AUTOMATIC INTERPRETATION OF MAGNETIC DATA USING EULER DECONVOLUTION WITH MODIFIED ALGORITHM

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AUTOMATIC INTERPRETATION OF MAGNETIC DATA USING EULER DECONVOLUTION WITH MODIFIED ALGORITHM

by

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A	Angle between positive x-axis and magnetic north (0° to 90°) of dike model
a	order of differentiation
B	Background field/ base level of the total magnetic field
c	Magnetic constant
∂	Partial differentiation
δ:	$90 - I$
d	dip
d_1	Depth to top of the structure of contact model
d_2	Depth to the bottom of the structure of contact model
E	True model parameter
$\hat{E}$	Estimated model parameter
f	field
F	Force
$f(x)$	Observed field at x
$f(p)$	General function
H	magnetizing field
h	depth of the element (box model) below the level of observation
I	inclination of the geomagnetic field
i	Positive and negative in the Northern and Southern Hemisphere respectively
I_p	Polarization
I'_o	effective inclination of cylinder model
I_o	real inclination of cylinder model

k	Magnetic susceptibility
K	Dipole moment
L_1	direction cosine characterizing polarization vector of the volume element (box model)
L_2	direction sine characterizing polarization vector of the volume element (box model)
l	$sin\,\delta$
M	Magnetization
m	magnetic induction
N	Structural index
$\boldsymbol{n}$	direction cosine vector of the earth’s field (box model)
n	Degree of homogeneity
O’	Origin
p	$I_p \int F(\beta_1)\, d\beta_1$
Φ	magnetic flux
R	Adjacent distance from the observation point to the depth to the top of the dike model
$R_{o'p}$	vector originate at O’ towards P
r	Radius between the poles
r_o^2	$p^2 + \alpha_1^2$
$\bar{r}$	Vector directed from the source point to observation point
S	the cross-sectional area of cylinder model
S^{-N}	operator of the differential similarity transform
T_o':	effective total intensity of cylinder model
T	Tesla
t	Number of observations

u	Inclination
v	Declination
u_i	Set of variables
w	Width of the sheet
x	observation point along x
x_0	Position of source in x direction
$\mathrm{X_i}$	Independent variable in MLR
y_0	Position of source in y direction
$\mathrm{Y_i}$	Dependent variable in MLR
y	observation point along y
z	observation point along z
z_0	Position of source in z direction/depth
α	Coordinate of the volume element dα
ρ	Density of magnetic flux
β_0	intercept of dependent variable
$\mathrm{\beta_i}$	Slope of regression/ coefficient of variable
$\mathrm{\varepsilon_i}$	Regression error
$\Delta T(x)$	Total magnetic intensity at x
μ_0	Permeability of free space
μ	1+K
τ	threshold value of Euler depth solutions
θ	the azimuth angle of box model
λ	strike angle of the structure measured clockwise from north of contact model
Δn	Absolute error

LIST OF ABBREVIATIONS

AN-EUL	Analytical Euler
AS	Analytic Signal
AAS	Amplitude of Analytic Signal
AS_tot	Analytic signal of total field
AS_rtp	Analytic signal of RTP data
AUTO-EUD	A technique of magnetic data interpretation for the fully automation (AUTO) of Euler deconvolution (EUD) relation
CED	Conventional Euler Deconvolution
cw	distance from the centre of convolution window
dz	Vertical derivative
DST	Differential Similarity Transform
ED	Euler deconvolution
f_d	declination of the ambient field
ft	Feet
IAF	Integrated and Automated Filter
IGRF	International Geomagnetic Reference Field
Lap	Laplacian
m_d	declination of magnetization
MLR	Multiple Linear Regression
Ng	Geographic north
nT	Nano Tesla
O_e	Reduction to the equator
O_p	Reduction to the pole
THDR	Total Horizontal Derivative

RTP	Reduction to the pole
RTE	Reduction to The Equator
S.I	International System of Units
SI dev	Deviation of structural index from the integer value
SD	Standard Deviation
TA	Tilt Angle
TD	Tilt derivative
THD_TD	Total horizontal derivative of tilt derivative
TDR	Tilt Derivative
TMI	Total Magnetic Field Intencity
USGS	United States Geological Survey
ws	Window size
2D	Two dimensional
3D	Three dimensional
p_{mic}	Micro scale clustering
p_{mac}	Macro scale clustering

