IMPROVED SWITCHING-BASED MEDIAN FILTER FOR IMPULSE NOISE REMOVAL

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by

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF ABBREVIATIONS

1-D	One-Dimensional
2-D	Two-Dimensional
ABDND	Advanced Boundary Discrimination Noise Detector
AFSF	Adaptive Fuzzy Switching Median Filter
BDND	Boundary Discrimination Noise Detector
DSMFBDNDE	Directional Switching Median Filter using Boundary Discrimination Noise Detection by Elimination
IIR	Infinite Impulse Response
IMF	Improved Median Filter
MDBUTMF	Modified Decision Based Unsymmetric Trimmed Median Filter
PCINRF	Pixel Correlation-based Impulse Noise Reduction Filter
SMF	Standard Median Filter
WMF	Weighted Median Filter

LIST OF SYMBOLS

θ	the intensity value of the damaged image
$\mathbf{C}$	the damaged or corrupted image
C	the intensity value of the damaged image
$C(i, j)$	the intensity value of the damaged image at spatial position (i, j)
$\mathbf{D}$	the damaged or corrupted image
D	the intensity value of the damaged image
$D(i, j)$	the intensity value of the damaged image at spatial position (i, j)
$\mathbf{F}$	the restored or filtered image
F	the intensity value of the filtered image
$F(i, j)$	the intensity value of the filtered image at spatial position (i, j)
i, j, k, l	the spatial coordinates on image
J, K	the dimensions of the image
L	the discrete gray level
n_s	the number of samples for median value calculation
n_i	the number iterations
$\mathbf{M}$	the noise mask
M	the value of the noise mask
$M(i, j)$	the condition of the noise mask at spatial position (i, j)

$\mathbf{N}$	the noise
N	the value of the noise
$N(i, j)$	the strength of the noise at spatial position (i, j)
$p()$	the probability density function
P	the noise density
P_1	the density of pepper noise
P_2	the density of salt noise
$\mathbf{T}$	the temporary digital image
T	the intensity value of the temporary image
$T(i, j)$	the intensity value of the temporary image at spatial position (i, j)
$\mathbf{X}$	the digital image (in general)
X	the intensity value of the image (in general)
$X(i, j)$	the intensity value of the image at spatial position (i, j) (in general)
W	the sliding window
$w \times h$	the dimensions of the sliding window or filter

PENAPIS MEDIAN BERASAS-PENSUISAN DIPERBAIKI UNTUK PENYINGKIRAN HINGAR DEDENYUT

ABSTRAK

Tesis ini mencadangkan satu algoritma baharu untuk mengurangkan hingar dedenyut daripada imej digital. Bagi mencapai matlamat ini, tinjauan bacaan yang menyeluruh ke atas model hingar dedenyut dan rangka kerja penapis median telah dilaksanakan dengan jayanya. Algoritma yang dicadangkan adalah berdasarkan pendekatan penapisan median pensuisan. Kaedah ini secara amnya boleh dibahagikan kepada dua peringkat utama, iaitu peringkat pengesanan hingar dedenyut dan peringkat pembatalan hingar. Pengubahsuaian terhadap kaedah pengesanan hingar berdiskriminasikan sempadan (BDND) telah dibuat. Pertamanya, tanpa menggunakan sebarang algoritma pengisihan, nilai-nilai median setempat ditentukan daripada histogram tempatan yang dimanipulasi. Seterusnya, diperingkat pengesanan hingar, disamping pendekatan pembezaan jarak keamatan yang asal, kaedah baharu turut menggunakan pendekatan pembezaan tinggi keamatan bagi mengurangkan kadar pengesanan palsu. Kemudian, tanpa menggunakan pendekatan penyesuaian untuk peringkat pembatalan hingar, kaedah yang dicadangkan menggunakan pendekatan lelaran. Model hingar dedenyut lebar telah digunakan untuk proses penilaian, bagi menyiasat keteguhan kaedah. Berdasarkan penilaian dari segi punca kuasa perbezaan purata (RMSE), kadar pengesanan positif palsu, kadar pengesanan negatif palsu, indek purata kesamaan struktur (MSSIM), masa pemprosesan, dan pemeriksaan visual, menunjukkan bahawa kaedah yang dicadangkan adalah kaedah terbaik apabila dibandingkan dengan tujuh kaedah penapisan median terkini yang lain.

IMPROVED SWITCHING-BASED MEDIAN FILTER FOR IMPULSE NOISE REMOVAL

ABSTRACT

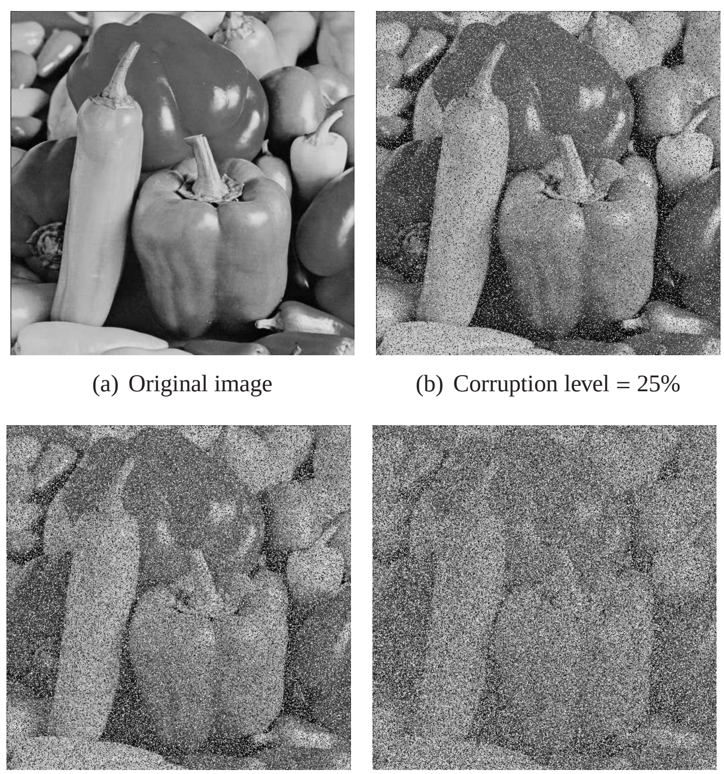
This thesis proposed a new algorithm to reduce impulse noise from digital images. In order to achieve this, thorough literature surveys on impulse noise models and median filtering frameworks have been carried out successfully. The proposed algorithm is based on switching median filtering approaches. The method can be generally divided into two main stages, which are impulse noise detection stage and impulse noise cancellation stage. Modifications towards a well known boundary discriminative detection (BDND) method have been made. First, rather than using any sorting algorithm, the local median values were determined from manipulated local histograms. Next, in the noise detection stage, in addition to the originally proposed intensity distance differential approach, the new method includes intensity height differential approach to reduce false detection rate. Then, instead of using adaptive approach for noise cancellation stage, the proposed method utilizes iterative approach. Broad impulse noise model has been employed for the evaluation process, to investigate the robustness of the method. Based on the evaluations from root mean square error (RMSE), false positive detection rate, false negative detection rate, mean structure similarity index (MSSIM), processing time, and visual inspection, it is shown that the proposed method is the best method when compared with seven other state-of-the art median filtering methods.

