First Semester Examination
2017/2018 Academic Session

January 2018

MAT223 - Differential Equations I
[Persamaan Pembezaan I]

Duration : 3 hours
(Masa : 3 jam)

Please check that this examination paper consists of **SEVEN (7)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH (7)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini].*

Instructions : Answer **all five (5)** questions.

*[**Arahan :** Jawab **semua lima (5)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

Question 1

(a) Verify that each given function is a solution to the differential equations:

(i) $y'' = 16y; \quad y_1 = e^{-4x}, \; y_2 = e^{4x}.$

(ii) $y'' + 4y' + 4y = 0; \quad y_1 = e^{-2x}, \; y_2 = xe^{-2x}.$

(b) A first order differential equation is of the form

$$\frac{dy}{dx} = f(x, y),$$

where f is a function of two variables. If f depends linearly on the dependent variable y, then this differential equation is linear. Give a proper definition of a linear differential equation in standard form.

(c) Find general solution of the following differential equation.

$$\frac{dy}{dx} = y^2 \sin x - y^2 \cos x, \quad y\left(\frac{\pi}{2}\right) = \frac{1}{3}.$$

[100 marks]

Soalan 1

(a) Tentusahkan bahawa setiap fungsi yang diberi adalah penyelesaian kepada persamaan pembezaan berikut:

(i) $y'' = 16y; \quad y_1 = e^{-4x}, \; y_2 = e^{4x}.$

(ii) $y'' + 4y' + 4y = 0; \quad y_1 = e^{-2x}, \; y_2 = xe^{-2x}.$

(b) Persamaan pembezaan peringkat pertama adalah dalam bentuk

$$\frac{dy}{dx} = f(x, y),$$

di mana f adalah fungsi dua pembolehubah. Jika f bersandar secara linear terhadap pembolehubah bersandar y, maka persamaan pembezaan ini adalah linear. Berikan definisi yang sesuai bagi persamaan pembezaan linear dalam bentuk umum.

(c) Cari penyelesaian am kepada persamaan pembezaan berikut.

$$\frac{dy}{dx} = y^2 \sin x - y^2 \cos x, \quad y\left(\frac{\pi}{2}\right) = \frac{1}{3}.$$

[100 markah]

Question 2

(a) By applying the Wronskian test, show that the functions $f(x) = x$ and $g(x) = e^{2x}$ are linearly independent at $x_0 = 0$.

(b) Find the solution of the initial value problem

$$y'' - y' + \frac{y}{4} = 0, \quad y(0) = 2, \quad y'(0) = \frac{1}{3}.$$

(c) Find a particular solution of the non-homogeneous differential equation

$$y'' - 3y' - 4y = 3e^{3x} - 8e^{x}\cos(2x).$$

[100 marks]

Soalan 2

(a) Dengan menggunakan ujian Wronskian, tunjukkan bahawa fungsi-fungsi $f(x) = x$ dan $g(x) = e^{2x}$ tak bersandar linear pada $x_0 = 0$.

(b) Cari penyelesaian bagi masalah nilai awal

$$y'' - y' + \frac{y}{4} = 0, \quad y(0) = 2, \quad y'(0) = \frac{1}{3}.$$

(c) Cari suatu penyelesaian khusus bagi persamaan pembezaan tak homogen

$$y'' - 3y' - 4y = 3e^{3x} - 8e^{x}\, cos(2x).$$

[100 markah]

Question 3

(a) The logistic equation for population growth can be written as

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{C}\right),$$

where $P(t)$ is the population at time t and C is the carrying capacity of the environment. Solve the given logistic equation.

(b) During the period from 1850 to 1900, the population of a country grew from 3.9 million to 5.3 million. Based on the given information, answer the following questions:

(i) Predict its population in 1950 and in 2000 using the exponential model of population growth.

(ii) Then considering that the population of the country in 1950 was actually 6.9 million people, correct your prediction for 2000 using the logistic model of population growth
(Hint: use $k = 0.00596$ in the model.)

(iii) What is the carrying capacity C of the country according to this logistic model?

[100 marks]

Soalan 3

(a) *Persamaan logistik untuk pertumbuhan populasi boleh ditulis sebagai*

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{C}\right),$$

di mana P (t) adalah populasi penduduk pada masa t dan C adalah daya tampung persekitaran. Selesaikan persamaan logistik yang diberi.