PENAFSIRAN AUTOMATIK DATA MAGNETIK MENGGUNAKAN KAEDAH DEKONVOLUSI EULER DENGAN ALGORITMA TERUBAH SUAI

ABSTRAK

Dekonvolusi Euler konvensional mempunyai lima parameter yang tidak diketahui untuk diselesaikan iaitu lokasi sumber (x_0, y_0 and z_0), medan latar belakang (B) dan indeks struktur (N). Antara lima perkara yang tidak diketahui ini, indeks struktur dipilih secara manual oleh pengguna. Input manual indeks struktur ke dalam persamaan Euler menjadikan teknik ini semi-automatik dan tafsiran menjadi subjektif. Untuk menangani masalah ini, penyelidikan ini bertujuan untuk mengautomasikan teknik dekonvolusi Euler dan memperkenalkan teknik penurasan untuk membezakan penyelesaian yang boleh dipercayai daripada output penyongsangan. Ia juga merupakan sebahagian daripada objektif kajian ini, untuk menilai kesan kecondongan teknik baru dan menyiasat ketepatan algoritma yang diubahsuai. Regresi linear berganda digunakan untuk menyelesaikan lima parameter hubungan dekonvolusi Euler yang tidak diketahui untuk data magnetik bergrid. Untuk menyediakan penurasan yang berkesan, enam penuras dianalisis untuk memilih yang terbaik yang akan digunakan sebagai bantuan untuk penuras penyelesaian Euler. Kriteria lain yang digunakan untuk penurasan output penyongsangan ialah jarak dari pusat tetingkap konvolusi, sisihan indeks struktur dan ralat regresi. Kriteria ini disepadukan, automatik dan digunakan untuk memilih penyelesaian yang boleh dipercayai dari output penyongsangan. Kesan kecondongan pada teknik ini dinilai menggunakan kajian model sintetik (mudah dan gabungan) dan model lapangan. Setiap model disimulasikan menggunakan kecondongan yang belainan (0°, 15°, 30°, 45°, 60°, 75° dan 90°) dengan parameter lain yang tetap. Terbitan bagi setiap set data dikira dan disongsangkan. Penyelesaian yang boleh dipercayai dipilih dan hasilnya dibandingkan. Untuk data sebenar, keputusan tersongsang dan terturas dari jumlah medan dan data yang dikurangkan kepada kutub juga dibandingkan. Kajian model sintetik dan lapangan atas sumber magnet digunakan untuk menunjukkan keupayaan algoritma yang diubah suai untuk menyelesaikan lokasi sumber dan sifat sasaran. Hasil songsangan (fail) terdiri daripada 5 parameter yang tidak diketahui yang terdapat dalam persamaan dekonvolusi Euler. Isyarat analitik didapati mempunyai banyak kelebihan terhadap penuras yang dianalisis dan ia dipilih sebagai salah satu kriteria (sebagai tambahan kepada tiga kriteria yang disebutkan) untuk penapisan. Hasil model sintetik menggunakan kecondongan yang berlainan adalah sama. Hasil yang diperolehi dari penyongsangan jumlah medan dan data yang dikurangkan kepada kutub dari model medan pelbagai sumber juga adalah sama. Anggaran kedalaman min yang diperoleh dari penyongsangan jumlah medan dan data yang dikurangkan kepada kutub bagi data aeromagnet Nevada adalah 801 dan 787 m masing-masing. Keputusan yang diperolehi daripada analisis data Nevada telah memperkuatkan hasil yang diperolehi daripada pemodelan sintetik. Dalam kebanyakan ujian dijalankan, algorithma yang diperkenalkan menentukan kedudukan sasaran dengan kejituan yang baik dan teknik ini tidak bergantung pada kecondongan. Teknik ini adalah mod cepat tafsiran data magnetik dan mudah dilaksanakan kerana ia melibatkan terbitan tertib pertama medan tersebut.

AUTOMATIC INTERPRETATION OF MAGNETIC DATA USING EULER DECONVOLUTION WITH MODIFIED ALGORITHM

ABTRACT

The conventional Euler deconvolution has five unknown parameters to be solve which are the location of source (x_0, y_0 and z_0), the background field (B) and the structural index (N). Among these 5 unknowns, the structural index is to be manually selected by the user. The manual input of structural index into the Euler equation makes the technique to be subjective and semi-automated. The objectives of this research are, to automate Euler deconvolution equation and introduce a filter for discriminating reliable solution from the inversion output. It is also part of the objectives of this research, to **a**ssess the effect of inclination on the new technique and investigate the accuracy of the introduced algorithm. Multiple linear regression was used to solve the five unknown parameters of Euler deconvolution relation for gridded magnetic data. To provide an effective filtering, six filters were analysed in order to select a best one that would be used as an aid for filtering Euler solutions. Other criteria used for filtering of the inversion output are distance from the centre of convolution window, deviation of structural index and regression error. These criterions are integrated, automated and used for selecting more reliable solutions from the inversion output. The effect of inclination on this technique is assessed using synthetic (simple and combined) and field model's studies. Each model is simulated using different inclinations (0°, 15°, 30°, 45°, 60°, 75° and 90°) with other parameters kept constant. The derivatives of each data set were computed, inverted, more reliable solutions are selected and the results were compared. For real data, the inverted and filtered results from the total field and it's reduced to the pole data were also compared. The synthetic and field models studies over magnetic sources were used to demonstrate the ability of the modified technique to solve for the source location and nature of the target. The inversion (file) result comprises of 5 unknown parameters contained in Euler deconvolution equation. The inversion can be achieved by prescribing the window size which is the only choice a user has to make. Analytic signal is found to have so many advantages over the filters analysed and it is chosen as one of the criteria (in addition to the three mentioned criteria) for filtering. The results of synthetic models using different inclinations are about the same. The result obtained from the inversion of total field and it's reduced to the pole data of multiple source field model are about the same. The mean depth estimates obtained from the inversion of total field and reduced to the pole data of aeromagnetic data from Nevada are 801 and 787 m respectively. The results obtained from the analysis of Nevada data have further corroborated the result obtained from the synthetic modeling. In most of the tests carried out, the introduced algorithm located the position of the target with good precision and the technique does not depend on inclination. The technique is fast mode of magnetic data interpretation and easy to implement as it involves first order derivatives of the field.

