Simplex Method:

$$\text{minimise} \quad z = 2x_1 + x_2$$

subject to

$$\begin{aligned} 3x_1 + x_2 &\geq 3 \\ 4x_1 + 3x_2 &\geq 6 \\ x_1 + 2x_2 &\leq 3 \\ x_1, \quad x_2 &\geq 0. \end{aligned}$$

(c) Consider the following LP:

$$\text{minimise} \quad z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

subject to

$$\begin{aligned} x_1 + \quad x_2 + \quad \ldots + \quad x_n &= 1 \\ x_1, \quad x_2, \quad \ldots, \quad x_n &\geq 0. \end{aligned}$$

where

$$c_1, c_2, \ldots, c_n \in \mathbb{R}.$$

(i) Find the dual of the LP.

(ii) Given that $c_3 < c_i \ \forall i$ such that $i \neq 3$, solve the dual LP.

(iii) Use the solution in (ii) to solve the primal LP. State all theorems used.

[25 marks]

Soalan 4

(a) *HealthyPeople Sdn. Bhd. membuat 2 jenis bar protein (Chocoloco dan Nuttynuts) menggunakan 3 bahan ramuan (cip coklat, kacang dan kismis). Mereka merumuskan suatu model Pengaturcaraan Linear (PL) yang memaksimumkan pendapatan (dalam RM), dan memperoleh baris-z bagi tablo optimum, kedua-duanya diberikan di bawah:*

$$\textit{maksimumkan} \quad z = x_1 + 4x_2$$

terhadap

$$\begin{aligned} 2x_1 + \quad x_2 &= 6 \quad && \textit{(cip coklat dalam kg)} \\ -x_1 + \quad x_2 &\geq 3 \quad && \textit{(kacang dalam kg)} \\ x_1 + 2x_2 &\leq 10 \quad && \textit{(kismis dalam kg)} \\ x_1, \quad x_2 &\geq 0. \end{aligned}$$

Asas	x_1	x_2	s_3	e_2	a_1	a_2	*Penyelesaian*
z	*0*	*0*	$^7/_3$	*0*	$M - {}^2/_3$	M	$^{58}/_3$

Dapatkan dan tafsirkan harga dual bagi setiap kekangan. Nyatakan semua teorem dan kaedah yang anda gunakan.

(b) *Selesaikan model PL berikut menggunakan Kaedah Simpleks Dual:*

$$\textit{minimumkan} \quad z = 2x_1 + x_2$$

terhadap

$$\begin{aligned} 3x_1 + x_2 &\geq 3 \\ 4x_1 + 3x_2 &\geq 6 \\ x_1 + 2x_2 &\leq 3 \\ x_1, \quad x_2 &\geq 0. \end{aligned}$$

(c) *Pertimbangkan model PL berikut:*

$$\textit{minimumkan} \quad z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

terhadap

$$\begin{aligned} x_1 + x_2 + \ldots + x_n &= 1 \\ x_1, \quad x_2, \quad \ldots, \quad x_n &\geq 0. \end{aligned}$$

di mana

$$c_1, c_2, \ldots, c_n \in \mathbb{R}.$$

(i) *Dapatkan dual bagi PL.*

(ii) *Diberi* $c_3 < c_i \ \forall i$ *di mana* $i \neq 3$*, selesaikan model PL dual.*

(iii) *Gunakan penyelesaian dalam (ii) untuk menyelesaikan model PL primal. Nyatakan semua teorem yang anda gunakan.*

[25 markah]