

SULIT



First Semester Examination
2017/2018 Academic Session

January 2018

MAT382 - Introductory Numerical Methods
[Pengenalan Kaedah Berangka]

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **EIGHT (8)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN (8)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **all four (4)** questions.

[Arahan : Jawab **semua empat (4)** soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunakan].

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Question 1

(a) Consider the following linear system:

$$\begin{pmatrix} -1 & 6 & 2 \\ 4 & 2 & 0 \\ 2 & 2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 16 \\ 20 \end{pmatrix}$$

- (i) By using the starting vector, $p_0 = (x_0, y_0, z_0) = (0, 0, 0)$, compute the first two Jacobi iterates of p_1 and p_2 .
- (ii) By using the starting vector, $p_0 = (x_0, y_0, z_0) = (0, 0, 0)$, compute the first two Gauss-Seidel iterates of p_1 and p_2 .
- (iii) Compare your answers in (i) and (ii). Which method performs better? Why?

[60 marks]

(b) Determine the root of $f(x) = -0.9x^2 + 1.7x + 2.5$ where $x_0 = 5$ by using

- (i) Fixed-Point Iteration method
- (ii) Newton-Raphson method

Perform the computation up to 7 successive iterations .

[40 marks]

Soalan 1

(a) *Pertimbangkan sistem linear berikut:*

$$\begin{pmatrix} -1 & 6 & 2 \\ 4 & 2 & 0 \\ 2 & 2 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 16 \\ 20 \end{pmatrix}$$

- (i) *Dengan menggunakan vektor permulaan, $p_0 = (x_0, y_0, z_0) = (0, 0, 0)$, kirakan dua lelaran pertama Jacobi iaitu p_1 dan p_2 .*
- (ii) *Dengan menggunakan vector permulaan, $p_0 = (x_0, y_0, z_0) = (0, 0, 0)$, kirakan dua lelaran pertama Gauss-Seidel iaitu p_1 dan p_2 .*
- (iii) *Bandingkan jawapan anda dalam (i) dan (ii). Kaedah manakah lebih baik? Kenapa?*

[60 markah]

(b) *Cari punca bagi $f(x) = -0.9x^2 + 1.7x + 2.5$ di mana $x_0 = 5$ dengan menggunakan*

- (i) *Kaedah Lelaran Titik Tetap*
- (ii) *Kaedah Newton-Raphson*

Lakukan pengiraan sehingga 7 lelaran berterusan.

[40 markah]

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Question 2

- (a) (i) Show that the iteration equation for the Secant method can be written in the following form (Hint: Use the Secant iteration formula):

$$p_n = \frac{f(p_{n-1})p_{n-2} - f(p_{n-2})p_{n-1}}{f(p_{n-1}) - f(p_{n-2})}$$

[30 marks]

- (iii) Let $f(x) = -x^3 - \cos(x)$, and $p_0 = -1, p_1 = 0$, find p_3 using the Secant method.
- (b) By using Regula-Falsi method, find an approximate root for $x \log_{10} x = 1.2$. Let $x_0 = 2$ and compute up to 4 successive iterations.

[30 marks]

- (c) The velocity of a body is given by

$$v(t) = \begin{cases} 2t, & 1 \leq t \leq 5 \\ 5t^2 + 3, & 5 < t \leq 14 \end{cases}$$

where t is given in seconds, and v is given in m/s . Use the two-segment trapezoidal rule to find the distance covered by the body from $t = 2$ seconds to $t = 9$ seconds .