CHAPTER 1

INTRODUCTION

1.1 Background

Now a days geophysical methods are widely applied to investigate subsurface of the Earth in order to explore geological structures of economic interest (in most cases) in areas of hydrology, solid minerals (Arisona et al. 2016), hydrocarbons, engineering, archaeological, geothermal studies (Khalil et al. 2017), geo-hazard assessment, geochemical (Yang et al. 2015) and environmental studies (Loke et al. 2013; Yang et al. 2015). The choice of geophysical methods over other techniques is partly due their nondestructive nature and cost effective in large area investigation. Geophysical survey can be carried out on land, from the air or over water because of the improved sensitivity of the measuring instruments. The speed of operation from air geophysical survey is another feature that attracted many Earth scientists to these techniques. The use of geophysical methods permits geophysicist to investigate the conceal features beneath the Earth's surface. These features appear in the form of anomaly due to different physical properties in the subsurface that need to be interpreted in to its geological relevance. The methods are Seismic, Electrical, Ground penetration radar, Transient electromagnetic (TEM), Gravity and Magnetic method among others.

Geophysical methods are classified as those that make use of the natural field of the Earth e.g. gravity and magnetic methods, and methods that require the input of artificially generated energy, e.g. seismic reflection and electrical methods. The geophysical surveying methods, measured parameters together with their respective operative physical properties are shown in the Table 1.1. It is the operative physical property that determines the specific use of any method. Thus for example, seismic or electrical method are suitable for locating water table because saturated rock may be distinguished from dry rock by its higher seismic velocity and higher electrical conductivity. Nevertheless, other considerations also determine the type of methods employed in a geophysical exploration program. For example, reconnaissance surveys are often carried out from air because of the high speed of operation. In such cases the electrical or seismic methods are not applicable since these require physical contact with the ground for the direct input of energy.

Table 1.1: Geophysical methods (Kearey et al., 2002)

Method	**Measured Parameters**	**Operative Physical Properties**
Seismic	Travel times of reflected/ refracted seismic waves	Density and elastic moduli, which determine the propagation velocity of seismic waves
Gravity	Spatial variations in the strength of the gravitational field of the Earth	Density
Magnetics	Spatial variations in the strength of the geomagnetic field of the Earth	Magnetic susceptibility
Electrical -Resistivity -Induced polarization	-Earth resistance -Polarization voltages or frequency-dependent ground resistance	-Electrical conductivity -Electrical capacitance
Self-potential	Electrical potential	Electrical conductivity
Electromagnetic	Response to electromagnetic radiation	Electrical conductivity and inductance
Radar	Travel times of reflected radar pulses	Dielectric constant

Measurement of geomagnetic field can be used to determine the structure of the Earth since many rocks have magnetization. Magnetic method can be used as a tool for detecting shallow structure of local, regional and global scales. With the aid of techniques used for the inversion of magnetic data (Gerovska, and Bravo 2003; Gerovska, et al. 2010;Cooper, 2014; Cooper and Whitehead 2016; Salem, 2007), it is possible to determine the horizontal and vertical position of concealed metallic objects in the near vicinity of the earth's surface in addition to the delineation of deep-seated structures. The advantages of magnetic method include its ability to detect near surface weak magnetic signal produced by the buried objects and its relative ease of operation.

The choice of a geophysical method to locate a particular geological structure depends on its mineral content. Some of the reasons for choosing the magnetic method are:

i. This method is widely used in mineral and petroleum explorations, engineering, environmental, geothermal and global applications. Magnetic method is the most versatile of geophysical prospecting techniques.
ii. Magnetic measurements are made more easily and cheaply than most geophysical measurements (Telford et al., 1990).
iii. In order to understand this field (geophysics) very well, magnetic method needs to be studied since the study of the Earth's magnetism is the oldest branch of geophysics.
iv. Magnetic method is one of the methods that use the natural field of the Earth, unlike some other methods that requires the artificially generated field. It is therefore provide room to understand the variation of a certain phenomenon on the Earth.
v. Aeromagnetic maps of most of the areas around the globe are available for free or at nominal amount.

Aeromagnetic survey is carried out in order to detect rocks or minerals that have abnormal magnetic properties which can be identified by causing anomalies in the geomagnetic field. It is fast, cost effective and accessible technique used for regional geological mapping, mineral and petroleum exploration (Chinwuka, 2012). Euler deconvolution can assist in the interpretation of aeromagnetic data by indicating the nature of the basement topography (undulating), depth and the direction of steepness. Overburden thickness of the sedimentary sediment is very essential in hydrocarbon exploration.

Generally, potential field data interpretation can be categorized into three sections; forward modeling, inverse modeling, data enhancement and display (Blakely, 1996). Modeling is an essential aspect of geophysics because it can be used to predict a particular geological structure based on known model parameters. It can also be used to determine feasible subsurface distribution of physical properties of the target. The former and latter processes are known as forward and inversion modeling. Mathematical modeling can be divided into three main groups which are analytic, empirical and numerical models. Analytical modeling applies to simple cases only and it provides error free solution. Analytical modeling is a vital tool used in potential field data inversion. In general, modeling of geophysical data is addressed in terms of depth to simple magnetic or gravity sources. Modeling leads to a distinct inversion techniques as a result of non-uniqueness nature of the causative sources.

