First Semester Examination
2017/2018 Academic Session

January 2018

MAT264 - Non-Parametric Statistics
[Statistik Tak Berparameter]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **TWENTY TWO (22)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH DUA (22)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all eight (8)** questions.

*[**Arahan:** Jawab **semua lapan (8)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai.]

Question 1

Under what circumstances is the sign test preferred to the *t*-test for making inferences about the central tendency of a population?

[7 marks]

Soalan 1

Dalam keadaan apakah ujian tanda lebih sesuai digunakan berbanding dengan ujian-t dalam membuat kesimpulan tentang kecenderungan pusat populasi?

[7 markah]

Question 2

(a) A student who typically does not do his homework was asked to toss a coin 20 times and write down the sequence of results. Instead of tossing the coin, the student simply wrote down the following sequence (reading from left to right) of hypothetical outcomes.

H T H T H T H H T H
T T H T H T H T H T

Use an appropriate test to show that the professor was justified in accusing the student of not actually tossing the coin.

(b) The following data represent the 2015 and 2016 net earnings of common stocks in 20 representative corporations.

Corporations	2015 (RM)	2016 (RM)
1	1680	1710
2	1920	502170
3	2500	2250
4	2900	2430
5	3110	2320
6	3350	3150
7	3800	3300
8	3850	5520
9	3890	3320
10	4360	3760
11	4640	4790
12	4760	4330
13	5350	6050
14	5810	7090
15	6110	6380
16	6350	6000
17	6690	6010
18	8410	7410
19	8830	9330
20	8970	9250

Does there seem to be significant trend in earning from 2015 to 2016?

[15 marks]

Soalan 2

(a) *Seorang pelajar yang biasanya tidak melakukan kerja rumahnya diminta melambung duit syiling 20 kali dan menuliskan keputusan urutan. Pelajar tersebut tidak melambung duit syiling, tetapi hanya menulis jujukan (dibaca dari kiri ke kanan) berikut.*

H T H T H T H H T H
T T H T H T H T H T

Gunakan ujian yang sesuai untuk menunjukkan bahawa profesor itu wajar menuduh pelajar tidak melambung duit syiling itu.

(b) *Data berikut mewakili pendapatan bersih saham biasa bagi 20 syarikat perwakilan pada tahun 2015 dan 2016.*

Syarikat	***2015 (RM)***	***2016 (RM)***
1	1680	1710
2	1920	502170
3	2500	2250
4	2900	2430
5	3110	2320
6	3350	3150
7	3800	3300
8	3850	5520
9	3890	3320
10	4360	3760
11	4640	4790
12	4760	4330
13	5350	6050
14	5810	7090
15	6110	6380
16	6350	6000
17	6690	6010
18	8410	7410
19	8830	9330
20	8970	9250

Adakah terdapat trend yang signifikan dalam pendapatan dari tahun 2015 hingga 2016?

[15 markah]

Question 3

The English department at a college has hired a new instructor to teach the composition course to first-year students. The department head is concerned that the new instructor's grading practices might not be consistent with those of the professor (Professor A) who taught this course previously. She randomly selects 10 essays written by students for this class and make two copies of each essay. She asks Professor A and this instructor (working independently) to assign a numerical grade to each of the 10 essays. The results are shown in the following table.

Essay	**Professor A**	**Instructor**
1	75	80
2	62	50
3	90	85
4	48	55
5	67	63
6	82	78
7	94	89
8	76	81
9	78	75
10	84	83

(a) Suppose the department head wants to determine whether the instructor tends to grade higher or lower than Professor A. Which of the statistical tests could she use? Note that more than one test may be appropriate.

(b) Using an appropriate test from your answer in part (a), can you conclude that the instructor tends to grade higher or lower than Professor A?

(c) Suppose the department head wants to determine whether the instructor is consistent with Professor A in the sense that they tend to agree on which paper is the best, which is the second best, and so forth. Which test would be appropriate to use? State the relevant null and alternative hypotheses.

(d) Using the test you chose in part (c), can you conclude that Professor A and the instructor are consistent in their grading?

[20 marks]

Soalan 3

Jabatan Bahasa Inggeris di sebuah kolej telah mengupah seorang pengajar baru untuk mengajar kursus menulis esei kepada pelajar tahun pertama. Ketua Jabatan prihatin bahawa amalan penggredan pengajar baru mungkin tidak konsisten dengan profesor (Profesor A) yang mengajar kursus ini sebelumnya. Beliau secara rawak memilih 10 esei yang ditulis oleh pelajar kelas ini dan membuat dua salinan bagi setiap esei. Beliau meminta Professor A dan pengajar baru (bekerja secara berasingan) untuk memberikan gred berangka kepada setiap esei. Hasilnya ditunjukkan dalam jadual berikut.