CHAPTER 1

INTRODUCTION

1.1 Overview

Similar to other digital signal, digital images sometime could be corrupted by noise. One of the noise types normally related to digital image is impulse noise. Impulse noise appears as a sprinkle of bright or dark spots on the image, and normally these spots have relatively high contrast towards their surrounding areas [1, 2]. An example is shown in Figure 1.1. As shown by this figure, even at low corruption level, impulse noise can significantly degrade the appearance and quality of the image.



(a) Original image

(b) Corruption level = 25%

(c) Corruption level = 50%

(d) Corruption level = 75%

Figure 1.1: Example of image corrupted by impulse noise

There are many causes that can contribute to impulse noise. Impulse noise can be generated when the digital image is transmitted over a noisy channels, especially in the over-the-air transmission channels. These channels include standard broadcasting and satellite communication [3, 4]. Impulse noise also can be resulted from faulty pixels on image sensor array, malfunctioning memory location, and timing errors in analog-to-digital conversion [5–7]. Other sources of impulse noise include lightning, industrial machines, faulty or dusty insulation of high-voltage powerlines and various unprotected electric switches [4].

Impulse noise not only degrades the appearance of the image, it can also significantly affect the results of image segmentation, feature extraction, edge detection and object recognition. Therefore, it is very essential to restore the corrupted digital images before they are supplied to any automated computer-vision based system [8, 9]. One of the popular methods used to deal with impulse noise is standard median filter (SMF)* [10, 11].

1.2 Problem Statements

There are a few disadvantages of standard median filter (SMF):

- SMF does not differentiate between uncorrupted pixel from corrupted pixel. Therefore, even the pixels are uncorrupted, SMF still changing their intensity level. Therefore, SMF introduces another distortion to the image.
- Most of the implementations of SMF are employing sorting algorithm to determine the median value. As SMF works by using local window, and the median value need to be found at every pixel position, sorting algorithm makes SMF relatively computational expensive, especially when the size of filter is large.
- Although there are many variations of median filter have been proposed, these filters

*Description on SMF will be given in Section 2.2.1.

focus only on one type of impulse noise which is fixed value impulse noise. Therefore, a general impulse noise detector which can detect the noise accurately is needed.

1.3 Objective of the Research

The key objective of this research is to improve median filter based method for the removal of impulse noise. In order to achieve this objective, several goals have been set. They are:

1. To develop a new robust impulse noise detection method which will be able to cater a wide range of impulse noise models.
2. To improve the performance of impulse noise cancellation algorithm.

1.4 Scope of the Research

The research carried out in this thesis has been limited to the following scopes:

1. The research only deals with impulse noise reduction technique. Other types of noise, such as Gaussian noise, and Rician noise, will not be covered by this thesis.
2. The thesis studies on impulse noise reduction methods related to median filter only. Just spatial domain-based methods are implemented for comparison purpose.
3. The input image for the system is limited to the natural images (e.g. scanned photograph, or pictures taken by optical camera). The research not cover images from other imaging modalities, such as computed tomography, or satellite images.
4. This research concentrates on restoration of grayscale images, which can be considered as a 2D signal data. Higher dimensional data, such as color images, or video, is not of the research interest.

1.5 Summary of Contributions

The contributions from this thesis can be generally divided into two parts, which are:

1. A robust impulse noise detector: A height and distance decision making technique has been utilized in boundary discriminative based impulse noise detection method.

2. A better impulse noise cancellation method: A modification towards the originally proposed noise cancellation stage in boundary discriminative noise detection method has been proposed.

1.6 Organization of Thesis

This thesis is divided into five chapters. Chapter 1 gives an overview of the work done in this research. Next, Chapter 2 presents a literature review on impulse noise models and methods related to median filter. The proposed method is then described in Chapter 3. Following that, the results and discussions are presented in Chapter 4. In this chapter, the performance of the proposed filter has been benchmarked with several other median based filters, and the results are evaluated based on some quality measures. Finally, Chapter 5 concludes the findings, and give some ideas for future work.

CHAPTER 2

LITERATURE REVIEW

This chapter is divided into three sections. First, in order to understand the nature of impulse noise, a survey on impulse noise models has been carried out. From this survey, it is found that impulse noise can be defined in several ways. Therefore, these mathematical formulas, which are used by researchers to present impulse noise, are presented in Section 2.1. From the survey, it is also found that there are thousands of median filter variations currently available in literature. Therefore, Section 2.2 categorized these filters into eight common median filter variations. Some examples of the recent median filtering technologies are presented in the last section, which is Section 2.3.

2.1 Impulse Noise Models

Impulse noise is considered as an additive noise [10–16]. The additive noise is defined as follows. If (i, j) are the spatial coordinates on the image, and $\mathbf{C} = \{C(i, j)\}$ is the ideal uncorrupted and clean image, the damaged image by additive noise $\mathbf{D} = \{D(i, j)\}$ can be described as:

$$\mathbf{D} = \begin{cases} \mathbf{C} & : \text{ with probability } 1 - P \\ \mathbf{C} + \mathbf{N} & : \text{ with probability } P \end{cases} \tag{2.1}$$

where P (i.e. $0 \leq P \leq 1$) presents the noise density, and $\mathbf{N}$ is the noise intensity value or the noise amplitude. In this Equation, $\mathbf{N}$ can have a positive or a negative value.

Impulse noise can be characterized by a long-tail probability distribution* of N, and thus impulse noise can be considered as an additive long-tailed noise [17–20]. The probability density function p for impulse noise can be modelled by some skewed distributions. Furutsu and Ishida in 1961 [17] used a combinations of Poisson distributions to present impulse noise. In 1988, Lin and Willson Jr [18] presented impulse noise by using a log-normal distribution as given as:

$$p(|N|) = \frac{1}{\sqrt{2\pi|N|}} \exp\left(-\frac{1}{2}\ln^2\left(\frac{|N|\exp(0.5)}{|\overline{N}|}\right)\right) \tag{2.2}$$

where $|\overline{N}|$ is the average value of $|N|$. However, the popularity of impulse noise models that are described by skewed probability distributions, such as the work by [17] and [18], are decreasing in recent research papers.

From Equation (2.1), for the case of the corrupted pixels, the results of $\mathbf{C}+\mathbf{N}$ actually can take any value because $\mathbf{N}$ is a random value. Therefore, in recent literatures, Equation (2.1) has been simplified and replaced with Equation (2.3)† [21–23].

$$\mathbf{D} = \begin{cases} \mathbf{C} & : \quad \text{with probability } 1-P \\ \mathbf{N} & : \quad \text{with probability } P \end{cases} \tag{2.3}$$

This Equation shows that for most of the current impulse noise models, the corrupted pixels are directly replaced with the noise intensity values [24–28]. Unlike Equation (2.1), in this Equation, the value of $\mathbf{N}$ is restricted to positive values only. If the image is quantized into L intensity levels, $\mathbf{N}$ can take any values between 0 to $L-1$ [25].

The distribution p of noise $\mathbf{N}$ in Equation (2.3) can be defined in many ways. Some re-

*A long-tailed probability distribution is a distribution which has relatively high probability regions far from the mean or median values.

†Due to this simplification, in some literatures, impulse noise is neither considered as an additive noise nor a multiplicative noise, but as another class of its own.

searchers define p as a uniform distribution, which is:

$$p(N) = P/L \qquad 0 \le N \le L-1 \tag{2.4}$$

For this case, the noise amplitudes occupy all possible intensity values, from 0 to $L-1$. This type of impulse noise is widely known as random-valued impulse noise. Random-valued impulse noise have been studied in many works, such as [5, 25, 26, 29–34].

