

SULIT



First Semester Examination
2017/2018 Academic Session

January 2018

MAT201 - Advanced Calculus
[Kalkulus Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **FIVE (5)** pages of printed materials before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LIMA (5)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all ten** (10) questions.

[Arahan: Jawab **semua sepuluh** (10) soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

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Question 1

Show that $\lim_{x \rightarrow 0} (x + e^x)^{1/x} = e^2$.

[6 marks]

Soalan 1

Tunjukkan bahawa $\lim_{x \rightarrow 0} (x + e^x)^{1/x} = e^2$.

[6 markah]

Question 2

Determine whether each of the following sequence converges or diverges:

$$(a) \quad \left\{ \frac{n + (-1)^n}{n} \right\} \qquad (b) \quad \left\{ \sqrt{n^4 + 2n^2} - n^2 \right\}$$

[8 marks]

Soalan 2

Tentukan sama ada setiap jujukan berikut menumpu atau mencapah:

$$(a) \quad \left\{ \frac{n + (-1)^n}{n} \right\} \qquad (b) \quad \left\{ \sqrt{n^4 + 2n^2} - n^2 \right\}$$

[8 markah]

Question 3

Test the convergence of the following series:

$$(a) \quad \sum_{n=1}^{\infty} \frac{n + 2^n}{n^2 2^n} \qquad (b) \quad \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$$

[10 marks]

Soalan 3

Uji penumpuan siri berikut:

$$(a) \quad \sum_{n=1}^{\infty} \frac{n + 2^n}{n^2 2^n} \qquad (b) \quad \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$$

[10 markah]

Question 4

Find all values of x for which $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} (3x-1)^n}{n^2}$ converges absolutely.

[10 marks]

Soalan 4

Dapatkan semua nilai x supaya $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} (3x-1)^n}{n^2}$ menumpu secara mutlak.

[10 markah]

Question 5

Estimate the maximum error in using Taylor polynomial $T_3(x)$ about $a = 4$ to approximate $f(x) = x^{5/2}$ when $2 \leq x \leq 6$.

[7 marks]

Soalan 5

Anggarkan ralat maksimum apabila menggunakan polinomial Taylor $T_3(x)$ sekitar $a = 4$ untuk menganggar $f(x) = x^{5/2}$ apabila $2 \leq x \leq 6$.

[7 markah]

Question 6

Determine whether $\int_{\pi}^{\infty} \frac{1 + \sin x}{x^2} dx$ converges or diverges.

[5 marks]

Soalan 6

Tentukan sama ada $\int_{\pi}^{\infty} \frac{1 + \sin x}{x^2} dx$ menumpu atau mencapah.

[5 markah]

Question 7

(a) Determine the continuity of f at $(0,0)$:

$$f(x, y) = \begin{cases} \frac{2x^2y}{x^4 + y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0). \end{cases}$$

- (b) If f is differentiable, $w = f\left(ts^2, \frac{s}{t}\right)$, $\frac{\partial f}{\partial x}(x, y) = xy$, and $\frac{\partial f}{\partial y}(x, y) = \frac{x^2}{2}$, find $\frac{\partial w}{\partial t}$.

[13 marks]

Soalan 7

- (a) Tentukan keselajaran bagi f pada $(0,0)$:

$$f(x, y) = \begin{cases} \frac{2x^2y}{x^4 + y^2} & , (x, y) \neq (0,0) \\ 0 & , (x, y) = (0,0). \end{cases}$$

- (b) Jika f terbezakan, $w = f\left(ts^2, \frac{s}{t}\right)$, $\frac{\partial f}{\partial x}(x, y) = xy$, dan $\frac{\partial f}{\partial y}(x, y) = \frac{x^2}{2}$, dapatkan $\frac{\partial w}{\partial t}$.

[13 markah]

Question 8

Find

- (a) the direction at which the derivative of $f(x, y, z) = 2xy + yz$ at the point $P(1, -1, 1)$ is -4 .
- (b) $\frac{dy}{dx}$ at $(0, \ln 2)$ if $xe^y + \sin(xy) + y - \ln 2 = 0$.

[13 marks]

Soalan 8

Dapatkan

- (a) arah supaya terbitan bagi $f(x, y, z) = 2xy + yz$ pada titik $P(1, -1, 1)$ ialah -4 .
- (b) $\frac{dy}{dx}$ pada $(0, \ln 2)$ jika $xe^y + \sin(xy) + y - \ln 2 = 0$.

[13 markah]

Question 9

Find the extreme values of $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ on the disk $x^2 + y^2 \leq 16$.

[12 marks]

Soalan 9

Dapatkan nilai-nilai extrema bagi $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ pada cakera $x^2 + y^2 \leq 16$.

[12 markah]

Question 10

(a) Evaluate $\iint_D y \, dA$, where D is the region in the first quadrant bounded by the curves $xy = 1$, $y = x$, and $y = 2$.

(b) Find the volume of the solid enclosed by the cylinder $x^2 + y^2 = 9$, bounded above by the sphere $x^2 + y^2 + z^2 = 16$, and bounded below by the xy -plane.

[16 marks]

Soalan 10

(a) Nilaikan $\iint_D y \, dA$, yang mana D merupakan rantau di sukuan pertama yang dibatasi oleh lengkung-lengkung $xy = 1$, $y = x$, and $y = 2$.

(b) Dapatkan isipadu pepejal yang dibatasi oleh silinder $x^2 + y^2 = 9$, bahagian atasnya dibatasi oleh sfera $x^2 + y^2 + z^2 = 16$, dan bahagian bawahnya dibatasi oleh satah xy .

[16 markah]

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