Esei	***Profesor A***	***Pengajar***
1	75	80
2	62	50
3	90	85
4	48	55
5	67	63
6	82	78
7	94	89
8	76	81
9	78	75
10	84	83

(a) *Katakan ketua jabatan ingin menentukan sama ada pengajar baru cenderung menilai lebih tinggi atau lebih rendah daripada Profesor A. Nyatakan ujian-ujian statistik yang boleh digunakannya? Perhatikan bahawa lebih daripada satu ujian mungkin sesuai.*

(b) *Gunakan ujian yang sesuai dari jawapan anda dalam bahagian (a), adakah anda dapat membuat kesimpulan bahawa pengajar baru cenderung menilai lebih tinggi atau lebih rendah daripada Profesor A?*

(c) *Katakan ketua jabatan ingin menentukan sama ada pengajar adalah konsisten dengan Profesor A dalam erti kata bahawa mereka cenderung bersetuju manakah kertas yang terbaik, yang kedua terbaik, dan sebagainya. Ujian mana yang patut digunakan? Nyatakan hipotesis nol dan alternatif yang berkaitan.*

(d) *Gunakan ujian yang anda pilih dalam bahagian (c), adakah anda dapat membuat kesimpulan bahawa Profesor A dan pengajar adalah konsisten dalam penilaian mereka?*

[20 markah]

Question 4

The following table shows the self-confidence test scores of seven employees before and after they attended a course designed to build self-confidence.

Before	**After**
8	10
5	8
4	5
9	11
6	6
9	7
5	9

(a) At the 5% level of significance, can you conclude that attending this course increase the median self-confidence test score of employees?

(b) Determine the *p*-value.

(c) Construct a 95% confidence interval for median difference using the Wilcoxon signed-rank test.

[10 marks]

Soalan 4

Jadual berikut menunjukkan skor ujian keyakinan diri bagi tujuh orang pekerja sebelum dan selepas mereka mengikuti kursus yang direka untuk membina keyakinan diri.

Sebelum	***Selepas***
8	10
5	8
4	5
9	11
6	6
9	7
5	9

(a) Pada aras keertian 5%, dapatkah anda menyimpulkan bahawa mengikuti kursus ini akan meningkatkan median skor ujian keyakinan diri bagi pekerja-perkerja?

(b) Tentukan nilai-p.

(c) Binakan suatu selang keyakinan 95% bagi perbezaan median dengan menggunakan ujian Wilcoxon pangkat-bertanda.

[10 markah]

Question 5

A group of student at USM wants to compare textbook costs for students majoring in economics, history, and psychology. The group obtained data from random samples of 10 economics majors, 9 history majors, and 11 psychology majors, all in the second semester of their first year. The total textbook costs of the 30 students were recorded and ranked. The rank sums for economics and history majors were 134 and 157 respectively.

(a) Find the rank sum for psychology majors. [Hint: The sum of integers from 1 through n is given by $n(n+1)/2$.]

(b) At the 2.5% level of significance, can you reject the null hypothesis that the median textbook costs are the same for students in all three majors who are in the second semester of the first year?

[10 marks]

Soalan 5

Suatu kumpulan pelajar di USM ingin membandingkan kos buku teks untuk pelajar dalam bidang ekonomi, sejarah, dan psikologi. Kumpulan ini memperoleh data daripada sampel rawak 10 jurusan ekonomi, 9 jurusan sejarah, dan 11 jurusan psikologi, semuanya dalam semester kedua tahun pertama mereka. Jumlah kos buku teks bagi 30 pelajar tersebut telah direkodkan dan disenaraikan. Jumlah pangkat untuk jurusan ekonomi dan sejarah adalah 134 dan 157 masing-masing.

(a) Dapatkan julat pangkat jurusan psikologi. [Petunjuk: Jumlah integer dari 1 hingga n diberikan oleh $n(n+1)/2$*.]*

(b) Pada aras keertian 2.5%, bolehkah anda menolak hipotesis nol bahawa kos median buku teks adalah sama untuk pelajar dalam ketiga-tiga jurusan yang berada pada semester kedua tahun pertama?

[10 markah]

Question 6

The ages and blood pressure of 15 women are recorded as follows. Does there seem to be a significant monotonic relation between age and blood pressure?

Age	Blood Pressure
48	144
60	168
35	135
38	125
55	159
51	148
49	128
38	134
54	151
56	152
31	141
24	144
77	170
63	157
67	162

[8 marks]

Soalan 6

Umur dan tekanan darah bagi 15 wanita dicatatkan seperti berikut. Adakah terdapat hubungan monotonik yang signifikan antara umur dan tekanan darah?

Umur	*Tekanan darah*
48	144
60	168
35	135
38	125
55	159
51	148
49	128
38	134
54	151
56	152
31	141
24	144
77	170
63	157
67	162

[8 markah]

Question 7

A certain town consists of five wards. Then houses are selected at random from each wards and given a score from 0 to 100, depending on the level of deterioration of the house and yard (0 = no deterioration, 100 = no redeeming social value). The results are listed in the table below.

House	Ward 1	Ward 2	Ward 3	Ward 4	Ward 5
1	08	74	92	03	37
2	45	42	79	09	28
3	43	77	99	22	42
4	64	09	38	06	44
5	03	32	31	26	01
6	85	66	83	20	32
7	74	16	27	56	65
8	48	45	76	20	02
9	19	15	82	04	80
10	57	24	37	29	93

(a) Which test that you feel is the best one to analyze these data to detect difference from ward to ward?

(b) Using an appropriate test from your answer in part (a) to test the hypothesis of no differences from ward to ward.

[10 marks]

Soalan 7

Sebuah pekan tertentu terdiri daripada lima wad. Kemudian rumah-rumah dipilih secara rawak dari setiap wad dan diberikan markah dari 0 hingga 100, bergantung pada tahap kemerosotan rumah dan halaman (0 = tidak merosot, 100 = tidak ada nilai sosial penebusan). Hasilnya disenaraikan dalam jadual di bawah.

