
UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2007/2008

Jun 2008

MAT 222 – Differential Equations II
[Persamaan Pembezaan II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer all four [4] questions.

Arahan: Jawab semua empat [4] soalan.]

1. Consider the system of equations

$$\left. \begin{aligned} \frac{dx}{dt} &= a_1(t)x + b_1(t)y \\ \frac{dy}{dt} &= a_2(t)x + b_2(t)y \end{aligned} \right\} (*)$$

Prove the following theorem:

If the system (*) has the two solutions

$$\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases} \text{ and } \begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$$

in the interval $[a, b]$, then

$$\begin{cases} x = c_1x_1(t) + c_2x_2(t) \\ y = c_1y_1(t) + c_2y_2(t) \end{cases}$$

is also a solution for (*) in $[a, b]$ for any constants c_1 and c_2 .

[100 marks]

2. Consider the first order partial differential equation $u_x - u_y = \frac{\cos(x+y)}{y^2}$ where $u = u(x, y)$.

(a) Suppose $\hat{u}(x, y)$ is a particular solution for the equation above. Consider now the two possible choices for \hat{u}_x below and obtain a suitable expression for \hat{u}_y in each case. Determine which choice produces a particular solution for the differential equation.

$$(i) \quad \hat{u}_x = \frac{\cos(x+y)}{y^2}$$

$$(ii) \quad \hat{u}_x = -\frac{\sin(x+y)}{y}$$

(b) Hence, solve the given partial differential equation using the initial condition

$$u(x, \pi) = -\frac{\cos x}{\pi}.$$

[100 marks]

1. Pertimbangkan sistem persamaan

$$\left. \begin{aligned} \frac{dx}{dt} &= a_1(t)x + b_1(t)y \\ \frac{dy}{dt} &= a_2(t)x + b_2(t)y \end{aligned} \right\} (*)$$

Buktikan teorem berikut:

Jika sistem (*) mempunyai dua penyelesaian

$$\left\{ \begin{aligned} x &= x_1(t) \\ y &= y_1(t) \end{aligned} \right\} \text{ dan } \left\{ \begin{aligned} x &= x_2(t) \\ y &= y_2(t) \end{aligned} \right\}$$

pada selang $[a, b]$, maka

$$\left\{ \begin{aligned} x &= c_1 x_1(t) + c_2 x_2(t) \\ y &= c_1 y_1(t) + c_2 y_2(t) \end{aligned} \right\}$$

juga merupakan suatu penyelesaian bagi (*) pada $[a, b]$ bagi sebarang pemalar c_1 dan c_2 .

[100 markah]

2. Pertimbangkan persamaan pembezaan separa peringkat pertama $u_x - u_y = \frac{\cos(x+y)}{y^2}$

di mana $u = u(x, y)$.

(a) Katakan $\hat{u}(x, y)$ merupakan suatu penyelesaian khusus bagi persamaan di atas. Pertimbangkan kedua-dua pilihan yang mungkin bagi \hat{u}_x seperti di bawah dan dapatkan suatu ungkapan yang mungkin untuk \hat{u}_y dalam setiap kes. Tentukan pilihan yang mana dapat menghasilkan suatu penyelesaian khusus bagi persamaan pembezaan yang diberi.

$$(i) \quad \hat{u}_x = \frac{\cos(x+y)}{y^2}$$

$$(ii) \quad \hat{u}_x = -\frac{\sin(x+y)}{y}$$

(b) Dengan itu, selesaikan persamaan pembezaan separa di atas dengan menggunakan syarat awal $u(x, \pi) = -\frac{\cos x}{\pi}$.

[100 markah]

3. Consider the function below:

$$f(x) = \frac{1}{2}(x - |x|) + 1 \text{ for } a < x < \pi \text{ with } f(x + 2\pi) = f(x).$$

- (a) Given $a = -\pi$, find the Fourier series for $f(x)$.
 (b) Given $a = 0$, find the Fourier Cosine series for $f(x)$.

[100 marks]

4. Consider the following one-dimensional wave equation for $u = u(x, t)$:

$$u_{xx} = c^{-2}u_{tt}, \quad 0 < x < \pi, t > 0,$$

with the initial conditions

$$u(0, t) = u(\pi, t) = 0, t > 0, \text{ and}$$

$$u(x, 0) = f(x), u_t(x, 0) = 0, 0 < x < \pi.$$

- (i) Solve the given problem using the method of separation of variables.
 (ii) Hence, simplify your solution given that

$$f(x) = \begin{cases} \frac{1}{4}x, & 0 < x \leq \frac{\pi}{2} \\ \frac{1}{4}(\pi - x), & \frac{\pi}{2} < x < \pi \end{cases}.$$

[100 marks]

3. Pertimbangkan fungsi berikut:

$$f(x) = \frac{1}{2}(x - |x|) + 1 \text{ bagi } a < x < \pi \text{ dengan } f(x + 2\pi) = f(x).$$

(a) Diberi $a = -\pi$, cari siri Fourier bagi $f(x)$.

(b) Diberi $a = 0$, cari siri Kosinus Fourier bagi $f(x)$.

[100 markah]

4. Pertimbangkan persamaan gelombang satu dimensi bagi $u = u(x, t)$ yang berikut:

$$u_{xx} = c^{-2}u_{tt}, \quad 0 < x < \pi, t > 0,$$

dengan syarat awal

$$u(0, t) = u(\pi, t) = 0, \quad t > 0, \text{ dan}$$

$$u(x, 0) = f(x), \quad u_t(x, 0) = 0, \quad 0 < x < \pi.$$

(i) Selesaikan masalah di atas dengan menggunakan kaedah pemisahan pembolehubah.

(ii) Dengan demikian, ringkaskan penyelesaian anda diberikan

$$f(x) = \begin{cases} \frac{1}{4}x, & 0 < x \leq \frac{\pi}{2} \\ \frac{1}{4}(\pi - x), & \frac{\pi}{2} < x < \pi \end{cases}$$

[100 markah]