

**FORECASTING PERFORMANCE OF
NONLINEAR AND NONSTATIONARY STOCK
MARKET DATA USING EMPIRICAL MODE
DECOMPOSITION**

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by

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LIST OF ABBREVIATIONS

AIC	Akaike Information Criterion
EMD	Empirical Mode Decomposition
EXP	Exponential smoothing method
FFT	Fast Fourier transform
HW	Holt-Winter model
IMF	Intrinsic Mode Function
MA	Moving Average
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MASE	Mean Absolute Scale Error
MSE	Mean Square Error
RMSE	Root Mean Square Error
RW	Random Walk
TheilU	Theil's U-statistic

LIST OF SYMBOLS

B	The number of bootstrap
e	Natural exponential constant (2.718)
ε_t	The random error at time t
h	Number of forecasting trials
iid	The independent and identically distributed
ℓ	Time series level
l	Block length in MBB
ω	Frequency value
$r(t)$	Residue of the original time series data from EMD process
t	Continuous time
τ	The quantile(s) to be estimated
$x(t)$	Original time series data
\hat{y}_t	Forecast value of y at time t

PRESTASI PERAMALAN DATA PASARAN SAHAM TAK LINEAR DAN TAK PEGUN MENGGUNAKAN PENGHURAIAAN MOD EMPIRIK

ABSTRAK

Indeks pasaran saham biasanya tidak linear dan tidak pegun dengan data berheteroskedastisiti tinggi, yang mana mempengaruhi ketepatan dan kesahihan dapatan dari kaedah ramalan tradisional. Oleh itu, kajian ini memberi tumpuan kepada kaedah penguraian untuk menyelesaikan masalah tak linear dan tak pegun dalam data dengan tingkah laku heteroskedastisiti yang tinggi untuk meningkatkan ketepatan ramalan pasaran saham. Kebelakangan ini, kaedah penguraian mod empirikal (EMD) telah diperkenalkan sebagai teknik yang berkesan untuk mengatasi masalah tidak linear dan tidak pegun dalam data siri masa. EMD mempunyai beberapa ciri yang tidak ditunjukkan oleh kaedah penguraian yang lain. Oleh itu, tesis ini mencadangkan lima teknik yang berbeza untuk meramalkan pasaran saham dengan menggabungkan EMD dengan teknik peramal tradisional atau butstrap EMD dengan teknik tradisional untuk menyelesaikan masalah ketidaktepatan ramalan dalam data siri masa kewangan dan oleh itu untuk mendapatkan hasil ramalan yang lebih baik. Lima teknik baru, iaitu EMD dengan purata bergerak, EMD dengan Holt-Winter (EMD-HW), EMD dengan perjalanan rawak (EMD-RW), EMD dengan kaedah pelicinan eksponen (EMD-EXP), dan butstrap EMD dengan HW (pengedaran EMD-HW) dicadangkan. Teknik-teknik yang dicadangkan ini dibandingkan dengan lapan kaedah ramalan tradisional, sepuluh indeks pasaran saham harian yang lebih daripada seribu lima ratus cerapan untuk setiap set digunakan. Berdasarkan lima sukatan ralat (iaitu RMSE, MAE, MAPE, MASE,

dan TheilU), dapatan menunjukkan bahawa lima teknik yang dicadangkan lebih tepat daripada teknik ramalan yang sedia ada dalam ramalan pasaran saham. Kajian ini menunjukkan bahawa kaedah yang dicadangkan boleh digunakan dengan jayanya dalam ramalan pasaran saham, kerana teknik ini terbukti menjadi kaedah yang berguna dari segi ramalan prestasi untuk data pasaran saham. Oleh itu, lima teknik yang dicadangkan ini merupakan sumbangan utama kepada literatur untuk kajian ramalan pasaran saham bukan linear dan tidak pegun dengan heteroskedastisiti yang tinggi.