Rumah	Kawasan 1	Kawasan 2	Kawasan 3	Kawasan 4	Kawasan 5
1	08	74	92	03	37
2	45	42	79	09	28
3	43	77	99	22	42
4	64	09	38	06	44
5	03	32	31	26	01
6	85	66	83	20	32
7	74	16	27	56	65
8	48	45	76	20	02
9	19	15	82	04	80
10	57	24	37	29	93

(a) Apakah ujian yang anda rasakan terbaik untuk menganalisis data ini untuk mengesan perbezaan dari kawasan ke kawasan?

(b) Gunakan ujian dari jawapan anda dalam bahagian (a) untuk menguji hipotesis tidak ada perbezaan dari lingkungan ke wad.

[10 markah]

Question 8

A psychology research team set up a temporary research facility in a busy shopping mall, and conducted experiments using volunteers over 12 years of age in groups of 10. They paid each volunteer RM20 for participating. The table below shows the frequency distribution of number of males in each group of 10 volunteers.

Number of males	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	1	5	7	21	27	20	12	4	2	1

(a) Use the Kolmogorov-Smirnov one sample test to test the null hypothesis that number of males per group of 10 is distributed as the binomial with $p = 0.50$.

(b) Perform the test of part (a) using the Chi-square test.

(c) Compare your conclusions from parts (a) and (b).

[20 marks]

Soalan 8

Pasukan penyelidikan psikologi menubuhkan satu pusat penyelidikan sementara di pusat membeli-belah yang sibuk, dan menjalankan eksperimen dengan menggunakan sukarelawan yang lebih dari 12 tahun dalam satu kumpulan dengan 10 orang. Mereka membayar setiap sukarelawan RM20 untuk penyertaan mereka. Jadual di bawah menunjukkan taburan kekerapan bilangan lelaki dalam setiap kumpulan dengan 10 orang sukarelawan.

Bilangan Lelaki	0	1	2	3	4	5	6	7	8	9	10
Kekerapan	0	1	5	7	21	27	20	12	4	2	1

(a) Gunakan ujian Kolmogorov-Smirnov satu sampel untuk menguji hipotesis nol bahawa bilangan lelaki setiap kumpulan dengan 10 orang adalah ditaburkan sebagai binomial dengan $p = 0.50$.

(b) Laksanakan ujian pada bahagian (a) dengan menggunakan ujian Khi Kuasa dua.

(c) Bandingkan kesimpulan anda di bahagian (a) dan (b).

[20 markah]

APPENDIX

1. Sign Test**:**

Small sample: X = Number of (+) signs [or (–) signs]

Large sample: $z = \dfrac{(k+0.5)-n/2}{\sqrt{n}/2}$

2. Wilcoxon Signed-rank:

Small sample: $W = \min\left(\sum(+), \sum(-)\right)$

Large sample: $Z = \dfrac{W-\mu_W}{\sigma_W}$, $\mu_W = \dfrac{n(n+1)}{4}$, $\sigma_W = \sqrt{\dfrac{n(n+1)(2n+1)}{24}}$

3. Mann-Whitney Test:

Small sample:

$$U_1 = n_1 n_2 + \frac{n_2(n_1+1)}{2} - R_2$$

$$U_2 = n_1 n_2 + \frac{n_1(n_2+1)}{2} - R_1$$

Large sample:

$$z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1+n_2+1)}{12}}}$$

4. The Median Test:

$$T = \frac{A/n_1 - B/n_2}{\sqrt{\hat{p}(1-)\hat{p}\left(1/n_1 + 1/n_2\right)}}$$

5. Chi-square Test:

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$$

6. Fisher's Exact Test:

$$P = \frac{(n_{11}+n_{12})!(n_{11}+n_{21})!(n_{21}+n_{22})!(n_{12}+n_{22})!}{n_{11}!n_{12}!n_{21}!n_{22}!n!}$$

7. McNemar's Test:

$$z = \frac{n_{12}-n_{21}}{\sqrt{n_{12}+n_{21}}}$$

8. Run Test:

Large sample: $$z = \frac{r - \left\{\left[\left(2n_1n_2\right)/\left(n_1+n_2\right)\right]+1\right\}}{\sqrt{\frac{2n_1n_2\left(2n_1n_2-n_1-n_2\right)}{\left(n_1+n_2\right)^2\left(n_1+n_2-1\right)}}}$$

9. Wald-Walfowitz Runs Test:

Large Sample: $$z = \frac{r - \left(\frac{2n_1n_2}{n_1+n_2}+1\right)}{\sqrt{\frac{2n_1n_2\left(2n_1n_2-n_1-n_2\right)}{\left(n_1+n_2\right)^2\left(n_1+n_2-1\right)}}}$$

10. Cox-Stuart Test:

X = Number of (+) signs [or (–) signs]

