First Semester Examination
2017/2018 Academic Session

January 2018

MAT161 - ELEMENTARY STATISTICS
[STATISTIK PERMULAAN]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **NINE (9)** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEMBILAN (9)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all ten (10)** questions.

*[**Arahan**: Jawab **semua sepuluh (10)** soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

Question 1

The following table shows the daily allowance for a sample of 60 employees in Company A.

Daily allowance (RM)	No. of employees
50-59	6
60-69	10
70-79	16
80-89	14
90-99	10
100-109	4

(a) Calculate the mean and standard deviation of the distribution of the daily allowance.

(b) Estimate the percentage of workers who get an allowance of less than RM90 a day.

(c) Use Chebyshev's Theorem to obtain an interval of the daily allowance for at least 75% of the employess of the company.

[10 marks]

Soalan 1

Jadual berikut menunjukkan elaun harian bagi sampel 60 orang pekerja di Syarikat A.

Elaun harian (RM)	*Bilangan pekerja*
50-59	6
60-69	10
70-79	16
80-89	14
90-99	10
100-109	4

(a) Hitungkan min dan sisihan piawai bagi taburan elaun harian.

(b) Anggarkan peratusan pekerja yang mendapat elaun kurang daripada RM90 sehari.

(c) Gunakan Teorem Chebyshev untuk mendapatkan suatu selang elaun harian bagi sekurang-kurangnya 75% daripada pekerja-pekerja syarikat tersebut.

[10 markah]

Question 2

(a) The probability that Henry will like a new movie is 0.70 and the probability that Jean, his girlfriend, will like it is 0.60. If the probability is 0.28 that he will like it and she will dislike it, what is the probability that he will like it given that she is not going to like it?

(b) If A and B are two nonempty and independent events, show that $\overline{A}$ and $\overline{B}$ are also independent.

[10 marks]

Soalan 2

(a) *Kebarangkalian Henry menyukai wayang gambar baru adalah 0.70 dan kebarangkalian teman wanitanya, Jean menyukai wayang gambar tersebut adalah 0.60. Sekiranya kebarangkalian Henry suka dan teman wanitanya tidak suka wayang gambar tersebut adalah 0.28, apakah kebarangkalian bahawa Henry suka diberikan Jean tidak menyukai wayang gambar itu?*

(b) *Sekiranya A dan B adalah dua peristiwa bukan sifar dan tidak bersandar, tunjukkan* $\overline{A}$ *dan* $\overline{B}$ *juga adalah tidak bersandar.*

[10 markah]

Question 3

(a) Complete the following probability distribution if $P(6)$ equals two-thirds of $P(4)$. Give the values of x and y. Then, find μ, σ^2 and σ for the distribution.

X	1	2	4	6	9
$P(X)$	0.23	0.18	x	y	0.015

(b) The quality assurance engineer of a television (TV) manufacturer inspects TVs in lots of 100. He selects 5 of the 100 TVs at random and inspects them thoroughly. Assuming that 6 of the 100 TVs in the current lot are defective, find the probability that exactly 2 of the 5 TVs selected by the engineer are defective.

[10 marks]

Soalan 3

(a) *Lengkapkan taburan kebarangkalian berikut sekiranya P(6) bersamaan dua pertiga P(4). Nyatakan nilai x dan y. Seterusnya, dapatkan* μ, σ^2 *dan* σ *bagi taburan tersebut.*

X	1	2	4	6	9
$P(X)$	0.23	0.18	x	y	0.015

(b) *Seorang jurutera kualiti dalam pembuatan televisyen (TV) telah memeriksa TV dalam lot sebanyak 100. Beliau memilih 5 daripada 100 TV secara rawak dan memeriksanya secara terperinci. Andaikan 6 daripada 100 TV dalam lot adalah rosak, dapatkan kebarangkalian tepat 2 daripada 5 TV yang dipilih oleh jurutera tersebut adalah rosak.*

[10 markah]

Question 4

In a reality program show, it has been studied that Mawi's winning votes are according to a Poisson distribution with 5 votes per minute. Meanwhile, Siti's winning votes are following the Poisson distribution with 4.5 votes per minute.

(a) Calculate the probability that Mawi wins 5 votes or more within three minutes.

(b) Within 1 hour of the program, calculate the probability that Siti wins more than 300 votes.[Hint: use normal approximation to solve the question]

(c) After two hours running of the reality program, using the normal approximation and assuming that Mawi's votes and Siti's votes are mutually independent, find the probability that total Siti's votes is higher than Mawi's.

[10 marks]

Soalan 4

Dalam suatu program realiti, kajian menunjukkan bahawa Mawi memenangi undian berdasarkan taburan Poisson dengan 5 undian per minit. Manakala Siti meraih undian berdasarkan taburan Poisson dengan 4.5 undian per minit.

(a) Kira kebarangkalian bahawa Mawi meraih 5 atau lebih undian dalam tempoh tiga minit.