FORECASTING PERFORMANCE OF NONLINEAR AND NONSTATIONARY STOCK MARKET DATA USING EMPIRICAL MODE DECOMPOSITION

ABSTRACT

The stock market indices are typically non-linear and non-stationary with high heteroscedasticity data, which affect the accuracy and validity of the results of traditional forecasting methods. Therefore, this study focuses on decomposition method to solve the problem of non-linearity and non-stationarity in data with high heteroscedasticity behavior to improve the accuracy of stock market forecasting. Recently, Empirical mode decomposition (EMD) method has been introduced as an effective technique for overcoming the non-linearity and non-stationarity in time series data. EMD presents several characteristics that other decomposition methods do not have. Thus, this thesis proposes five different techniques to forecast the stock markets by combining EMD with traditional forecasting techniques or bootstrapping EMD with traditional technique to solve the forecasting inaccuracy problem in financial time series data and therefore to obtain improving forecasting results. The five new techniques, namely, EMD with moving average, EMD with Holt-Winter (EMD-HW), EMD with random walk (EMD-RW), EMD with exponential smoothing method (EMD-EXP), and bootstrapping of EMD with HW (EMD-HW bagging) are proposed. These proposed five techniques are compared with eight traditional forecasting methods, ten different daily stock market indexes of over one thousand five hundred observations for each set are used. Based on five error measures (i.e. RMSE, MAE, MAPE, MASE, and

TheilU), the results show that the five proposed techniques are more accurate than the existing forecasting techniques on the stock market forecasting. This study indicates that the proposed methods can be applied successfully in stock market forecasting, as these techniques are proven to be useful methods in terms of forecasting performance for stock market data. Therefore, these five proposed techniques are the main contribution to the literature of studying the forecasting of the nonlinear and non-stationary stock market with high heteroscedasticity.

CHAPTER 1

INTRODUCTION

1.1 General Introduction

Forecasting, as presented by Hyndman and Athanasopoulos (2014) and Box et al. (2015), is one of the most important tools that aid an organization in its attempt to survive future uncertainties. Forecasting relies on understanding historical and present data and analyzing data trends. Moreover, the forecasted future values of a variable start with a certain hypothesis based on the experience, knowledge, and judgment of researchers.

A time series is a collection of recordings of observations for a particular event that changes over time (Bloomfield, 2004), such as over short term (minute-by-minute wind speed and day-by-day stock market index) or long term (annual unemployment). Time series observations are taken at either continuous or discrete times.

Forecasting is important, especially in the field of financial time series. Moreover, forecasting has become a dynamic area of study and has drawn considerable attention of researchers due to relevance to investment and financial decision making (Yang and Lin, 2017). However, over the past 50 years, stock market forecasting has been regarded as one of the most challenging applications of modern time series forecasting (Lin et al., 2012). This phenomenon may be a result of several reasons, the most challenging of which is the stock market data being nonlinear, non-stationary (Wang et al., 2015), highly heteroscedastic (Kazem et al., 2013), and with random walk (RW)

(Patel et al., 2015). The methods reported in literature focus mostly on forecasting linear and stationary processes (Oh et al., 2009).

Current forecasting methods cannot accurately predict stock market data, such as investor targets (Wang et al., 2011). Therefore, stock market forecasting has been regarded as generally volatile among those interested in this area, such as decision makers, governments, and investors. Consequently, many researchers continue to develop new hybrid forecasting models to improve the efficiency of stock market forecasting. Therefore, an intelligent information processing technology must be used to forecast stock market data.

Many methods are used to eliminate the nonlinear and non-stationary behavior (with high heteroscedasticity) of forecasting. Some of these methods include the transformation from the time domain to frequency domain and the differentiation between data and decomposition methods. However, transformation and differentiation problems result in some lost data features (Parsons et al., 2000). Thus, decomposition data methods have been preferred. Many decomposition methods are found in the literature, and one of the best methods is the empirical mode decomposition (EMD) by Huang et al. (1998) on the basis of comparison of recent research.