11. Kruskal-Wallis Test:

$$H = \frac{12}{n(n+1)}\sum n_j\left(\overline{R}_j - \overline{R}\right)^2$$

12. Friedman F_r -Test:

$$F_r = \frac{12b}{k(k+1)}\sum\left(\overline{R}_j - \overline{R}\right)^2$$

13. Spearman’s Rank Correlation Coefficient:

$$r_s = 1 - \frac{6\sum d^2}{n\left(n^2-1\right)}$$

14. Kendall’s Tau Test

$$\hat{\tau} = \frac{S}{n(n-1)/2}$$

15. Kolmogorov-Smirnov One-Sample Test:

$$D = \sup_x |\,S(x) - F_o(x)\,|$$

16. Kolmogorov-Smirnov Two-Sample Test:

$$D = \max\left|\,S_1(x) - S_2(x)\,\right|$$

LIST OF TABLES

TABLE A-7 Critical Values for the Sign Test

n	α: .005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)	.05 (one tail) .10 (two tails)
1	*	*	*	*
2	*	*	*	*
3	*	*	*	*
4	*	*	*	*
5	*	*	*	0
6	*	*	0	0
7	*	0	0	0
8	0	0	0	1
9	0	0	1	1
10	0	0	1	1
11	0	1	1	2
12	1	1	2	2
13	1	1	2	3
14	1	2	2	3
15	2	2	3	3
16	2	2	3	4
17	2	3	4	4
18	3	3	4	5
19	3	4	4	5
20	3	4	5	5
21	4	4	5	6
22	4	5	5	6
23	4	5	6	7
24	5	5	6	7
25	5	6	7	7

NOTES:

1. * indicates that it is not possible to get a value in the critical region.
2. Reject the null hypothesis if the number of the less frequent sign (x) is less than or equal to the value in the table.
3. For values of n greater than 25, a normal approximation is used with

$$z = \frac{(x + 0.5) - \left(\frac{n}{2}\right)}{\frac{\sqrt{n}}{2}}$$

Table A.3 *d*-factors for Wilcoxon signed-rank test and confidence intervals for the median (α' = one-sided significance level, α'' = two-sided significance level)

n	*d*	Confidence coefficient	α''	α'
3	1	.750	.250	.125
4	1	.875	.125	.063
5	1	.938	.062	.031
	2	.875	.125	.063
6	1	.969	.031	.016
	2	.937	.063	.031
	3	.906	.094	.047
	4	.844	.156	.078
7	1	.984	.016	.008
	2	.969	.031	.016
	4	.922	.078	.039
	5	.891	.109	.055
8	1	.992	.008	.004
	2	.984	.016	.008
	4	.961	.039	.020
	5	.945	.055	.027
	6	.922	.078	.039
	7	.891	.109	.055
9	2	.992	.008	.004
	3	.988	.012	.006
	6	.961	.039	.020
	7	.945	.055	.027
	9	.902	.098	.049
	10	.871	.129	.065
10	4	.990	.010	.005
	5	.986	.014	.007
	9	.951	.049	.024
	10	.936	.064	.032
	11	.916	.084	.042
	12	.895	.105	.053
11	6	.990	.010	.005
	7	.986	.014	.007
	11	.958	.042	.021
	12	.946	.054	.027
	14	.917	.083	.042
	15	.898	.102	.051
12	8	.991	.009	.005
	9	.988	.012	.006
	14	.958	.042	.021
	15	.948	.052	.026
	18	.908	.092	.046
	19	.890	.110	.055
13	10	.992	.008	.004
	11	.990	.010	.005
	18	.952	.048	.024
	19	.943	.057	.029
	22	.906	.094	.047
	23	.890	.110	.055
14	13	.991	.009	.004
	14	.989	.011	.005
	22	.951	.049	.025
	23	.942	.058	.029
	26	.909	.091	.045
	27	.896	.104	.052
15	16	.992	.008	.004
	17	.990	.010	.005
	26	.952	.048	.024
	27	.945	.055	.028
	31	.905	.095	.047
	32	.893	.107	.054
16	20	.991	.009	.005
	21	.989	.011	.006
	30	.956	.044	.022
	31	.949	.051	.025
	36	.907	.093	.047
	37	.895	.105	.052
17	24	.991	.009	.005
	25	.989	.011	.006
	35	.955	.045	.022
	36	.949	.051	.025
	42	.902	.098	.049
	43	.891	.109	.054
18	28	.991	.009	.005
	29	.990	.010	.005
	41	.952	.048	.024
	42	.946	.054	.027
	48	.901	.099	.049
	49	.892	.108	.054
19	33	.991	.009	.005
	34	.989	.011	.005
	47	.951	.049	.025
	48	.945	.055	.027
	54	.904	.096	.048
	55	.896	.104	.052
20	38	.991	.009	.005
	39	.989	.011	.005
	53	.952	.048	.024
	54	.947	.053	.027
	61	.903	.097	.049
	62	.895	.105	.053
21	43	.991	.009	.005
	44	.990	.010	.005
	59	.954	.046	.023
	60	.950	.050	.025
	68	.904	.096	.048
	69	.897	.103	.052
22	49	.991	.009	.005
	50	.990	.010	.005
	66	.954	.046	.023
	67	.950	.050	.025
	76	.902	.098	.049
	77	.895	.105	.053
23	55	.991	.009	.005
	56	.990	.010	.005
	74	.952	.048	.024
	75	.948	.052	.026
	84	.902	.098	.049
	85	.895	.105	.052
24	62	.990	.010	.005
	63	.989	.011	.005
	82	.951	.049	.025
	83	.947	.053	.026
	92	.905	.095	.048
	93	.899	.101	.051
25	69	.990	.010	.005
	70	.989	.011	.005
	90	.952	.048	.024
	91	.948	.052	.026
	101	.904	.096	.048
	102	.899	.101	.051

Source: F. Wilcoxon, S. Katti, and R. A. Wilcox, *Critical Values and Probability Levels for the Wilcoxon Rank Sum Test and the Wilcoxon Signed Rank Test*, Pearl River, N.Y.; American Cyanamid Co., 1949; used by permission of American Cyanamid Company

Note: For $n > 25$ use $d \approx \frac{1}{2}[\frac{1}{2}n(n+1)+1-z\sqrt{n(n+1)(2n+1)/6}]$, where z is read from Table A.2.