[40 marks]

Soalan 2

- (a) (i) *Tunjukkan bahawa persamaan lelaran untuk Kaedah Sekan boleh ditulis dalam bentuk berikut (Petunjuk: Gunakan formula lelaran Sekan):*

$$p_n = \frac{f(p_{n-1})p_{n-2} - f(p_{n-2})p_{n-1}}{f(p_{n-1}) - f(p_{n-2})}$$

[30 markah]

- (ii) *Biarkan $f(x) = -x^3 - \cos(x)$ dan $p_0 = -1$, $p_1 = 0$, cari p_3 menggunakan Kaedah Sekan.*
- (b) *Dengan menggunakan Kaedah Kedudukan Palsu, dapat anggaran punca bagi $x \log_{10} x = 1.2$. Biar $x_0 = 2$ dan kira sehingga 4 lelaran.*

[30 markah]

- (c) *Halaju sebuah objek diberikan oleh*

$$v(t) = \begin{cases} 2t, & 1 \leq t \leq 5 \\ 5t^2 + 3, & 5 < t \leq 14 \end{cases}$$

di mana t adalah dalam saat, dan v diberikan dalam m/s . Gunakan Kaedah Trapezoid dua segmen untuk mencari jarak yang dilitupi oleh objek tersebut dari $t = 2$ saat hingga $t = 9$ saat.

[40 markah]

Question 3

- (a) Use the Power Method to calculate an approximation to the dominant eigen pair for

$$A = \begin{pmatrix} -7 & 2 \\ 8 & -1 \end{pmatrix}$$

[30 marks]

- (b) Solve the following system of linear equations by LU factorization.

$$\begin{aligned} 9x_1 + 3x_2 + 3x_3 + 3x_4 &= 24 \\ 3x_1 + 10x_2 - 2x_3 - 2x_4 &= 17 \\ 3x_1 - 2x_2 + 18x_3 + 10x_4 &= 45 \\ 3x_1 - 2x_2 + 10x_3 + 10x_4 &= 29 \end{aligned}$$

[70 marks]

Soalan 3

- (a) *Gunakan Kaedah Kuasa untuk mengira penganggaran kepada pasangan dominan eigen untuk*

$$A = \begin{pmatrix} -7 & 2 \\ 8 & -1 \end{pmatrix}$$

[30 markah]

- (b) *Selesaikan sistem persamaan linear berikut dengan menggunakan kaedah penfaktoran LU.*

$$\begin{aligned} 9x_1 + 3x_2 + 3x_3 + 3x_4 &= 24 \\ 3x_1 + 10x_2 - 2x_3 - 2x_4 &= 17 \\ 3x_1 - 2x_2 + 18x_3 + 10x_4 &= 45 \\ 3x_1 - 2x_2 + 10x_3 + 10x_4 &= 29 \end{aligned}$$

[70 markah]

Question 4

- (a) The following data are taken from an experiment:

Temp, T ($^{\circ}C$)	140	150	160	170	180
Pressure (N/m^2)	3.685	4.854	6.302	8.076	10.225

Find the pressure at temperature $T = 142^{\circ}C$ and $T = 175^{\circ}C$.

[30 marks]

- (b) Find
- $f(8)$
- by Newton's Divided Difference formula for the data:

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

[30 marks]

- (c) By using Newton's Forward Interpolation formula, find polynomial
- $f(x)$
- satisfying the following data.

x	4	6	8	10
y	1	3	8	10

Hence evaluate y at $x = 5$.

[40 marks]

Soalan 4

(a) Data berikut diambil dari satu eksperimen:

Suhu, T ($^{\circ}\text{C}$)	140	150	160	170	180
Tekanan (N/m^2)	3.685	4.854	6.302	8.076	10.225

Dapatkan tekanan pada suhu $T = 142^{\circ}\text{C}$ and $T = 175^{\circ}\text{C}$.

[30 markah]

(b) Cari $f(8)$ dengan formula Beza Terbahagi Newton untuk data:

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

[30 markah]

(c) Dengan menggunakan formula Interpolasi Newton ke Hadapan, cari polinomial $f(x)$ yang memenuhi data berikut.

x	4	6	8	10
y	1	3	8	10

Dengan itu nilaikan y pada $x = 5$.

[40 markah]

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