Other widely used practical impulse noise model is known as fixed-valued impulse noise. In this model, $\mathbf{N}$ in Equation (2.3) is restricted to the minimum or the maximum intensity value (i.e. 0 or $L-1$) [‡]. As the noise with intensity 0 appears as black pixels on the image, this noise is referred as pepper noise. On the other hand, the noise with intensity $L-1$ appears as white pixels on the image. This type of noise if referred as salt noise. Therefore, fixed-valued impulse noise is also known as salt-and-pepper noise [§]. If D is the intensity of image $\mathbf{D}$, fixed-valued impulse noise is given by the following probability density function:

$$p(D) = \begin{cases} \frac{1}{2}P & : \quad pepper; D = 0 \\ 1-P & : \quad \text{noise free pixels}; 0 \le D \le L-1 \\ \frac{1}{2}P & : \quad salt; D = L-1 \end{cases} \tag{2.5}$$

Because of its simplicity and practicality, this noise model is the most popular impulse noise model used in literatures, such as the works in [35–56].

A simple modification can be done to Equation (2.5) by allowing unequal densities of salt

[‡]It is easier to understand the idea behind this noise model by using equation (2.1). In this equation, impulse noise N can take positive or negative value. It is assumed that the magnitude of N is very large, such that $C(i, j) + N(i, j)$ will produce values either greater than $L-1$ or lower than 0. Due to quantization process, these values are truncated to $L-1$ or 0 [11].

[§]Sometimes, this type of noise also been referred as data-drop-out or spike noise [11].

noise and pepper noise, as given as:

$$p(D) = \begin{cases} P_1 & : \quad pepper; D = 0 \\ 1 - P & : \quad \text{noise free pixels}; 0 \leq D \leq L - 1 \\ P_2 & : \quad salt; D = L - 1 \end{cases} \tag{2.6}$$

where $P_1 + P_2 = P$. This noise model has been used in some recent literatures, such as [34, 49–56]. This type of noise is called unipolar when either P_1 or P_2 is zero [11].

A variation to Equation (2.6) is obtained by allowing salt noise and pepper noise to be presented by two intensity ranges. Salt noise occupies high intensity range, while pepper noise occupies low intensity range. Each range is presented by m intensity levels. This noise model is given as:

$$p(N) = \begin{cases} P_1/m & : \quad pepper; 0 \leq N < m \\ 1 - P & : \quad \text{noise free pixels}; 0 \leq N \leq L - 1 \\ P_2/m & : \quad salt; L - 1 - m < N \leq L - 1 \end{cases} \tag{2.7}$$

Some example of works that are using this noise model can be found in [34, 52–56].

Equation (2.7) can be simplified by assuming that the density of salt noise is equal to the density of pepper noise. This is given as:

$$p(D) = \begin{cases} P/2m & : \quad pepper; 0 \leq D < m \\ 1 - P & : \quad \text{noise free pixels}; 0 \leq D \leq L - 1 \\ P/2m & : \quad salt; L - 1 - m < D \leq L - 1 \end{cases} \tag{2.8}$$

Actually, when m is equal to one, this equation is equivalent to Equation (2.5), which is fixed-valued impulse noise. On the other hand, when m is equal to $L/2$, this model resembles Equation (2.4), which is random-valued impulse noise. This noise model has been used in [52–56].

Universal impulse noise, or also known as mixed impulse noise, is given as a combination between random-valued impulse noise as defined by Equation (2.4), with fixed-valued impulse noise as defined by Equation (2.5). In this model, the contamination of the image is equally contributed by these two well-known impulse noise models, which is $\frac{1}{2}P$ random-valued impulse noise and $\frac{1}{2}P$ fixed-valued impulse noise. Some works related to universal impulse noise can be found in [57–59].

2.2 Median Filter

A popular solution to deal with impulse noise is by using rank-order filters, or also known as order-statistic filters. This type of filters is nonlinear and works in spatial domain. It uses sliding window approach, where at each sliding iteration, only the value of the pixel corresponds to the center of the window is changed. This value is obtained based on the ordered intensity values of the pixels contained in the area defined by the filtering window [10, 11].

Among these rank-order filters, median based filters are one of the techniques to reduce both bipolar and unipolar impulse noise [10, 11]. Generally, median filter uses median value in its filtering process. The median value $\tilde{X}$ of a sample is defined as [60]:

$$\tilde{X} = \begin{cases} X_{(n_s+1)/2} & : \ n_s \text{ is odd} \\ 0.5\left(X_{n_s/2} + X_{(n_s+1)/2}\right) & : \ n_s \text{ is even} \end{cases} \tag{2.9}$$

where $X_1, X_2, \ldots, X_{n_s}$ are the intensity values, arranged in either increasing or decreasing order, and n_s is the size of the sample. However, there are a lot of median filter variations. Therefore, this section reviews some of the median filter techniques.

2.2.1 Standard Median Filter (SMF)

Standard median filter (SMF), or also known as median smoother, has been introduced by Tukey in 1971 [19]. The filtered image $\mathbf{F} = \{F(i,j)\}$ from SMF can be defined by the following equation [11, 61]:

$$F(i,j) = \text{median}_{(k,l)\in W_{h,w}}\{D(i+k, j+l)\} \tag{2.10}$$

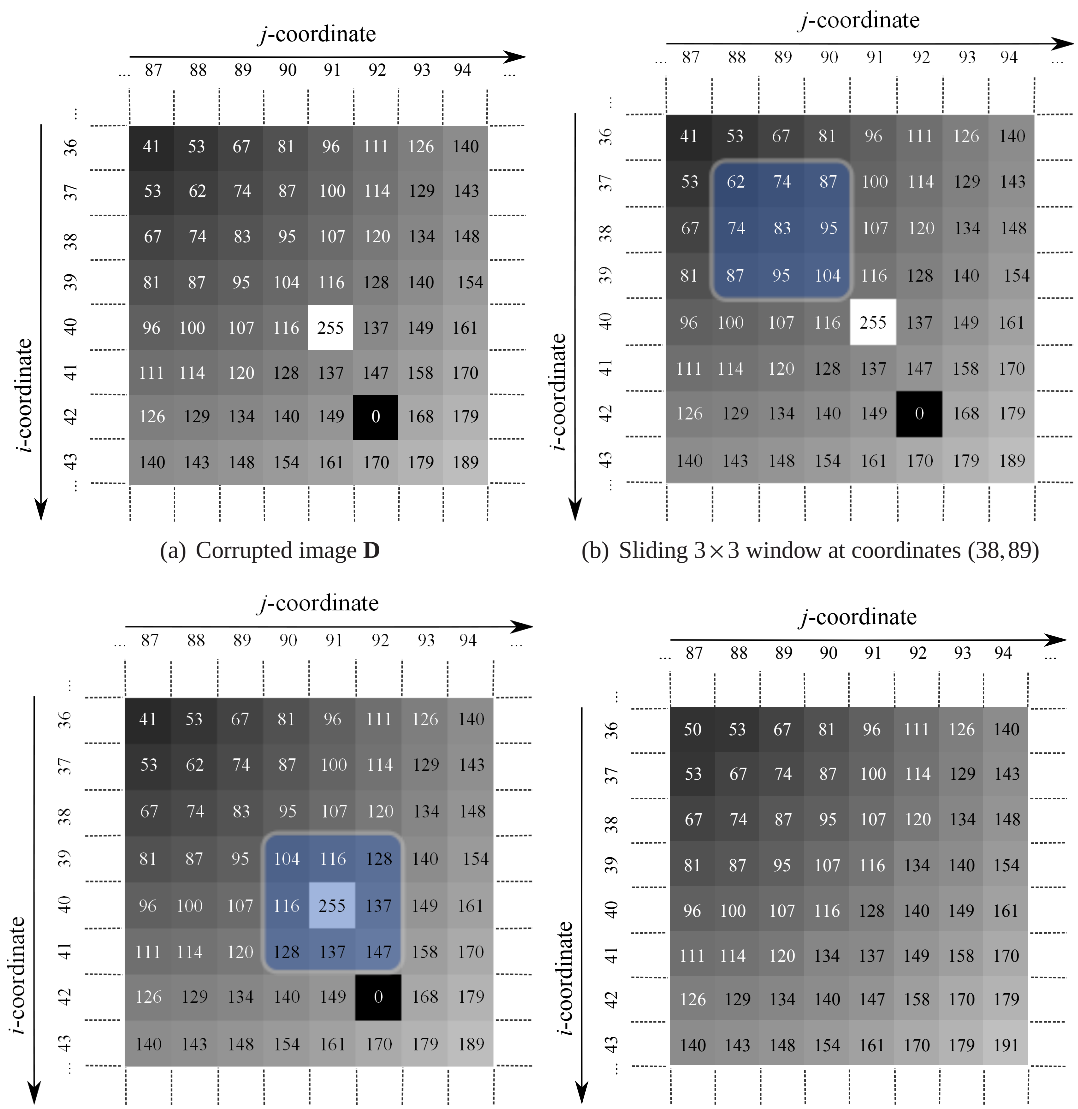
where $W_{h,w}$ is a sliding window of size $h\times w$ pixels centered at coordinates (i,j). The median value is calculated by using Equation (2.9) with $n_s = w\times h$.