(b) Dalam tempoh 1 jam, kira kebarangkalian Siti meraih undian lebih daripada 300 undian.[Petunjuk: gunakan penghampiran normal untuk menyelesaikan soalan ini]

(c) Selepas dua jam program tersebut berlangsung, dengan menggunakan penghampiran normal serta andaian bahawa undian Mawi dan Siti adalah tidak bersandar, cari kebarangkalian keseluruhan undian Siti adalah lebih tinggi daripada keseluruhan undian Mawi.

[10 markah]

Question 5

The continuous random variable X has the probability density function (pdf) given by:

$$f(x)=\begin{cases} k-5+x & 5<x<6 \\ 0 & \textit{otherwise} \end{cases}$$

where k is a constant. Determine the median.

[10 marks]

Soalan 5

Pemboleh ubah rawak selanjar X mempunyai fungsi ketumpatan kebarangkalian (fkk) yang diberikan sebagai:

$$f(x)=\begin{cases}k-5+x & 5<x<6\\ 0 & \textit{otherwise}\end{cases}$$

yang mana k adalah pemalar. Tentukan nilai median.

[10 markah]

Question 6

A university is subdivided into faculties of Arts, Engineering, Humanities, Law and Science. The numbers of students in these faculties are 1300, 800, 1100, 500 and 1400 respectively. A questionnaire concerning library usage is sent to all the students. The first 300 replies opened contained 101 from Arts students, 30 from Engineering students and 69, 17 and 83 from students of the other three faculties respectively. Using a 0.1% significance level, test whether the replies appear to be providing an unbiased representation of the students in the university.

[10 marks]

Soalan 6

Sebuah universiti dibahagikan kepada fakulti Sastera, Kejuruteraan, Kemanusiaan, Undang-undang dan Sains. Bilangan pelajar bagi fakulti-fakulti tersebut adalah 1300, 800, 1100, 500 dan 1400 masing-masing. Soal selidik berkaitan penggunaan perpustakaan diedarkan kepada semua pelajar. 300 jawapan pertama yang diterima adalah daripada 101 pelajar Sastera, 30 pelajar Kejuruteraan, dan masing-masing 69, 17 dan 83 pelajar dari fakulti-fakulti lain. Menggunakan aras keertian 0.1%, uji sama ada jawapan yang diberikan adalah perwakilan saksama dengan bidang pelajar di universiti.

[10 markah]

Question 7

Ten students independently performed an experiment to estimate the value on π . Their results were:

3.12, 3.16, 2.94, 3.33, 3.00, 3.11, 3.50, 2.81, 3.02, 3.10

(a) Given that $\sum x^2 = 97.0011$, calculate the sample mean, $\bar{x}$ and sample variance, s^2.

(b) Stating any necessary assumption that you make, calculate a 95% confidence interval for π based on these data.

(c) Estimate the minimum number of results that would be needed if it is required that the width of the resulting 95% confidence interval should be at most 0.02.

[10 marks]

Soalan 7

Sepuluh pelajar menjalankan eksperimen secara tidak bersandar bagi menganggar nilai π*. Keputusan mereka adalah:*

3.12, 3.16, 2.94, 3.33, 3.00, 3.11, 3.50, 2.81, 3.02, 3.10

(a) *Diberikan bahawa* $\sum x^2 = 97.0011$*,hitung min sampel,* $\bar{x}$ *dan varians sampel,* s^2*.*

(b) *Nyatakan andaian bersesuaian yang anda lakukan, bina selang keyakinan 95% bagi* π *berdasarkan data tersebut.*

(c) *Anggarkan bilangan keputusan minimum yang diperlukan sekiranya lebar selang keyakinan 95% adalah paling banyak 0.02.*

[10 markah]

Question 8

In a particular river, a certain micro-organism occurs at an average rate of 10 per mililitre. A random sample of 0.5 litres of water is taken from a nearby stream and is found to contain 3478 micro-organisms. Does this provide significant evidence, at the 5% level, of a difference in the incidence of the micro-organisms between the stream and the river?

[10 marks]

Soalan 8

Dalam sebatang sungai, kewujudan sejenis mikroorganisma adalah pada kadar purata 10 per milliliter. Satu sampel rawak sebanyak 0.5 liter air daripada anak sungai berdekatan didapati mengandungi 3478 mikroorganisma. Adakah ini menunjukkan bukti yang bererti, pada aras keertian 5%, terdapat perbezaan dalam insiden kewujudan mikroorganisma diantara anak sungai dengan sungai?

[10 markah]

Question 9

The marks obtained in a Statistics paper by a random sample of 200 male students have $\bar{x} = 54.6$ and $s^2 = 101.3$. On the same paper, a random sample of 150 female students had a mean mark of 57.1, with $s^2 = 92.4$. Assuming a common population variance, obtained a pooled estimate of this variance, and test at 1% significance level, whether there is significant evidence of a difference in the two population means.