The difficulties attached on seeking an inverse solution are:

i. Scientifically, error is present in all the measurements collected due to instrumental and systematic error.

ii. The presence of sub-surface features that are not properly addressed by a model

iii. Superposition of close features. However, these effects can be constrained by using geological map and borehole information.

There are so many depth estimation techniques that assist geophysicist in potential field data interpretation such as Werner deconvolution, source parameter imaging (SPI), source location using total field homogeneity, depth from extreme points, tilt depth and so on. In addition to the mentioned manual or automatic/semi-automatic depth estimation techniques, Euler deconvolution is powerful technique designed to analyze large amount of potential field data. It has been applied extensively in delineating geologic boundaries (Hsu et al., 1996; Ugalde and Morris, 2010; Barbosa and Silva, 2011), and locating geothermal sources or hot springs (Nouraliee et al., 2015); and is combined with other geophysical methods to ensure enhanced interpretation of subsurface geology. It is one of the techniques that can be used to provide fast means of data interpretation. Euler deconvolution technique uses field and its derivatives in the system of linear equation in relation to the source coordinate to estimate depth and location of anomalous source. This technique can assist geoscientist by indicating portion of interest which can then further be analyzed in detail. Some of the justifications on the need of depth estimation technique are:

i. Large amount of potential field data sets (especially magnetic and gravity methods) have been collected using aeroplane, ships and satellites in regional/global scale. These data sets need to be process and interpreted in to its geological relevance.

ii. The thickness of the sedimentary section and depth of ore bodies (that contains magnetic minerals) are highly interested in hydrocarbon and mineral exploration respectively.
iii. Euler deconvolution technique (Thompson, 1982) has been popularly used by Chevron Oil Companies and within the Gulf, EULDPH is also applied by Durrheim (1983), Corner and Wisher (1989) to determine magnetic markers in search for gold in Witwatersrand Basin.

In addition, Euler deconvolution technique does not assume any geological model, but it requires (prior knowledge of the rate of decay of the field of a particular source) structural index which gives the nature of the geological structure. The anomaly source is considered as singular point that consists of elementary potential field distribution such as point poles or dipoles. An anomaly is considered as the field caused by local variation in the geomagnetic field given rise by a singular point of source. With the aid of Euler homogeneity relation, magnetic method can be used to delineate the presence of metallic structures in the subsurface. Therefore, some of the advantages of this technique are speed of operation, ability to interpret large data sets and its implementation is less tedious.

The conventional Euler deconvolution (Thompson 1982; Reid et al 1990; Ugalde and Morris 2010; Barbosa and Silva 2011; Oruç and Selim 2011; Chen et al. 2014) has 5 unknown parameters which are the location of source in x, y and z-directions (x_0, y_0 and z_0), the background field and the structural index (N). Among these 5 unknowns, N is to be manually selected by the interpreter/user. An interpreter has to solve the equation using different N and finally select the best set of solution. The interpreter is left with the decision that has the highest impact on the depth solutions: which *N* should be chosen? Much of the interpreter’s efforts will be exhausted on choosing the solution produced by the appropriate structural index.

1.2 Problem statement

i. The manual input of structural index into the Euler equation makes the technique to be semi-automated. This makes the procedure too subjective (Interpreters can make different decisions), tedious and time consuming. Moreover, the geology of the earth is comprises of different structures (it is very complex) which may not be fitted by a fixed *N*. Hsu (2002) stated that the use of wrong *N* can cause bias on depth estimate and scattered solution on target’s locations. Therefore, the used of fixed structural index may not estimate the parameters of different sources in the real geology with desired accuracy.

   One of the disadvantages of conventional Euler deconvolution is that the interpreter/user has to select *N* manually. This property is a setback to one of the most important attribute of the technique which is fast means of interpreting large volume of data. However, attempts made to address this problem using Differential Similarity Transform (DST) (Stavrev, 1997; Gerovska and Arouzo-Bravo 2003; Grerovska et al. 2010) and other related techniques that does not require the use of structural index (Mushayandebvu et al., 1999, 2001; Nabighian and Hansen, 2001; Salem and Ravat, 2003; FitzGeral et al, 2004; Keating and Pilkington, 2004; Salem et al., 2007) surfer some drawbacks. DST is less implemented because of the complexity involved in operation (Reid and Thurston, 2014). According to Florio et al., 2006, the estimation of N using AS (Salem and Ravat, 2003) could lead to error. Tilt depth (Salem et al., 2007) technique uses higher order derivatives (Reid and Thurston, 2014).

A procedure for solving five unknown parameters (including the structural index) of magnetic anomaly using Euler deconvolution that can be implemented without the use of complex mathematics, the use of analytic signal and higher order derivatives is missing in the literature.