In order to ease the explanation regarding to the operations in SMF, one simple example is given in Figure 2.1. Figure 2.1(a) shows an 8×8 portion of the damaged image $\mathbf{D}$. In this example, the pixels at locations (40,91) and (42,92) are considered damaged by impulse noise (i.e. salt-and-pepper noise). The image is then filtered by SMF, utilizing $W_{3,3}$, which is a sliding window of size 3×3 pixels. The origin of the filter , with coordinates $(0,0)$, is located at the centre of the window. The sliding window moves in raster fashion, starting from the top left pixel towards the bottom right pixel. At each sliding window position, the corresponding intensity of the output image $F(i,j)$ is calculated by using Equation (2.10). For example, when the centre of the filter is located at coordinates (38,89) as shown in Figure 2.1(b), $F(38,89)$ is determined as:

$$\begin{aligned} F(38,89) &= \text{median}_{(k,l)\in W_{3,3}}\{D(38+k, 89+l)\} \\ &= \text{median}\{62,74,87,74,83,95,87,95,104\} \\ &= \text{median}\{62,74,74,83,\boxed{87},87,95,95,104\} \\ &= 87 \end{aligned} \tag{2.11}$$

Similarly, for Figure 2.1(c):

$$
\begin{aligned}
F(40,91) &= \text{median}_{(k,l)\in W_{3,3}}\{D(40+k,91+l)\} &\quad (2.12)\\
&= \text{median}\{104,116,128,116,255,137,128,137,147\}\\
&= \text{median}\{104,116,116,128,\boxed{128},137,137,147,255\}\\
&= 128
\end{aligned}
$$



(a) Corrupted image **D**

(b) Sliding 3×3 window at coordinates (38,89)

(c) Sliding 3×3 window at coordinates (40,91)

(d) Restored image **F**

Figure 2.1: Example of restoring corrupted image using SMF

The corresponding output image portion is shown in Figure 2.1(d). In this figure, salt-and-pepper noise has successfully been removed . However, several uncorrupted intensity values (i.e. at positions $(36,87)$, $(37,88)$, $(38,89)$, $(39,90)$, $(39,92)$, $(40,92)$, $(41,90)$, $(41,92)$, $(42,91)$, $(42,93)$ and $(43,94)$) are also altered by SMF. This undesired situation happens because SMF does not differentiate between uncorrupted from corrupted pixels. Besides, large filter of SMF will introduce a significant distortion into the image [62].

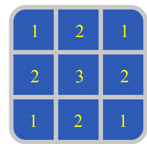
It is worth noting that Equation (2.9) is normally using sorting algorithm such as quick-sort or bubble-sort to arrange the samples in increasing or decreasing order. Even though sorting algorithm can be easily implemented, sorting procedure requires long computational time when $W_{h,w}$ is a large filter because the number of samples (i.e. $n_s = w \times h$) is big [61]. Thus, in order to avoid from using any direct sorting algorithm, the use of local histograms has been proposed for median value calculation. The time required to form local histogram can be reduced by using a method proposed by Huang et al. [63], where instead of updating $h \times w$ samples, only $2h$ samples need to be updated in each sliding iteration.

2.2.2 Weighted Median Filter (WMF)

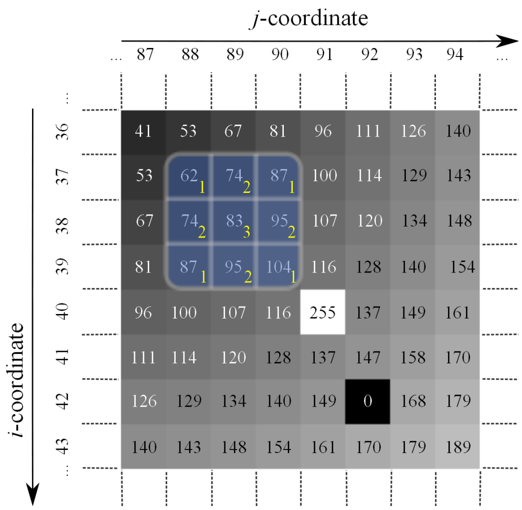
One of the branches of median filter is weighted median filter (WMF). WMF was first introduced by Justusson in 1981 [64], and further elaborated by Brownrigg [65]. The operations involved in WMF are similar to SMF, except that WMF has weight associated with each of its filter element. These weights correspond to the number of sample duplications for the calculation of median value. The filtered image $\mathbf{F} = \{F(i, j)\}$ from WMF can be defined by the following equation [5, 40, 62, 66]:

$$F(i, j) = \text{median}_{(k,l)\in W_{h,w}}\{W_{h,w}(k,l) \diamond D(i+k, j+l)\} \tag{2.13}$$

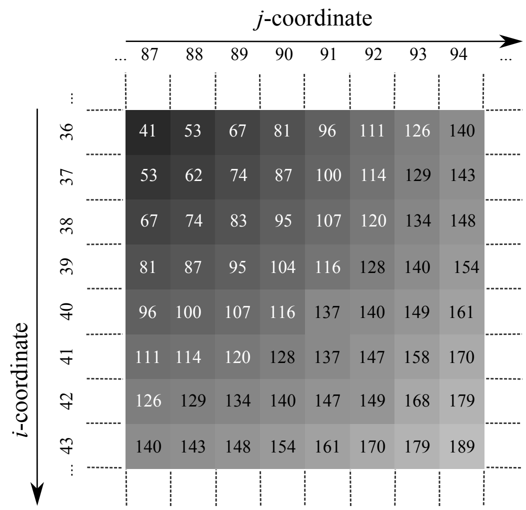
where operator $\diamond$ indicates repetition operation. The median value is calculated using Equation (2.9) with $n_s = \sum W_{h,w}(k,l)$. Normally, the filter weight $W_{h,w}(j,k)$ is set such that it will decrease when it is located away from the centre of the filtering window. By doing so, it is expected that the filter will give more emphasis to the central pixel, and thus improve the noise suppression ability while maintaining image details [66–69].