398 399

TABLE A-10 Critical Values for Number of Runs G

Value of n_1 \ Value of n_2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2
	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
3	1	1	1	1	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3
	6	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
4	1	1	1	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4
	6	8	9	9	9	10	10	10	10	10	10	10	10	10	10	10	10	10	10
5	1	1	2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5
	6	8	9	10	10	11	11	12	12	12	12	12	12	12	12	12	12	12	12
6	1	2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6
	6	8	9	10	11	12	12	13	13	13	13	14	14	14	14	14	14	14	14
7	1	2	2	3	3	3	4	4	5	5	5	5	5	6	6	6	6	6	6
	6	8	10	11	12	13	13	14	14	14	14	15	15	15	16	16	16	16	16
8	1	2	3	3	3	4	4	5	5	5	6	6	6	6	6	7	7	7	7
	6	8	10	11	12	13	14	14	15	15	16	16	16	16	17	17	17	17	17
9	1	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8
	6	8	10	12	13	14	14	15	16	16	16	17	17	18	18	18	18	18	18
10	1	2	3	3	4	5	5	5	6	6	7	7	7	7	8	8	8	8	9
	6	8	10	12	13	14	15	16	16	17	17	18	18	18	19	19	19	20	20
11	1	2	3	4	4	5	5	6	6	7	7	7	8	8	8	9	9	9	9
	6	8	10	12	13	14	15	16	17	17	18	19	19	19	20	20	20	21	21
12	2	2	3	4	4	5	6	6	7	7	7	8	8	8	9	9	9	10	10
	6	8	10	12	13	14	16	16	17	18	19	19	20	20	21	21	21	22	22
13	2	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	10
	6	8	10	12	14	15	16	17	18	19	19	20	20	21	21	22	22	23	23
14	2	2	3	4	5	5	6	7	7	8	8	9	9	9	10	10	10	11	11
	6	8	10	12	14	15	16	17	18	19	20	20	21	22	22	23	23	23	24
15	2	3	3	4	5	6	6	7	7	8	8	9	9	10	10	11	11	11	12
	6	8	10	12	14	15	16	18	18	19	20	21	22	22	23	23	24	24	25
16	2	3	4	4	5	6	6	7	8	8	9	9	10	10	11	11	11	12	12
	6	8	10	12	14	16	17	18	19	20	21	21	22	23	23	24	25	25	25
17	2	3	4	4	5	6	7	7	8	9	9	10	10	11	11	11	12	12	13
	6	8	10	12	14	16	17	18	19	20	21	22	23	23	24	25	25	26	26
18	2	3	4	5	5	6	7	8	8	9	9	10	10	11	11	12	12	13	13
	6	8	10	12	14	16	17	18	19	20	21	22	23	24	25	25	26	26	27
19	2	3	4	5	6	6	7	8	8	9	10	10	11	11	12	12	13	13	13
	6	8	10	12	14	16	17	18	20	21	22	23	23	24	25	26	26	27	27
20	2	3	4	5	6	6	7	8	9	9	10	10	11	12	12	13	13	13	14
	6	8	10	12	14	16	17	18	20	21	22	23	24	25	25	26	27	27	28

NOTE:

1. The entries in this table are the critical G values, assuming a two-tailed test with a significance level of $\alpha = 0.05$.
2. The null hypothesis of randomness is rejected if the total number of runs G is less than or equal to the smaller entry or greater than or equal to the larger entry.

From "Tables for testing randomness of groupings in a sequence of alternatives," *The Annals of Mathematical Statistics*, Vol. 14, No. 1. Reprinted with permission of the Institute of Mathematical Statistics.

Appendix 3

Kolmogorov–Smirnov Tables

Critical values, $d_{alpha};(n)^a$, of the maximum absolute difference between sample $F_n(x)$ and population $F(x)$ cumulative distribution.

Number of trials, n	Level of significance, α			
	0.10	0.05	0.02	0.01
1	0.95000	0.97500	0.99000	0.99500
2	0.77639	0.84189	0.90000	0.92929
3	0.63604	0.70760	0.78456	0.82900
4	0.56522	0.62394	0.68887	0.73424
5	0.50945	0.56328	0.62718	0.66853
6	0.46799	0.51926	0.57741	0.61661
7	0.43607	0.48342	0.53844	0.57581
8	0.40962	0.45427	0.50654	0.54179
9	0.38746	0.43001	0.47960	0.51332
10	0.36866	0.40925	0.45662	0.48893
11	0.35242	0.39122	0.43670	0.46770
12	0.33815	0.37543	0.41918	0.44905
13	0.32549	0.36143	0.40362	0.43247
14	0.31417	0.34890	0.38970	0.41762
15	0.30397	0.33760	0.37713	0.40420
16	0.29472	0.32733	0.36571	0.39201
17	0.28627	0.31796	0.35528	0.38086
18	0.27851	0.30936	0.34569	0.37062
19	0.27136	0.30143	0.33685	0.36117
20	0.26473	0.29408	0.32866	0.35241
21	0.25858	0.28724	0.32104	0.34427
22	0.25283	0.28087	0.31394	0.33666
23	0.24746	0.27490	0.30728	0.32954
24	0.24242	0.26931	0.30104	0.32286

Practical Reliability Engineering, Fifth Edition. Patrick D. T. O'Connor and Andre Kleyner.