Multiple linear regression (MLR) methodology can be used to solve positions (x_0, y_0 and z_0) of magnetic source, background (B) and structural index (N) simultaneously. The use of multiple linear regression to solve the unknown parameters of Euler deconvolution technique of magnetic anomaly is not available in the geophysical literature. Unlike the past works, this technique allows the use of first order derivatives, the inversion is independent of analytic signal (AS) and it does not involve complex mathematical operations. It is simple to apply and the derivatives are computed directly from the total field grid.

ii. Euler deconvolution treats the potential field sources as consisting of elementary points with different parameters (such as N) as such large number of solutions is usually obtained and it needs effective filtering technique. Because of the complicated nature of the Earth subsurface, some of these solutions are spurious/artifacts caused by interference of other sources. Many studies have been carried out to address this issue and they come out with different procedures in determining the correct solution. Reid and Thurston (2014) has advocated that when depth and

N of the source are to be estimated simultaneously, rigorous means of data filtering is required to choose the valid solutions.

The use of standard deviation of estimated depth and clustered solutions had been the preferred means of selecting valid solutions (Thompson, 1982; Reid et al., 1990, Grerovska et al., 2010). Other researchers used various traditional filtering techniques (FitzGerald et al., 2004) to discriminate the most accurate solutions. However, rigorous filtering technique still remained one of the challenges of using Euler deconvolution technique. Euler deconvolution method is built based on the potential field and its derivatives; so, the accuracy of Euler deconvolution method relies largely on the derivatives. Thus, Euler deconvolution solutions should be filtered based on the area of the data to be convolve, rather than focusing on the sprays of solutions. It is crucial to study how potential derivatives based filters can be used as an aid of choosing the correct range of depth solutions. The coupling problem that exists between depth and structural index can be avoided by identifying and using the locations immediately above the source body's critical points (Reid and Thurston, 2014).

iii. The pattern of magnetic anomaly depends on its position on the earth surface. The same structure placed at different geographical locations would give different anomaly's shape because of the variation in magnetic latitude. The dipolar nature of the magnetic field causes distortion in the anomaly's shape and as a result of this effect, error will be introduced to the data and there by affecting the estimate of the anomaly's location (Araffa et al, 2012). While the use of RTP is recommended to be applied on the data prior to the application of Euler deconvolution (Thompson, 1982), other researchers (Reid et al., 1990) are of the opinion that it should not be applied.

However, this problem remain unsolved, no attempts have been made to investigate this effect on Euler deconvolution related techniques. Also, no inclination's assessment was carried out on the present technique. The use of synthetic models and real data is very essential in understanding the effect of inclination of the introduced technique. Because, the introduced technique is not available in the literature, evaluating the effect of inclination will definitely be added or otherwise to the strength of the technique.

iv. The limit of the accuracy of depth estimation technique from magnetic data is well established in the literature (Breiner, 1973; 1999). The accuracy of conventional Euler deconvolution (Thompson, 1982; Reid et al., 1990) and other related techniques have been evaluated. The traditional approach for evaluating the performance of Euler deconvolution technique has been the use of deviation from a certain referenced value (mean).

However, in this research where a new approach is introduced, its accuracy remains unknown. Moreover, the accuracy of interpretation techniques determines its applicability in various geophysical applications. Therefore, there is need to assess the present technique in order to know its accuracy. Synthetic modeling using different models such as box, contact, cylinder, dike and sphere can be used to assess the accuracy of the introduced technique. In this case, the theoretical basis of the technique can be established. The assessment can also be carried out using field model data and the site where the detail geological information of the area is known. In addition to the deviation of the parameters, percentage of minimum/maximum error permits easier assessment of the output parameters.

1.3 Research question

i. Which approach shall be adopted to automate Euler deconvolution technique?

ii. What are the criteria for choosing valid Euler solutions?

iii. What is the effect of inclination on AUTO-EUD?

iv. How accurate is AUTO-EUD?

1.4 Research objective

An algorithm/procedure based on Euler's homogeneity relation for fully automation (hence the acronym, AUTO-EUD) of magnetic data interpretation is presented in this research. Some of the objectives of this research are:

i. to automate Euler deconvolution equation in order to estimate the horizontal coordinates (x_0 and y_0), depth, background (B) and structural index (N) of a magnetic source,

ii. to propose a filter for discriminating reliable solution from the inversion output of Euler homogeneity equation,

iii. to assess the effect of inclination on the introduced algorithm (AUTO-EUD) and

iv. to investigate the accuracy of AUTO-EUD’s solutions using synthetic and real magnetic data.

1.5 Novelty of the study

i. The introduced algorithm for solving the unknown parameters of magnetic anomaly using multiple linear regression is not available in the geophysical literature.

ii. The integrated and automated filter introduced in this study is unique in the geophysical literature and therefore, it is a novel.

iii. This study has empirically deduced the structural index of a box which is also not available in the literature.

iv. An application of the technique in engineering and environmental site has been demonstrated. This application is rarely found in the literature and it is therefore a new contribution to the knowledge.

1.6 Scope and limitation of the study

The scope of this study is limited to forward modeling and inversion of 3D magnetic field only. The accuracy of the introduced technique is determined using synthetic models and field model data. The test of this inversion program using synthetic model (in this research) is also limited to certain type of structures, namely box, contact, dike, horizontal cylinder and sphere. These structures are designed with the intention to simulate field with simple geologic structures. For synthetic and field model data, the solutions provided by the introduced technique are compared to true parameters of the models, whereas for real magnetic data, the depth solutions are compared to thickness of rocks available in the literature.