(a) Weights



(b) Sliding 3×3 window at coordinates $(38, 89)$



(c) Restored image **F**

Figure 2.2: Example of restoring corrupted image using WMF

Figure 2.2 presents an example of an image processed by using WMF. In this example, the same corrupted input image as shown in Figure 2.1(a) is used. This image is processed by WMF of size 3×3 pixels, with the filter's weights or coefficients are given by Figure 2.2(a). In this example, the weight gives more emphasis to the centre pixel. The corresponding output pixel, $F(i, j)$ is found by using Equation (2.13). For example, when the window is located at coordinates $(38,89)$, as shown in Figure 2.2(b), the corresponding output intensity is:

$$
\begin{aligned}
F(38,89) &= \text{median}_{(k,l)\in W_{3,3}}\{W_{3,3}(k,l) \diamond D(38+k,89+l)\} \qquad (2.14)\\
&= \text{median}\{1 \diamond 62, 2 \diamond 74, 1 \diamond 87, 2 \diamond 74, 3 \diamond 83, 2 \diamond 95, 1 \diamond 87, 2 \diamond 95, 1 \diamond 104\}\\
&= \text{median}\{62, \underline{74,74}, 87, \underline{74,74}, \underline{83,83,83}, \underline{95,95}, 87, \underline{95,95}, 104\}\\
&= \text{median}\{62,74,74,74,74,83,83,\boxed{83},87,87,95,95,95,95,104\}\\
&= 83
\end{aligned}
$$

The corresponding output image is shown in Figure 2.2(c). Compared with the related result from SMF shown in Figure 2.1(d), both SMF and WMF successfully remove the impule noise (i.e. salt-and-pepper noise) from the corrupted image. Yet, unlike SMF, WMF does not change most of the uncorrupted pixels. Therefore this filter is better in preserving image details. As can be seen in Figure 2.2(c), only uncorrupted pixels at locations (40,92) and (42,91) are modified by WMF. However, the successfulness of weighted median filter in preserving image details is highly dependent on the weighting coefficients, and the nature of the input image itself. Unfortunately, in practical situations, it is difficult to find the suitable weighting coefficients for this filter, and this filter requires high computational time when the weights are large [70–72].

Some researchers [62, 73], proposed adaptive weighted median filters (AWMF), which is an extension to WMF. By using a fixed filter size $W_{h,w}$, the weights of the filter will be adapted accordingly base on the local noise content. This adaptation can be done in many ways, mostly based on the local statistics of the damaged image. For example, in [73], the weights of the filter are defined as:

$$W_{h,w}(j,k) = \left\langle W_{h,w}(0,0) - c\left(\frac{\sigma^2 d}{\bar{x}}\right)\right\rangle \tag{2.15}$$

where $W_{h,w}(0,0)$ is a preset weight for the central filter element, c is a preset scaling factor, d is the distance of location (j,k) to coordinates $(0,0)$, and σ^2 and $\bar{x}$ are the local variance and local mean, respectively, defined by a sliding window of size $h \times w$. The operator $< . >$ presents the rounding operation if the argument inside it is a positive value, otherwise it will truncate the value to zero.

Centre weighted median filter (CWMF) is a special type of WMF. CWMF has the weights defined as follow:

$$W_{h,w}(k,l) = \begin{cases} n_w & : \quad (k,l) = (0,0) \\ 1 & : \quad \text{otherwise} \end{cases} \tag{2.16}$$

where n_w is an odd integer, with value greater or equal to one. Coordinates $(k,l) = (0,0)$ presents the centre of the filter. When n_w is set to one, CWMF becomes SMF. Large value of n_w is good in preserving details but worse in noise cancellation. When n_w is greater or equal to $h \times w$ (i.e. the area covered by filter $W_{h,w}$), CWMF turns into the identity filter. In this condition, CWMF does not filter the image, and thus the output image will become exactly the same as its corresponding input [68].

2.2.3 Iterative Median Filter

Several impulse noise filtering methods require iterative filtering procedure in their implementation [29, 42, 74–76]. Iterative method requires the same procedure to be repeated several times. In general, iterative median filter with n_i iterations, requires $n_i - 1$ temporary images **T** (i.e. $\mathbf{T}_1, \mathbf{T}_2, \ldots \mathbf{T}_{n_i-1}$). Iterative SMF can be defined by Equation (2.17) and depicted by Figure 2.3. Iteration procedure enables median filtering process to use smaller filter size and reduce the computational time, while maintaining local features or edges of the image.

$$\begin{aligned} T_1(i,j) &= \text{median}_{(k,l)\in W_{h,w}}\{D(i+k, j+l)\} \\ T_2(i,j) &= \text{median}_{(k,l)\in W_{h,w}}\{T_1(i+k, j+l)\} \\ &\vdots \\ F(i,j) &= \text{median}_{(k,l)\in W_{h,w}}\{T_{n_i-1}(i+k, j+l)\} \end{aligned} \tag{2.17}$$