Critical values, $d_{alpha};(n)^{a}$, of the maximum absolute difference between sample $F_n(x)$ and population $F(x)$ cumulative distribution.

Number of trials, n	Level of significance, α			
	0.10	0.05	0.02	0.01
25	0.23768	0.26404	0.29516	0.31657
26	0.23320	0.25907	0.28962	0.31064
27	0.22898	0.25438	0.28438	0.30502
28	0.22497	0.24993	0.27942	0.29971
29	0.22117	0.24571	0.27471	0.29466
30	0.21756	0.24170	0.27023	0.28987
31	0.21412	0.23788	0.26596	0.28530
32	0.21085	0.23424	0.26189	0.28094
33	0.20771	0.23076	0.25801	0.27677
34	0.20472	0.22743	0.25429	0.27279
35	0.20185	0.22425	0.26073	0.26897
36	0.19910	0.22119	0.24732	0.26532
37	0.19646	0.21826	0.24404	0.26180
38	0.19392	0.21544	0.24089	0.25843
39	0.19148	0.21273	0.23786	0.25518
40[b]	0.18913	0.21012	0.23494	0.25205

[a] Values of $d_\alpha(n)$ such that $p(\max)|F^n(x) - F(x)|d^\alpha(n) = \alpha$.

[b] $N > 40 \approx \frac{1.22}{N^{1/2}}, \frac{1.36}{N^{1/2}}, \frac{1.51}{N^{1/2}}$ and $\frac{1.63}{N^{1/2}}$ for the four levels of significance.

TABLE A19 Quantiles of the Smirnov Test Statistic for Two Samples of Equal Size n[a]

One-Sided Test: p =	0.90	0.95	0.975	0.99	0.995	One-Sided Test: p =	0.90	0.95	0.975	0.99	0.995
Two-Sided Test: p =	**0.80**	**0.90**	**0.95**	**0.98**	**0.99**	**Two-Sided Test: p =**	**0.80**	**0.90**	**0.95**	**0.98**	**0.99**
n = 3	2/3	2/3				n = 22	7/22	8/22	8/22	10/22	10/22
4	3/4	3/4	3/4			23	7/23	8/23	9/23	10/23	10/23
5	3/5	3/5	4/5	4/5	4/5	24	7/24	8/24	9/24	10/24	11/24
6	3/6	4/6	4/6	5/6	5/6	25	7/25	8/25	9/25	10/25	11/25
7	4/7	4/7	5/7	5/7	5/7	26	7/26	8/26	9/26	10/26	11/26
8	4/8	4/8	5/8	5/8	6/8	27	7/27	8/27	9/27	11/27	11/27
9	4/9	5/9	5/9	6/9	6/9	28	8/28	9/28	10/28	11/28	12/28
10	4/10	5/10	6/10	6/10	7/10	29	8/29	9/29	10/29	11/29	12/29
11	5/11	5/11	6/11	7/11	7/11	30	8/30	9/30	10/30	11/30	12/30
12	5/12	5/12	6/12	7/12	7/12	31	8/31	9/31	10/31	11/31	12/31
13	5/13	6/13	6/13	7/13	8/13	32	8/32	9/32	10/32	12/32	12/32
14	5/14	6/14	7/14	7/14	8/14	33	8/33	9/33	11/33	12/33	13/33
15	5/15	6/15	7/15	8/15	8/15	34	8/34	10/34	11/34	12/34	13/34
16	6/16	6/16	7/16	8/16	9/16	35	8/35	10/35	11/35	12/35	13/35
17	6/17	7/17	7/17	8/17	9/17	36	9/36	10/36	11/36	12/36	13/36
18	6/18	7/18	8/18	9/18	9/18	37	9/37	10/37	11/37	13/37	13/37
19	6/19	7/19	8/19	9/19	9/19	38	9/38	10/38	11/38	13/38	14/38
20	6/20	7/20	8/20	9/20	10/20	39	9/39	10/39	11/39	13/39	14/39
21	6/21	7/21	8/21	9/21	10/21	40	9/40	10/40	12/40	13/40	14/40
						Approximation for $n > 40$:	$\frac{1.52}{\sqrt{n}}$	$\frac{1.73}{\sqrt{n}}$	$\frac{1.92}{\sqrt{n}}$	$\frac{2.15}{\sqrt{n}}$	$\frac{2.30}{\sqrt{n}}$

SOURCE. Adapted from Birnbaum and Hall (1960), with permission from the Institute of Mathematical Statistics.

[a] The entries in this table are selected quantiles w_p of the Smirnov two-sample test statistic T defined by Equations 6.3.2 and 6.3.3 for the one-tailed test and defined by Equation 6.3.1 for the two-tailed test. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile of T as given in this table. The test statistic is a discrete random variable, so the exact level of significance may be less than the apparent α used in this table.

