UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2007/2008

Jun 2008

MSS 212 – Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all five** [5] questions.

*[**Arahan**: Jawab **semua lima** [5] soalan.]*

1. (a) Show that $\propto = \{e_1, e_2, e_3\}$ is a basis of $\mathbb{R}^3$ over $\mathbb{R}$ and also a basis of $\mathbb{C}^3$ over $\mathbb{C}$.

[50 marks]

(b) Let $W = \{(a,b,c,d) \in \mathbb{C}^4 | a = b, d = -c\}$

(i) Show that W is a subspace of $\mathbb{C}^4$.

[30 marks]

(ii) Find a basis of W over $\mathbb{C}$.

[30 marks]

(iii) Find a basis of W over $\mathbb{R}$.

[10 marks]

2. Let $T : \mathbb{C}^3 \rightarrow \mathbb{C}^4$ be a function such that

$$(x,y,z)T = (x,y,x+y,z).$$

(i) Show that T is a linear transformation over $\mathbb{C}$.

[40 marks]

(ii) Find $T_{\alpha,\beta}$ where $\alpha = \{ie_1, e_2, e_3\}$ and $\beta = \{e_1, ie_2, e_3, ie_4\}$.

[60 marks]

(iii) Give $T_{\mathbb{C}}$, the complexity of T.

[20 marks]

3. (a) Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ such that T has only 2 distinct eigen values λ_1 and λ_2 with algebraic multiplicity of λ_1 is 2.

(i) Give all passible forms of $JCF(T)$ if T can be diagonalised.

[12 marks]

(ii) Give all possible forms of $JCF(T)$ if T cannot be diagonalised.

[8 marks]

(b) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation such that

$$(x,y,z)\,T = (5x+4y+3z, -x-3x, x-2y+z)$$

Find the $JCF(T)$.

[100 marks]

4. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation such that

$$(x, y, z)\,T = (3x - z, 2y, -x + 3x)$$

(a) Show that T is self-adjoint.

[40 marks]

(b) Find an orthonomal basis α of $\mathbb{R}^3$ such that $T_{\alpha,\alpha}$ is a diagonal matrix.

[80 marks]

5. (a) Let $C = \begin{pmatrix} \frac{1}{2}a & d+g & 3g \\ -\frac{1}{2}b & e+h & 3h \\ c & f+i & 3i \end{pmatrix}$ and $det\begin{pmatrix} a & d & g \\ -b & e & h \\ 2c & f & i \end{pmatrix} = 3$

Find $det\,(2C^{-1})$.

[60 marks]

(b) Use Cramer's rule to solve the following system of linear equations

$$\begin{aligned} x_1 \qquad\;\; + 2x_3 &= 0 \\ -3x_1 + 4x_2 + 6x_3 &= 2 \\ -x_1 - 2x_2 + 3x_3 &= -1 \end{aligned}$$

[60 marks]

1. (a) *Tunjukkan* $\propto = \{e_1, e_2, e_3\}$ *ialah suatu asas bagi* $\mathbb{R}^3$ *atas* $\mathbb{R}$ *dan juga suatu asas bagi* $\mathbb{C}^3$ *atas* $\mathbb{C}$.

[50 markah]

(b) *Biar* $W = \{(a,b,c,d) \in \mathbb{C}^4 | a = b, d = -c\}$

(i) *Tunjukkan* W *ialah suatu subruang bagi* $\mathbb{C}^4$.

[30 markah]

(ii) *Cari suatu asas bagi* W *atas* $\mathbb{C}$.

[30 markah]

(iii) *Cari suatu asas bagi* W *atas* $\mathbb{R}$.

[10 markah]

2. *Biar* $T : \mathbb{C}^3 \to \mathbb{C}^4$ *ialah suatu fungsi sedemikian hingga*

$$(x,y,z)T = (x,y,x+y,z).$$

(i) *Tunjukkan* T *ialah suatu transformasi linear atas* $\mathbb{C}$.

[40 markah]

(ii) *Cari* $T_{\alpha,\beta}$ *dengan* $\alpha = \{ie_1, e_2, e_3\}$ *dan* $\beta = \{e_1, ie_2, e_3, ie_4\}$.

[60 markah]

(iii) *Berikan* $T_{\mathbb{C}}$, *kompleksiti bagi* T.

[20 markah]

3. (a) *Biar* T *suatu transformasi linear dari* $\mathbb{R}^3$ *ke* $\mathbb{R}^3$ *sedemikian hingga* T *hanya mempunyai 2 nilai eigen yang berbeza* λ_1 *dan* λ_2 *dengan pekali aljabar* λ_1 *ialah 2.*

(i) *Berikan semua bentuk yang mungkin bagi* $JCF(T)$ *jika* T *boleh diperpenjurukan.*

[12 markah]

(ii) *Berikan semua bentuk yang mungkin bagi* $JCF(T)$ *jika* T *tidak boleh diperpenjurukan.*

[8 markah]

(b) *Biar* $T : \mathbb{R}^3 \to \mathbb{R}^3$ *suatu transformasi linear sedemikian hingga*

$$(x,y,z)\,T = (5x + 4y + 3z, -x - 3x, x - 2y + z)$$

Cari $JCF(T)$.

[100 markah]

4. *Biar* $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ *suatu transformasi linear sedemikian hingga*

$$(x, y, z)\,T = (3x - z, 2y, -x + 3x)$$

(a) *Tunjukkan* T *ialah swadampingan.*

[40 markah]

(b) *Cari suatu asas ortonormal* α *bagi* $\mathbb{R}^3$ *sedemikian hingga* $T_{\alpha,\alpha}$ *ialah suatu matriks perpenjuru.*

[80 markah]

5. (a) *Biar* $C = \begin{pmatrix} \frac{1}{2}a & d+g & 3g \\ -\frac{1}{2}b & e+h & 3h \\ c & f+i & 3i \end{pmatrix}$ *dan* $\det\begin{pmatrix} a & d & g \\ -b & e & h \\ 2c & f & i \end{pmatrix} = 3$

Cari $\det\,(2C^{-1})$.

[60 markah]

(b) *Gunakan Petua Cramer's untuk menyelesaikan sistem persamaan linear yang berikut*

$$\begin{aligned} x_1 \quad\quad + 2x_3 &= 0 \\ -3x_1 + 4x_2 + 6x_3 &= 2 \\ -x_1 - 2x_2 + 3x_3 &= -1 \end{aligned}$$

[60 markah]

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