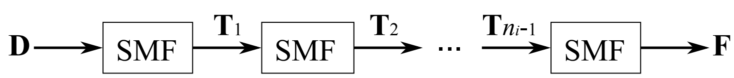
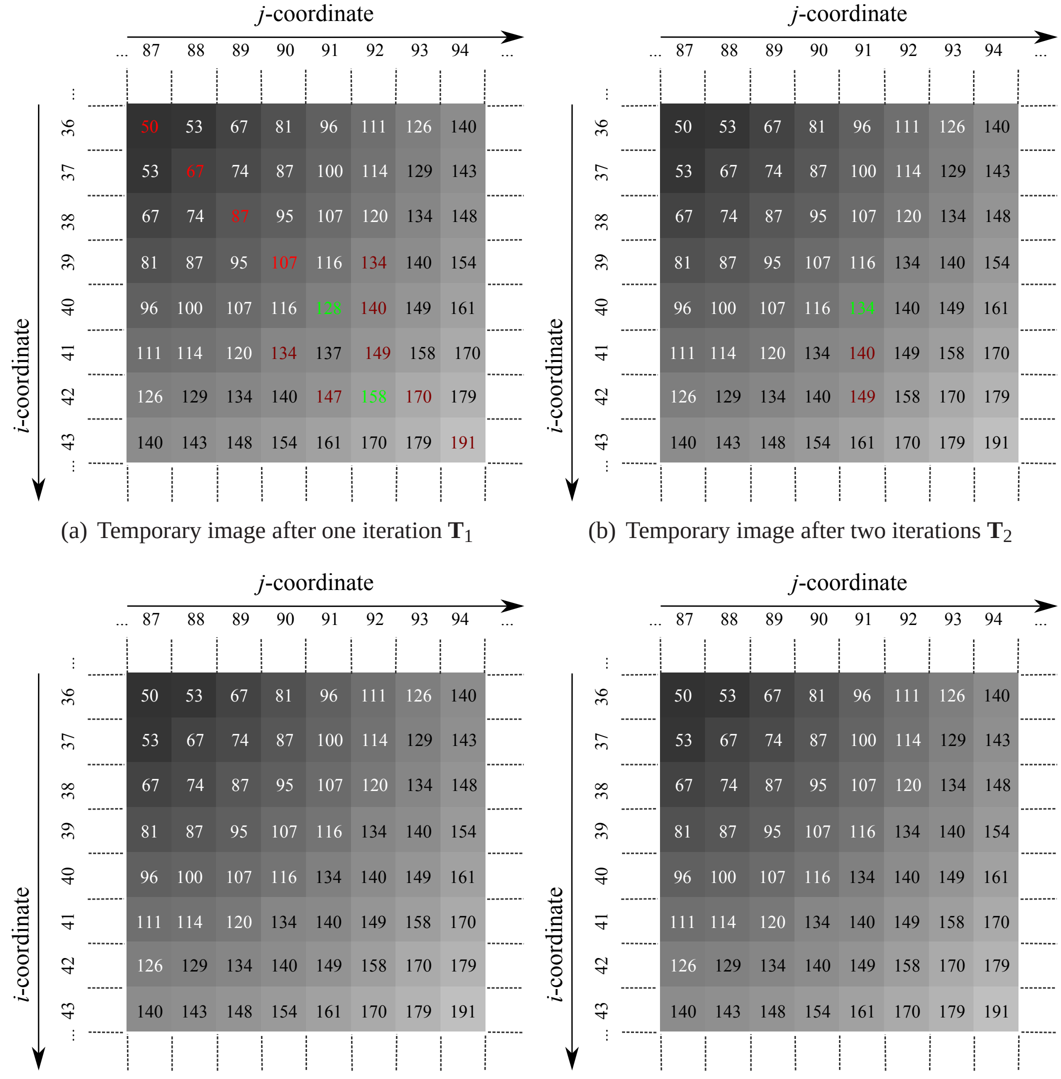


Figure 2.3: Block diagram presenting iterative median filter

The number of iterations n_i can be set by the user, or the iteration process stops when the output image converged (i.e. the current output image is equal to the previous output image). As an example, by taking the image shown in Figure 2.1(a) as the input image, the temporary images $\mathbf{T}$ from iterative median filter, which uses a sliding window of size 3×3 pixels $W_{3,3}$, are shown in Figure 2.4. The colour numbers in this figure indicate the intensity value of the pixels that have been changed with respect to the previous iteration. Thus, in this example, the processed image is already converged at the second iterations. Therefore, the output at the second iteration can be taken as the final output image. However, in practical, the number of iterations needed is dependent to the level of corruption and the nature of the input image itself.



(a) Temporary image after one iteration $\mathbf{T}_1$

(b) Temporary image after two iterations $\mathbf{T}_2$

(c) Temporary image after three iterations $\mathbf{T}_3$

(d) Temporary image after four iterations $\mathbf{T}_4$

Figure 2.4: Example of restoring corrupted image using iterative median filter

2.2.4 Recursive Median Filter

Several researches in median filtering [72, 77–81], use recursive approach in their methodology. Theoretically, recursive median filters can be considered analogous to infinite impulse response (IIR) filter because their outputs at certain position are determined not only from the input intensities, but also from the calculated outputs at previous locations. In implementation of recursive median filter, normally the degraded image and the filtered image share the same data array.

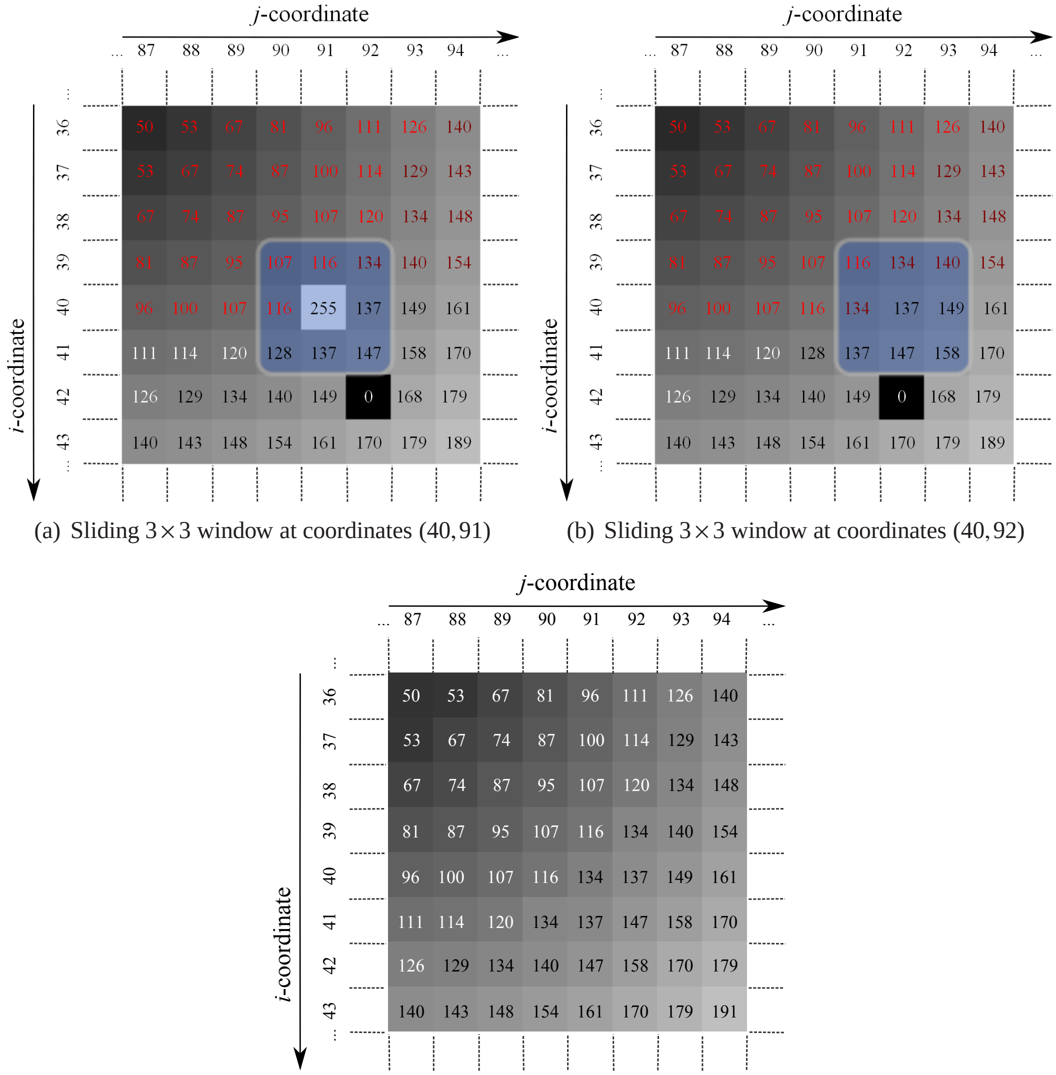
An example of image processed by recursive median filter of size 3×3 pixels, $W_{3,3}$, is shown in Figure 2.5. The corrupted image, which is the input for this filter, is shown in Figure 2.1(a). The coloured values in Figures 2.5(a) and (b) present the already processed pixels, which have been updated into the input image. For example, Figure 2.5(a) shows the filter when it is at coordinates $(40,91)$. At this position, the median value is calculated as:

$$\begin{aligned} F(40,91) &= \text{median}_{(k,l)\in W_{3,3}}\{D(40+k,91+l)\} \\ &= \text{median}\{107,116,134,116,255,137,128,137,147\} \\ &= \text{median}\{107,116,116,128,\boxed{134},137,137,147,255\} \\ &= 134 \end{aligned} \tag{2.18}$$