TABLE A20 Quantiles of the Smirnov Test Statistic for Two Samples of Different Size n and m[a]

One-Sided Test:		$p = 0.90$	0.95	0.975	0.99	0.995
Two-Sided Test:		$p = 0.80$	0.90	0.95	0.99	0.99
$N_1 = 1$	$N_2 = 9$	17/18				
	10	9/10				
$N_1 = 2$	$N_2 = 3$	5/6				
	4	3/4				
	5	4/5	4/5			
	6	5/6	5/6			
	7	5/7	6/7			
	8	3/4	7/8	7/8		
	9	7/9	8/9	8/9		
	10	7/10	4/5	9/10		
$N_1 = 3$	$N_2 = 4$	3/4	3/4			
	5	2/3	4/5	4/5		
	6	2/3	2/3	5/6		
	7	2/3	5/7	6/7	6/7	
	8	5/8	3/4	3/4	7/8	
	9	2/3	2/3	7/9	8/9	8/9
	10	3/5	7/10	4/5	9/10	9/10
	12	7/12	2/3	3/4	5/6	11/12
$N_1 = 4$	$N_2 = 5$	3/5	3/4	4/5	4/5	
	6	7/12	2/3	3/4	5/6	5/6
	7	17/28	5/7	3/4	6/7	6/7
	8	5/8	5/8	3/4	7/8	7/8
	9	5/9	2/3	3/4	7/9	8/9
	10	11/20	13/20	7/10	4/5	4/5
	12	7/12	2/3	2/3	3/4	5/6
	16	9/16	5/8	11/16	3/4	13/16
$N_1 = 5$	$N_2 = 6$	3/5	2/3	2/3	5/6	5/6
	7	4/7	23/35	5/7	29/35	6/7
	8	11/20	5/8	27/40	4/5	4/5
	9	5/9	3/5	31/45	7/9	4/5
	10	1/2	3/5	7/10	7/10	4/5
	15	8/15	3/5	2/3	11/15	11/15
	20	1/2	11/20	3/5	7/10	3/4
$N_1 = 6$	$N_2 = 7$	23/42	4/7	29/42	5/7	5/6
	8	1/2	7/12	2/3	3/4	3/4
	9	1/2	5/9	2/3	13/18	7/9
	10	1/2	17/30	19/30	7/10	11/15
	12	1/2	7/12	7/12	2/3	3/4
	18	4/9	5/9	11/18	2/3	13/18
	24	11/24	1/2	7/12	5/8	2/3

TABLE A20 (Continued)

One-Sided Test:		$p = 0.90$	0.95	0.975	0.99	0.995
Two-Sided Test:		$p = 0.80$	0.90	0.95	0.99	0.99
$N_1 = 7$	$N_2 = 8$	27/56	33/56	5/8	41/56	3/4
	9	31/63	5/9	40/63	5/7	47/63
	10	33/70	39/70	43/70	7/10	5/7
	14	3/7	1/2	4/7	9/14	5/7
	28	3/7	13/28	15/28	17/28	9/14
$N_1 = 8$	$N_2 = 9$	4/9	13/24	5/8	2/3	3/4
	10	19/40	21/40	23/40	27/40	7/10
	12	11/24	1/2	7/12	5/8	2/3
	16	7/16	1/2	9/16	5/8	5/8
	32	13/32	7/16	1/2	9/16	19/32
$N_1 = 9$	$N_2 = 10$	7/15	1/2	26/45	2/3	31/45
	12	4/9	1/2	5/9	11/18	2/3
	15	19/45	22/45	8/15	3/5	29/45
	18	7/18	4/9	1/2	5/9	11/18
	36	13/36	5/12	17/36	19/36	5/9
$N_1 = 10$	$N_2 = 15$	2/5	7/15	1/2	17/30	19/30
	20	2/5	9/20	1/2	11/20	3/5
	40	7/20	2/5	9/20	1/2	—
$N_1 = 12$	$N_2 = 15$	23/60	9/20	1/2	11/20	7/12
	16	3/8	7/16	23/48	13/24	7/12
	18	13/36	5/12	17/36	19/36	5/9
	20	11/30	5/12	7/15	31/60	17/30
$N_1 = 15$	$N_2 = 20$	7/20	2/5	13/30	29/60	31/60
$N_1 = 16$	$N_2 = 20$	27/80	31/80	17/40	19/40	41/80
Large sample approximation		$1.07\sqrt{\frac{m+n}{mn}}$	$1.22\sqrt{\frac{m+n}{mn}}$	$1.36\sqrt{\frac{m+n}{mn}}$	$1.52\sqrt{\frac{m+n}{mn}}$	$1.63\sqrt{\frac{m+n}{mn}}$

SOURCE. Adapted from Massey (1952), with permission from the Institute of Mathematical Statistics.

[a] The entries in this table are selected quantiles w_p of the Smirnov test statistic T for two samples, defined by Equations 6.3.1, 6.3.2, and 6.3.3. To enter the table let N_1 be the smaller sample size and let N_2 be the larger sample size. Reject H_0 at the level α if T exceeds $w_{1-\alpha}$ as given in this table. If n and m are not covered by this table, use the large sample approximation given at the end of the table, or consult exact tables by Kim and Jennrich, which appear in Harter and Owen (1970) for $n, m \leq 100$.