1.7 Thesis outline

This thesis consists of 5 chapters. The first chapter introduces geophysical prospecting methods and data interpretation with emphasize on Euler deconvolution. The introductory chapter also presents problem statements, research questions, objective of this study, novelty of the study, the scope and limitation of the study. Other component of this chapter, although not the least, is significance of findings, organization of the thesis.

The second chapter provides fundamentals of magnetic field and some background of Euler's homogeneity concept, which is the basis of Euler deconvolution methodology. This chapter also includes the previous works to give overview of how Euler deconvolution has evolved and modified through the past decades, and also to sort out the research gap in Euler deconvolution methodology.

The third chapter presents the methodology used in this study and it consists of (i) the introduction of the new technique using Multiple Linear Regression methodology, (ii) the accuracy assessment of AUTO-EUD using synthetic modeling and (iii) the accuracy assessment of AUTO-EUD using real magnetic data. This chapter also explains how the solutions are filtered based on analytic signal and the comparison between Conventional Euler Deconvolution (CED) and the present technique (AUTO-EUD).

The fourth chapter presents the results of the forward modeling and inversion of synthetic models as well as the inversion of real magnetic data of field models application site. This chapter also discusses the accuracy of AUTO-EUD based on the comparison between solutions from AUTO-EUD and the true models, and between solutions and geological map. Besides these, the discussion also includes the limitations of AUTO-EUD based on the results obtained.

The last chapter concludes the study by relating the findings to the objectives of this study, emphasizing the significance of AUTO-EUD in locating the source of magnetic field. This chapter includes some recommendation for future study as well.

CHAPTER 2

LITERITURE REVIEW

2.1 Introduction

This chapter introduces magnetic susceptibility and Euler homogeneity relation as a depth estimation technique. An overview on the development of Conventional Euler Deconvolution (CED) methodology through the past decades is included in order to provide theoretical basis of the present algorithm. Introduction on filtering and accuracy assessment of Euler deconvolution are presented.

2.2 Magnetic susceptibility

The quantity of magnetic moment per unit volume is called magnetization (also called magnetization intensity, dipole per unit volume or magnetic polarization) and it is denoted by a symbol M. It is the vector field that expresses the density of permanent dipole moments contained in a magnetic material. The arrangement/line-up of internal dipoles gives rise to a field M which is added to the magnetizing field H. The S.I unit for magnetization is ampere per meter. For low magnetic fields (Equation 2.1)

$$M \alpha H$$

$$M = kH \tag{2.1}$$

The constant in Equation 2.1 is called magnetic susceptibility (k), it is determined the degree to which a body is magnetized. The total field including the effect of magnetization is called magnetic induction (m) and it is given by (Equation 2.2)

$$m = \mu_0 \ (H+M) = \mu_0(1+k) \ H$$

$$= \mu\mu_0 H \tag{2.2}$$

The S.I and electromagnetic units for m is the tesla (T) and gauss (10^{-4}T) respectively. Gamma(γ) is the unit of magnetic induction that is generally used for geophysical work. The magnetic flux (Φ) is given by (Equation 2.3)

$$\Phi = \boldsymbol{m}.A \tag{2.3}$$

Where A is a vector area. Thus

$$|\boldsymbol{m}| = \frac{\Phi}{|A|}$$

A and B are parallel, that is, ***m*** is the density of magnetic flux. The S.I unit for magnetic flux is the Weber.

Magnetic susceptibility is the significant variable in magnetics. Although instruments are available for measuring susceptibilities in the field, they can only be used on outcrops or on rock samples and such measurement do not give the bulk susceptibility of the formation. Table 2.1 lists magnetic susceptibilities for a variety of rocks. Sedimentary and basic igneous rocks have the lowest and the highest average values of magnetic susceptibility respectively:

Table 2.1: Magnetic susceptibility of rocks and minerals (source: Telford et al., 2001)

Rock/mineral type	**Susceptibility × 10^3(S.I Unit)**	
	Range	**Average**
Metamorphic		
Schist	0.3-3	1.4
Gneiss	0.1-25	-
Slate	0-35	6
Igneous		
Granite	0-50	25
Porphyry	0.3-200	60
Peridotite	90-200	150
Diabase	1-160	55
Pyroxenite	-	125
Diorite	0.6-120	85

Minerals		
Magnetite	1200-19200	6000
Pyrrhotite	1-6000	1500
Ilmenite	300-3500	1800
Clays	-	0.2
Graphite	-	0.1
Casiterite	-	0.9
Limonite	-	2.5
Pyrite	0.05-5	1.5
Sedimentary		
Dolomite	0-0.9	0.1
Sandstones	0-20	0.4
Limestone	0-3	0.3

2.2.1 Magnetic Elements

i. Inclination of the geomagnetic field: It is the angle between magnetic north and the direction of the Earth field (Telford et al., 1990)

ii. Declination of the geomagnetic field: This is the angle between geographic north and magnetic north (Telford et al., 1990).

iii. The angle of dip at a place: Is the angle between the direction of earth's magnetic field and the horizontal component of the earth's magnetic field in the magnetic meridian at that place

iv. Strike angle of the cylinder: Is the angle between the cylinder axis and magnetic north

v. Azimuth angle: Is the angle between geographic north and horizontal of a plane of box model.