This obtained value is then updated directly into the input image. Therefore, the calculation of the median value at coordinates $(40,92)$ as shown in Figure 2.5(b) is:

$$\begin{aligned} F(40,92) &= \text{median}_{(k,l)\in W_{3,3}}\{D(40+k,92+l)\} \\ &= \text{median}\{116,134,140,134,137,149,137,147,158\} \\ &= \text{median}\{116,134,134,137,\boxed{137},140,147,149,158\} \\ &= 137 \end{aligned} \tag{2.19}$$

In this method, the already processed pixels are now considered as noise free input pixels. Thus, by replacing the input pixels with these values, it assumes that the median value calculation will be more accurate. However, if the filter fails to remove the noise at previous locations, the error might be propagated to other area of the image. Furthermore, it is worth noting that the result from recursive median filter is dependent to the direction of filtering.



(a) Sliding 3×3 window at coordinates $(40, 91)$

(b) Sliding 3×3 window at coordinates $(40, 92)$

(c) Restored image **F**

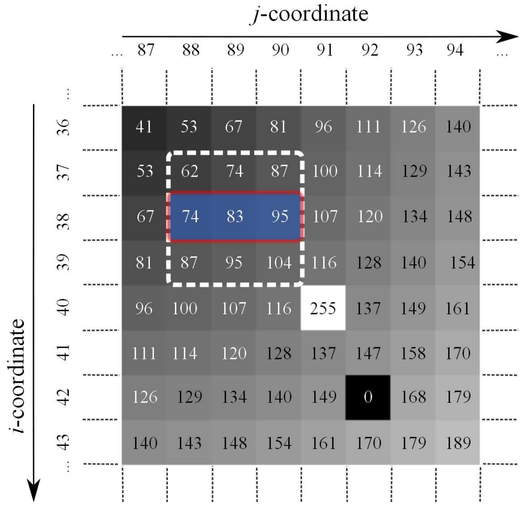
Figure 2.5: Example of restoring corrupted image using recursive median filter

2.2.5 Directional Median Filter

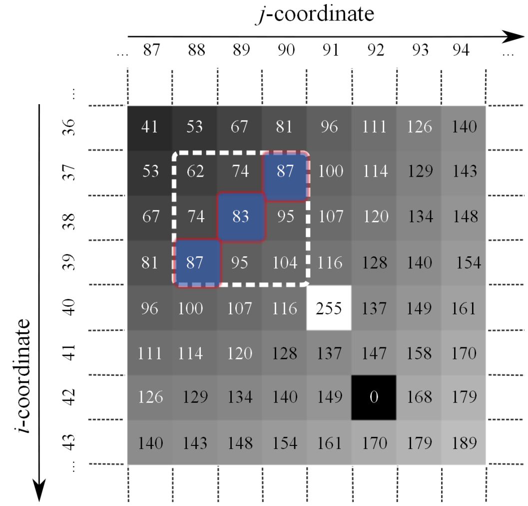
Directional median filter, or also known as stick median filter, works by separating its 2-D filter into several 1-D filter components [5, 53, 54, 59, 82]. Each filter component or stick, presented as a straight line, corresponds to a certain direction or angle θ. For a window of size $h \times w$ pixels, there are $h + w - 2$ sticks that will be used. The computed median values from these 1-D filters are then combined to obtain the final result. In [82], the output intensity is defined as:

$$F(i, j) = \max\{\text{median}_{(k,l)\in W_\theta}\{D(i+k, j+l)\}\} \tag{2.20}$$

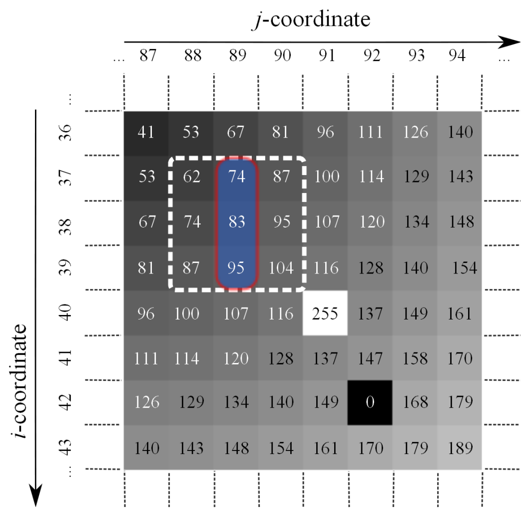
where W_θ is the stick. Here, the output intensity is defined as the largest median value determined at each location.



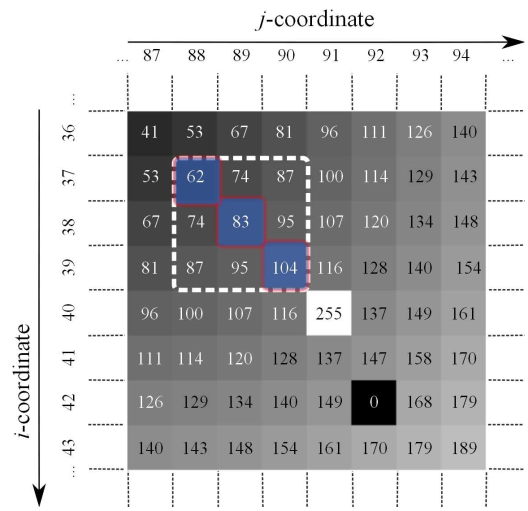
(a) Stick W_{0° at coordinates (38,89)



(b) Stick W_{45° at coordinates (38,89)



(c) Stick W_{90° at coordinates (38,89)



(d) Stick W_{135° at coordinates (38,89)

Figure 2.6: Example of restoring corrupted image using directional median filter

Figure 2.6 shows an example of the operation involved in calculating the output intensity at locations (38,89), by taking Figure 2.1(a) as the damaged image. The size of the filter used is 3×3, therefore there are $(3+3-2) = 4$ individual sticks are used. These sticks correspond to angle $\theta = 0^\circ, 45^\circ, 90^\circ$, and 135°. The output intensity at this position, $F(38,89)$ is calculated as follow:

$$
\begin{aligned}
F(38,89) &= \max\Big\{\text{median}_{(k,l)\in W_{0^\circ}}\{D(38+k,89+l),\text{median}_{(k,l)\in W_{45^\circ}}\{D(38+k,89+l),\\
&\quad \text{median}_{(k,l)\in W_{90^\circ}}\{D(38+k,89+l),\text{median}_{(k,l)\in W_{135^\circ}}\{D(38+k,89+l)\}\Big\} \qquad (2.21)\\
&= \max\{\text{median}\{74,83,95\},\text{median}\{87,83,87\},\text{median}\{95,83,74\},\\
&\quad \text{median}\{104,83,62\}\}\\
&= \max\Big\{\text{median}\{74,\boxed{83},95\},\text{median}\{83,\boxed{87},87\},\text{median}\{74,\boxed{83},95\},\\
&\quad \text{median}\{62,\boxed{83},104\}\Big\}\\
&= \max\{83,87,83,83\}\\
&= 87
\end{aligned}
$$

2.2.6 Switching Median Filter

Nowadays, one of the popular median filtering approaches is switching median filter, or also known as decision based median filter [4, 20, 25, 26, 30, 32, 34–44, 46, 47, 50–52]. Switching median filter tries to minimize the undesired alteration of uncorrupted pixels by the filter. Therefore, in order to overcome this problem, switching median filter checks each input pixel whether it has been corrupted by impulse noise or not. Then it changes only the intensity of noisy pixel candidates, while left the other pixels unchanged. Normally, switching median filter is built from two stages, as shown in Figure 2.7. The first stage is for noise detection, while the second stage is for noise cancellation.