Table XI Critical Values for the Spearman Rho Rank Correlation Coefficient Test

	One-tailed α			
	.05	.025	.01	.005
	Two-tailed α			
n	.10	.05	.02	.01
5	±.900	—	—	—
6	±.829	±.886	±.943	—
7	±.714	±.786	±.893	±.929
8	±.643	±.738	±.833	±.881
9	±.600	±.700	±.783	±.833
10	±.564	±.648	±.745	±.794
11	±.536	±.618	±.709	±.755
12	±.503	±.587	±.678	±.727
13	±.475	±.566	±.672	±.744
14	±.456	±.544	±.645	±.714
15	±.440	±.524	±.622	±.688
16	±.425	±.506	±.601	±.665
17	±.411	±.490	±.582	±.644
18	±.399	±.475	±.564	±.625
19	±.388	±.462	±.548	±.607
20	±.377	±.450	±.534	±.591
21	±.368	±.438	±.520	±.576
22	±.359	±.428	±.508	±.562
23	±.351	±.418	±.496	±.549
24	±.343	±.409	±.485	±.537
25	±.336	±.400	±.475	±.526
26	±.329	±.392	±.465	±.515
27	±.323	±.384	±.456	±.505
28	±.317	±.377	±.448	±.496
29	±.311	±.370	±.440	±.487
30	±.305	±.364	±.432	±.478

Upper Critical Values for Kendall's Rank Correlation Coefficient $\hat{\tau}$

Note: In the table below, the critical values give significance levels as close as possible to but not exceeding the nominal α.

	Nominal α					
n	0.10	0.05	0.025	0.01	0.005	0.001
4	1.000	1.000	-	-	-	-
5	0.800	0.800	1.000	1.000	-	-
6	0.600	0.733	0.867	0.867	1.000	-
7	0.524	0.619	0.714	0.810	0.905	1.000
8	0.429	0.571	0.643	0.714	0.786	0.857
9	0.389	0.500	0.556	0.667	0.722	0.833
10	0.378	0.467	0.511	0.600	0.644	0.778
11	0.345	0.418	0.491	0.564	0.600	0.709
12	0.303	0.394	0.455	0.545	0.576	0.667
13	0.308	0.359	0.436	0.513	0.564	0.641
14	0.275	0.363	0.407	0.473	0.516	0.604
15	0.276	0.333	0.390	0.467	0.505	0.581
16	0.250	0.317	0.383	0.433	0.483	0.567
17	0.250	0.309	0.368	0.426	0.471	0.544
18	0.242	0.294	0.346	0.412	0.451	0.529
19	0.228	0.287	0.333	0.392	0.439	0.509
20	0.221	0.274	0.326	0.379	0.421	0.495
21	0.210	0.267	0.314	0.371	0.410	0.486
22	0.203	0.264	0.307	0.359	0.394	0.472
23	0.202	0.257	0.296	0.352	0.391	0.455
24	0.196	0.246	0.290	0.341	0.377	0.449
25	0.193	0.240	0.287	0.333	0.367	0.440
26	0.188	0.237	0.280	0.329	0.360	0.428
27	0.179	0.231	0.271	0.322	0.356	0.419
28	0.180	0.228	0.265	0.312	0.344	0.413
29	0.172	0.222	0.261	0.310	0.340	0.404

	Nominal α					
n	0.10	0.05	0.025	0.01	0.005	0.001
30	0.172	0.218	0.255	0.301	0.333	0.393
31	0.166	0.213	0.252	0.295	0.325	0.389
32	0.165	0.210	0.246	0.290	0.323	0.379
33	0.163	0.205	0.242	0.288	0.314	0.375
34	0.159	0.201	0.237	0.280	0.312	0.369
35	0.156	0.197	0.234	0.277	0.304	0.361
36	0.152	0.194	0.232	0.273	0.302	0.359
37	0.150	0.192	0.228	0.267	0.297	0.351
38	0.149	0.189	0.223	0.263	0.292	0.346
39	0.147	0.188	0.220	0.260	0.287	0.341
40	0.144	0.185	0.218	0.256	0.285	0.338
41	0.141	0.180	0.215	0.254	0.280	0.334
42	0.141	0.178	0.213	0.250	0.275	0.329
43	0.138	0.176	0.209	0.247	0.274	0.324
44	0.137	0.173	0.207	0.243	0.268	0.321
45	0.135	0.172	0.204	0.240	0.267	0.317
46	0.132	0.169	0.202	0.239	0.264	0.314
47	0.132	0.167	0.199	0.236	0.260	0.310
48	0.129	0.167	0.197	0.232	0.257	0.307
49	0.129	0.163	0.196	0.230	0.253	0.303
50	0.127	0.162	0.192	0.228	0.251	0.300
51	0.126	0.161	0.191	0.225	0.249	0.297
52	0.124	0.158	0.189	0.223	0.246	0.294
53	0.123	0.157	0.187	0.221	0.244	0.290
54	0.122	0.156	0.185	0.219	0.241	0.287
55	0.121	0.154	0.182	0.216	0.239	0.285
56	0.119	0.152	0.181	0.214	0.236	0.282
57	0.118	0.152	0.179	0.212	0.234	0.279
58	0.117	0.149	0.177	0.210	0.232	0.276
59	0.116	0.148	0.176	0.209	0.230	0.274
60	0.115	0.147	0.174	0.207	0.228	0.272

-ooo000ooo-