2.3 Reduction of magnetic observations

The magnetic field readings measure from survey stations varies with time. Diurnal effect and magnetic storms are the most significant causes of the changes in magnetic field. This effect must be corrected from the data using appropriate techniques. The short term, spikes and erratic changes in magnetic field are known as micro-pulsations. These effects range from few to 100 nT in terms of intensity. When these changes become large (amplitude and period), they are called magnetic storm. It is a short term disturbances in the intensity of magnetic field associated with sun spot activity and charged particles from sun. To achieve a successful interpretation, the reduction has to be carried out to enhance and isolate the contribution of the field due to concealed structure (Ismail, 2015). The correction or reduction of magnetic data is necessary to remove all causes of magnetic variation and noises from the observations other than those arising from the magnetic anomalies in the subsurface.

2.3.1 Diurnal variation correction

Diurnal variation correction accounts for the temporal variation caused by the electromagnetic radiation of the sun, which disrupts the geomagnetic field and thus our survey, it can be checked by using two magnetometers with one acting as a base station and recording the magnetic field every 1 minute or so, while the other would be the primary machine to survey the area, however this method and magnetic survey in general must not be used during magnetic storms (Sharma, 1997). After the measurements, the primary machine readings would be time-synced with the base station and then subtracting the results would give us the corrected readings.

2.3.2 Geomagnetic correction

The earth’s magnetic field strength varies from 25000 nT at the magnetic equator to 69000 nT at the magnetic pole. This correction is carried out by subtracting the theoretical field from International Geomagnetic Reference Field (IGRF). This theoretical value changes with time (Sharma, 1997)

2.3.3 Reduction to the pole and equator

Reduction to the pole (RTP) is an operation use to transform the magnetic anomaly into an anomaly that would have been obtained if the measurements were taken at the magnetic pole, the area where the magnetic inclination is vertical, with the assumption that the source is magnetized by induction (Silva, 1986). This effect causes asymmetry and lateral shift of the anomaly of measured total magnetic field (Aina, 1986). It has some advantages when applied to magnetic data which include simplification of the interpretation of anomaly, it removes dipolar nature of magnetic anomaly and it changes the asymmetric shape of magnetic anomaly to its symmetric form. The dipolar nature of the magnetic field causes distortion in the anomaly's shape and as a result of this effect, error will be introduced to the data and there by affecting the estimate of the anomaly's location (Araffa et al., 2012). For reduction to the pole technique to be applied on magnetic data, the information about the remanent magnetization is required. However this information is not available in most areas. An alternative method to RTP is the reduction to the equator (RTE) which transforms the magnetic measurement as the one that would be observed if the anomaly were located at the equator.

The direction cosines of geomagnetic field vector are l, m and n. the geomagnetic field vector is assume to be parallel to the polarization vector, in this case, the remanent magnetization is zero. u and v are the Cartesian spatial coordinates of angular frequency given the reduction to the pole (O_p) and equator (Oe) operators as Equation 2.4 and 2.5 respectively:

$$O_p(u,v) = \frac{(u^2+v^2)\{n^2(u^2-v^2)-(lu+mv)^2\}-2j(u^2+v^2)^{3/2}n(lu+mv)}{[(lu+mv)^2+n^2(u^2+v^2)]^2} \tag{2.4}$$

$$O_e(u,v) = \frac{-(lu+mv)^2\left[\{n^2(u^2+v^2)-(lu+mv)^2\}+2jn(u^2+v^2)^{1/2}(lu+mv)\right]}{[(lu+mv)^2+n^2(u^2+v^2)]^2} \tag{2.5}$$

In Equation 2.4 and 2.5 above, j = -1. Both O_p and O_e can be used as:

1. Find the Fourier transform of the measured field, apply O_p/O_e on the transformed field and obtain the inverse Fourier transform.
2. Evaluate O_p/O_e for various values of u and v for the inclination and declination. Compute the inverse transform of various inclinations and declination of the magnetic field. The output is the space domain operator which can be convolved with the measured magnetic field.

   The first approach is more accurate, however, it can only be applied to relatively small areas. On the other hand, the second approach is less accurate, the operator requires to be truncated to manageable dimensions but it can be applied to maps of any size (Jain, 1988).

2.3.4 Applications of magnetic method

As for the applications of the magnetic survey, it can be used to map dikes blocking groundwater flow in the subsurface, structural trends and basement features. It can also be used for the detection of archaeological objects, buried metal drums and investigations over landfills.

2.3.5 Limitations of magnetic method

The magnetic survey has some limitations, it can only detect ferrous materials including volcanic rocks, its image resolution deteriorates quickly with the increase of depth, and it becomes almost useless near buildings, vehicles, or areas with reinforced concrete and where the ferrous materials or volcanic rocks are underlain by strongly magnetic rocks (Zohdy et al., 1974).