[10 marks]

Soalan 9

Markah yang diperoleh bagi kertas Statistik dari sampel rawak seramai 200 orang pelajar lelaki mempunyai $\bar{x} = 54.6$ *dan* $s^2 = 101.3$*. Bagi kertas yang sama, sampel rawak 150 pelajar wanita mempunyai min markah 57.1 dengan* $s^2 = 92.4$*. Diandaikan bahawa varians populasi adalah sama, dapatkan anggaran tergembeleng bagi varians, dan uji pada aras keertian 1% sama ada terdapat bukti yang bererti dalam perbezaan min kedua-dua populasi.*

[10 markah]

Question 10

In a large department store, the owner wishes to see whether the number of shoplifting incidents per day will change if the number of uniformed security officers is doubled. A random sample of 7 days before security is increased and 7 days after the increase shows the number of shoplifting incidents as follow:

Day	No. of shoplifting incidents	
	Before	After
Monday	7	5
Tuesday	2	3
Wednesday	3	4
Thursday	6	3
Friday	5	1
Saturday	8	6
Sunday	12	4

Is there enough evidence to support the claim, at $\alpha = 0.05$, that there is a difference in the number of shoplifting incidents before and after the increase in security?

[10 marks]

Soalan 10

Dalam sebuah pasaraya besar, pemiliknya ingin melihat sama ada bilangan kes kecurian dalam sehari akan berubah sekiranya bilangan pengawal keselamatan berpakaian seragam digandakan. Satu sampel rawak selama 7 hari sebelum dan selepas anggota keselamatan ditambah menunjukkan bilangan insiden kecurian seperti berikut:

Hari	*Bil insiden kecurian*	
	Sebelum	*Selepas*
Isnin	*7*	*5*
Selasa	*2*	*3*
Rabu	*3*	*4*
Khamis	*6*	*3*
Jumaat	*5*	*1*
Sabtu	*8*	*6*
Ahad	*12*	*4*

Adakah terdapat bukti yang mencukupi pada aras keertian $\alpha = 0.05$, bagi menyokong bahawa terdapat perbezaan dalam bilangan insiden kecurian sebelum dan selepas penambahan anggota keselamatan?

[10 markah]

FORMULA

$$\bar{x} = \frac{\sum xf}{\sum f}$$

$$s^2 = \frac{\sum (x^2 f) - \frac{(\sum xf)^2}{\sum f}}{\sum f - 1}$$

$$S_p{}^2 = \frac{(n_x - 1)s_x{}^2 + (n_y - 1)s_y{}^2}{n_x + n_y - 2}$$

$$\hat{p} = \frac{X + Y}{n_x + n_y}$$

$$m = b + \frac{c\left(\frac{n}{2} - l\right)}{f_m}$$

Confidence Intervals:

$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$\overline{X} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$$

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \quad to \quad \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

$$(\overline{X} - \overline{Y}) \pm z_{\alpha/2}\sqrt{\frac{\sigma_x{}^2}{n_x} + \frac{\sigma_y{}^2}{n_y}}$$

$$\left(\overline{X} - \overline{Y}\right) \pm z_{\alpha/2}\sqrt{\frac{s_x^2}{n_x} - \frac{s_y^2}{n_y}}$$

$$(\overline{X} - \overline{Y}) \pm t_{\alpha/2}\sqrt{S_p{}^2\left(\frac{1}{n_x} + \frac{1}{n_y}\right)}$$

$$\left(\hat{p}_x - \hat{p}_y\right) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_x(1-\hat{p}_x)}{n_x} + \frac{\hat{p}_y(1-\hat{p}_y)}{n_y}}$$

Test Statistics:

$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$T = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$$

$$T = \frac{\overline{d} - \mu_d}{\frac{s_d}{\sqrt{n_d}}}$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$$Z = \frac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x{}^2}{n_x} + \frac{\sigma_y{}^2}{n_y}}}$$

$$Z = \frac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{s_x{}^2}{n_x} + \frac{s_y{}^2}{n_y}}}$$

$$T = \frac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p{}^2\left(\frac{1}{n_x} + \frac{1}{n_y}\right)}}$$

$$Z = \frac{(\hat{p}_x - \hat{p}_y) - (p_x - p_y)}{\sqrt{\frac{p_x(1-p_x)}{n_x} + \frac{p_y(1-p_y)}{n_y}}}$$

$$Z = \frac{(\hat{p}_x - \hat{p}_y) - (p_x - p_y)}{\sqrt{\overline{p}(1-\overline{p})\left(\frac{1}{n_x} + \frac{1}{n_y}\right)}}$$

$$F = \frac{s_1^2}{s_2^2}$$

$$\chi^2 = \sum \frac{(O-E)^2}{E}, \quad E = np$$

Nonparametric Statistics:

Wilcoxon Signed-rank: $W = \sum R^+$, $W = \sum R^-$

$$Z = \frac{T - \mu_W}{\sigma_W}, \quad \mu_W = \frac{n(n+1)}{4}, \quad \sigma_W = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

Wilcoxon Rank Sum Test: $U = R - \frac{n(n+1)}{2}$

$$Z = \frac{T - \mu_T}{\sigma_T}, \quad \mu_T = \frac{n_1(n_1 + n_2 + 1)}{2}, \quad \sigma_T = \sqrt{\frac{n_1 n_2 (n+1)(n_1 + n_2 + 1)}{12}}$$

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