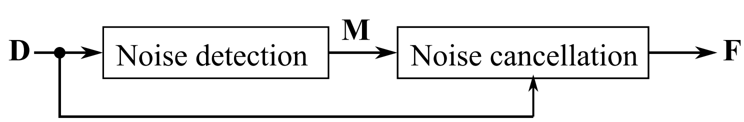
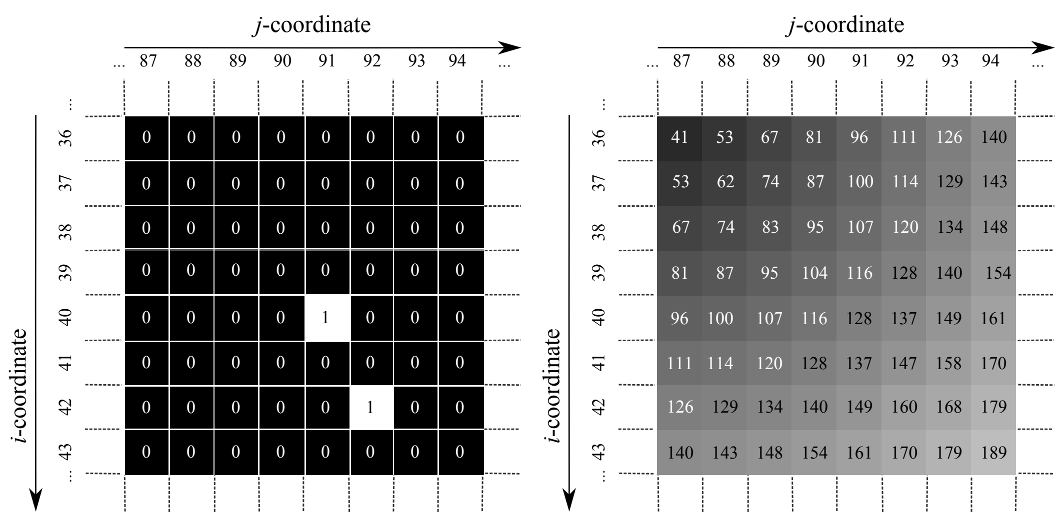


Figure 2.7: Block diagram presenting switching median filter

The output from the noise detection stage is a noise mask **M**. This mask is a binary mask, and normally defined as follow:

$$M(i,j) = \begin{cases} 1 & : \quad \text{impulse noise candidate} \\ 0 & : \quad \text{otherwise} \end{cases} \tag{2.22}$$

Noise detection procedure used by researchers are normally depending on the noise model been used. For fixed-valued impulse noise (i.e. salt-and-pepper noise), mostly the noise detection is done by thresholding the intensity values of the damaged image. Other popular noise detection methods include by checking the difference between intensity of the current pixel with its surrounding, inspecting the difference of the damaged image with its median filtered versions, or by applying some special filter. Next, mask **M** will be used in the noise cancellation stage.



(a) Noise mask **M** (b) Restored image **F**

Figure 2.8: Example of restoring corrupted image using switching median filter

In noise cancellation stage, only pixels with $M = 1$ are processed by the median filter. For the calculation of median, only "noise-free" pixels (i.e. pixels with $M = 0$) are taken as the sample. This can be defined as:

$$F(i,j) = \text{median}_{(k,l)\in W_{h,w};M(i+k,j+l)=0}\{D(i+k, j+l)\} \tag{2.23}$$

Figure 2.8 shows an example for this procedure. In this example, the damaged image is already shown in Figure 2.1(a). Let assume that Figure 2.8(a) shows a portion of mask **M**. In this figure, only pixels at positions (40,91) and (42,92) are detected as impulse noise from the noise detection stage. Therefore, only this two pixels are processed in the noise cancellation stage. For example, at coordinates (40,91), the output intensity calculated by this stage, by using a window of size 3×3 pixels $W_{3,3}$, is:

$$\begin{aligned} F(40,91) &= \text{median}_{(k,l)\in W_{h,w};M(40+k,91+l)=0}\{D(40+k,91+l)\} \qquad (2.24)\\ &= \text{median}\{104,116,128,116,137,128,137,147\}\\ &= \text{median}\{104,116,116,\boxed{128,\ 128},137,137,147\}\\ &= (128+128)/2 = 128 \end{aligned}$$

Similarly, at coordinates (42,92):

$$\begin{aligned} F(42,92) &= \text{median}_{(k,l)\in W_{h,w};M(42+k,92+l)=0}\{D(42+k,92+l)\} \qquad (2.25)\\ &= \text{median}\{137,147,158,149,168,161,170,179\}\\ &= \text{median}\{137,147,149,\boxed{158,\ 161},168,170,179\}\\ &= (158+161)/2 = 160 \end{aligned}$$

2.2.7 Adaptive Median Filter

Actually, the concentration of impulse noise on an image is varied because impulse noise is a random noise. Therefore, there are regions of the image with high level of corruption, and there are also regions with low level of corruption. For an effective noise filtering process, a larger filter should be applied to regions with high level of corruption. In contrast, a smaller filter should be applied to regions with low level of corruption. Therefore, many works, such as [35, 38–40, 50–52, 83–85], have proposed methods that are able to adjust the size of the filter accordingly based on the local noise content. Because the size of the filter is adapted to the local noise content, this type of median filter is known as adaptive median filter.

Commonly, the filter size at each processing locations is initially set to 3×3. The size of the filter is then gradually expanding until it met certain criteria. These criteria can include the number of potential noise free pixels, local mean, local maximum, local minimum or local median value. Sometimes, these criteria can never be met. Therefore, some methods restrict the expansion of the filter up to certain size only. Although adaptive median filters are good in restoring image corrupted by impulse noise, these filters normally require considerably long computational time when the image is highly corrupted.

2.2.8 Median Filter Incorporating Fuzzy Logic

In order to preserve the local details of the image, median filter should only change the intensity of corrupted pixels on the damaged image. However, it is very to difficult detect the corrupted pixels from this image correctly. Even for fixed-valued impulse noise (i.e. salt-and-pepper noise), where the noise only takes values 0 and $L-1$, simple thresholding method still cannot classify the pixels effectively. This is because some of the uncorrupted pixels are also been presented by these two values. Thus, some researchers incorporate fuzzy logic approach into median filtering process [3, 14–16, 21, 24–26, 37, 41, 53, 55].

There are several ways on how fuzzy logic been used in median filtering process. Fuzzy logic can be used to grade how high a pixel has been corrupted by impulse noise. Normally, based on this fuzzy degradation measure, a proper correction will be applied. On the other hand, some of the methods use fuzzy logic as a decision maker that selects a proper filter, from a filter bank, for a given input image.