2.4 Euler homogeneity

Euler Deconvolution method evolves from the Euler's homogeneity relationship (John, 1965); it is first initiated to solve 2D magnetic field by Thompson (1982). In the homogeneity concept (Equation 2.6), a function $f(u)$ is considered homogenous of degree n if

$$f\ (tu)\ =\ t^n f(u) \tag{2.6}$$

where $u = (u_1, u_2, \ldots, u_i)$ is the set of variables with respect to the homogeneity of the field f, t is a real number and n is the degree of homogeneity (Reid and Thurston, 2014). This function also satisfies the Euler's differential equation given in Equation 2.7. as

$$u\nabla f(u)\ =\ nf(u) \tag{2.7}$$

Equation 2.7 can be written in three Cartesian coordinates x, y and z form, such that

$$f\ (tx, ty, tx)\ =\ t^n f(x, y, z) \tag{2.8}$$

and the partial differential equation then would be (Equation 2.9)

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = nf \tag{2.9}$$

Based on this, the N is defined as (Equation 2.10)

$$N\ =\ -n \tag{2.10}$$

According to the power law, the field's fall off function for both $f\,(u)$ and $f\,(x, y,$ and $z)$ can be expressed in the form of (Equation 2.11)

$$f = C/r^N \tag{2.11}$$

where *C* is a constant which includes any factor that affects the field, r is the distance between the source and the observation point. The degree of homogeneity explains the rate at which the potential field strength reduces over increasing radius, and often referred to as structural index (N) in the literature.

One of the advantages of Euler deconvolution is that no geological assumption is required prior to inversion. However, some understanding of the study area is necessary since N represents type of source that produces the field anomaly (Thompson, 1982). Ideas about the geometry of the structure will elevate the confidence in choosing the correct N. Besides this, the application of Euler equation should be limited only to single-point sources, which give off potential field conforming to Equation 2.10. Sources with homogenous magnetic field are listed in Table 2.2.

Table 2.2: Structural index of magnetic source

Source	**Smellier model**	***N***
Sphere	Dipole	3
Vertical/horizontal cylinder	Line of dipoles	2
Dike/thin sheet	Line of poles	1
Infinite contact	-	0

In order to find the source location (x_0, y_0, z_0), Equation 2.9 is then further redefined as (Equation 2.12)

$$(x - x_0)\frac{\partial f}{\partial x} + (y - y_0)\frac{\partial f}{\partial y} + (z - z_0)\frac{\partial f}{\partial z} = N\,(B - f) \qquad (2.12)$$

where x, y, z are the observation point coordinates; x_0, y_0, and z_0 are the source locations; $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ are the potential derivatives; N is the structural index; B is the background of field f (Thompson, 1982). Equation (2.12) forms the basis for the methodology used in this research.

2.4.1 Previous work

Information that can be obtained from magnetic method can be used to estimate the position of a concealed body if it has enough magnetization. Among the parameters of the body that can be estimated is depth. Depth estimation techniques are divided into manual and automated techniques. Some of the manual depth estimation techniques include maximum slope, Peter half slope, half width, Sokolov distance and Hannel distance (Samuel, 2017). The computerized techniques can be further divided into 2; semi-automated and automated techniques. Euler deconvolution belongs to a group of semi-automated techniques, in its operation, it allows user to make choice of structural index. Numerous attempts have been made to make it fully automated.

Examples of other automated techniques include spectral depth technique Techniques for the estimation of depth to the bottom of magnetic sources has been presented (Bansal et al., 2011; Nwankwo, 2015). The techniques used for detailed analysis operates on limited amount of data and these are characteristics curve, iterative or inverse curve matching.

There were so many computer-assisted interpretation techniques in the literature that are belong to computerized class of depth estimation methods. Automatic interpretation techniques have advantages of operating directly on the recorded digital field data in addition to providing rapid means of analyzing large amount of data. The technique for locating vertices of polygon model has been presented (O’Brein, 1971). A technique based on vertical prism and thin plate models was described by Koulomzine et al. (1970). Werner (1953) simplified equation for the interpretation of two-dimensional thin dike. The position of the dike could be obtained/ devised by choosing appropriate point along a profile. This technique has been successfully applied in oil and mining companies as a result of the advancement in Aero Service and further researches based on Hartman et al. (1971). The analysis of magnetic discontinuities using derivatives (vertical and horizontal) of the total magnetic field intensity was incorporated into Werner's equation (Hartman et al., 1971).

A depth estimation technique using statistical approach that makes use of slope of the power spectral density has been presented (Vacqier et al., 1951). The location of the boundary is obtained through the computation of the horizontal gradient of the pseudo gravity that peaks over a vertical contact (Grauch and Cordell, 1987). For dipping contacts, the peak is somewhat offset (Thompson, 1982). Another related technique is the use of analytic signal and total gradient with the peaks directly over a contact model with arbitrary dip (Nabighian, 1972). Hansen et al. (1987) has shown that the peak over a contact model using total gradient and analytic signal were noisy estimator. Using this technique, the depth is obtained from the breadth of the peak (Hansen et al., 1987). So many automatic processing techniques that estimate both source location and depth have been presented. Naudy (1971) introduced an approach similar to Werner deconvolution (Jain, 1976) that makes use of prism and thin-plate models.

Euler deconvolution is a quick means of transforming field measurements into location and depth estimates of the magnetic source. The technique operates on a subset of data using a moving window in which the body coordinates are solved. Intermediate bodies have non-integer structural index, the technique is only approximate. The term Euler deconvolution was obtained by the analogy of Euler equation with the established Werner deconvolution technique. In strong term, Euler deconvolution is only valid for functions that are homogeneous. The field of most