(b) *Dalam tempoh 1850 hingga 1900, populasi bagi sebuah negara bertambah daripada 3.9 juta kepada 5.3 juta orang. Berdasarkan maklumat yang diberi, jawab soalan-soalan berikut:*

(i) *Ramalkan populasi bagi tahun 1950 dan 2000 dengan menggunakan model eksponen bagi pertumbuhan populasi.*

(ii) *Seterusnya, dengan mempertimbangkan populasi negara tersebut pada 1950 adalah 6.9 juta orang, perbaiki ramalan anda bagi tahun 2000 menggunakan model logistik bagi pertumbuhan populasi.*

(Petunjuk: guna k=0.00596 dalam model tersebut.)

(iii) *Apakah daya tampung C bagi negara tersebut berdasarkan model logistik ini?*

[100 markah]

Question 4

(a) Consider the first order initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(0) = y_0,$$

where $x_n = n\Delta x$ and $y_n = y(x_n)$. Derive the Euler's method by applying the Taylor series expansion formula.

(b) Use Euler's method with $h = 0.1$ to approximate the solution of the initial value problem

$$y' = x + y, \quad y(0) = 1.$$

(c) Compare the error of your solution in (b) with the exact solution.

[100 marks]

Soalan 4

(a) *Pertimbangkan masalah nilai awal peringkat pertama*

$$\frac{dy}{dx} = f(x, y), \quad y(0) = y_0,$$

di mana $x_n = n\Delta x$ *dan* $y_n = y(x_n)$. *Terbitkan kaedah Euler menggunakan rumus pengembangan siri Taylor.*

(b) *Gunakan kaedah Euler dengan* $h = 0.1$ *bagi menganggarkan penyelesaian bagi masalah nilai awal*

$$y' = x + y, \quad y(0) = 1.$$

(c) *Bandingkan ralat bagi penyelesaian di (b) dengan penyelesaian tepat.*

[100 markah]

Question 5

(a) Use power series method to solve the differential equation

$$x^2 y' = y - x - 1.$$

Determine the radius of convergence of the resulting series and explain about it.

(b) A mass-and-spring system with external force $f(x)$ as in Figure 1 satisfies the following initial value problem

$$y'' + 4y = \sin(3x), \quad y(0) = 0, \quad y'(0) = 0.$$

Solve the problem by using Laplace transforms method.

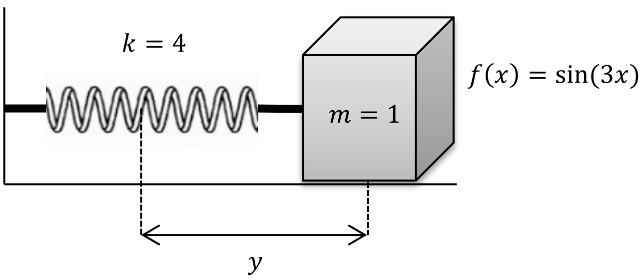


FIGURE 1: A mass-and-spring system

[100 marks]

Soalan 5

(a) *Gunakan kaedah siri kuasa untuk menyelesaikan persamaan pembezaan*

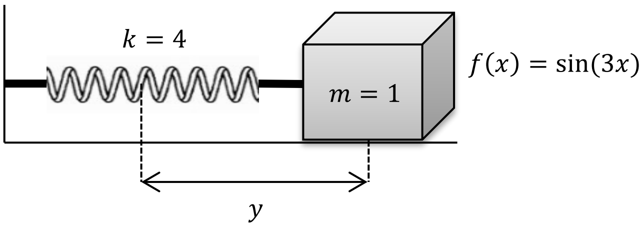
$$x^2y' = y - x - 1.$$

Tentukan jejari ketumpuan bagi siri yang diperoleh dan beri keterangan tentangnya.

(b) *Suatu sistem jisim-dan-spring dengan daya luar $f(x)$ seperti dalam Gambarajah 1 menepati masalah nilai awal*

$$y'' + 4y = \sin(3x), \quad y(0) = 0, \quad y'(0) = 0.$$

Selesaikan masalah tersebut menggunakan kaedah jelmaan Laplace.



GAMBARAJAH 1: Sistem jisim-dan-spring

[100 markah]

Table of Laplace Transform.

$f(x), \quad x \geq 0$	$F(s)$
$\delta(x)$	1
$u(x)$	$\frac{1}{s}$
x	$\frac{1}{s^2}$
x^n	$\frac{n!}{s^{n+1}}$
e^{-ax}	$\frac{1}{s+a}$
$\sin bx$	$\frac{b}{s^2+b^2}$
$\cos bx$	$\frac{s}{s^2+b^2}$
$e^{-ax}\sin \omega x$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-ax}\cos \omega x$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{df(x)}{dx}$	$sF(s)-f(0)$
$\int_0^x f(x)dx$	$\frac{1}{s}F(s)$
$f(x-a)$	$e^{-as}F(s)$

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