EMD is a time series decomposition method that breaks down non-stationary and nonlinear time series data, such as those for stock markets. EMD relies on smoothing algorithms and local characteristics of time-scale data, and it is adaptive and highly efficient without having to leave the time domain. EMD is also regarded as one of the most powerful signal processing techniques (Lei et al., 2013). As such, this present study focuses on employing EMD to aid stock market forecasting by integrating EMD

with existing forecasting methods. Subsequently, five forecasting models are developed. The experimental results of the five proposed methods show that these methods are superior to existing ones in terms of five accuracy forecasting measures. Furthermore, EMD has attracted the attention of researchers, especially when dealing with nonlinear and non-stationary time series in several fields.

1.2 Problem Statement

Time series forecasting methods have effectively solved most of forecasting problems on financial time series. However, three major problems currently exist (i.e., mostly related to stock market data), which include the following:

1. The first problem of the time series forecasting methods is that it assumes the linearity on the time series data. In other words, the mature forecasting methods assumed that there is a linear relationship between the time series observations. So, the linearity assumption in real-life time series data (such as stock market data) is not always right.
2. The second problem of time series forecasting methods is that assume the stationary on the time series data. In other words, the existed forecasting methods assumed that the properties of the time series (i.e. mean, variance, and autocorrelation) do not depend on the time where the series is observed. Some of the mature methods used the transformation, the differentiation between data, or (old) decomposition methods to overcome this problem. However, transformation and differentiation problems result in some lost data features, the old decomposition methods are not adaptive and highly efficient, which it will adversely affect the

accuracy of the forecasting values of these methods.

3. The stock market data do not follow statistical time series assumptions such as the normality and homogeneity of data. Moreover, the changes in market conditions, such as the supply and demand environment, can cause noise that is mainly involuted to stock market data. Subsequently, this phenomenon weakens the forecasting execution of the existing time series forecasting methods.

These problems motivated this study to develop five new forecasting methods, four methods are hybrid of traditional methods with adaptive and highly efficient (i.e. the EMD) and bootstrapping of traditional methods with adaptive and highly efficient (i.e. the EMD). The five new proposed forecasting methods developed in this study will overcome the problems of existing forecasting models (i.e. the nonlinearity, the non-stationarity, and the heteroscedasticity in time series data).

1.3 Research Objectives

The thesis is centered on the forecasting performance of non-linear and non-stationary stock market data with high heteroscedasticity based on EMD method with the following objectives:

1. To improve the forecasting accuracy of stock market data, which are nonlinear and non-stationary with high heteroscedasticity by combining EMD with traditional forecasting techniques.
2. To develop a bootstrap forecasting method based on EMD to outperform all existing forecasting methods that took part in this study in forecasting the stock

market.

3. To display empirical examples (i.e. stock market data) that explain how combining of the EMD with existing methods affect the forecasting results of these methods and to display empirical examples that represent how the bootstrapping of EMD with existing methods affect on the forecasting results, these by comparing with popular existing methods.

1.4 Scope of the Study

This thesis focuses on solving the problem of forecasting accuracy in the nonlinear nonstationary stock market series data with high heteroscedasticity by developing five new forecasting techniques based on EMD. The stock markets of 10 countries, namely Australia, Belgium, Denmark, Finland, France, India, Lithuania, Netherlands, Switzerland, and United Kingdom (UK) are used in this study.

1.5 Significance of the Study

The forecasting accuracy of the stock market is important and must be studied by researchers, investors, market regulators, decision-makers in their country. Stock market data are non-stationary and nonlinear behavior with high heteroscedasticity as well as random walk behavior; hence, capturing the dominant properties of their changes is difficult. Subsequently, this phenomenon weakens the forecasting execution of most time series forecasting methods. This study intends to address this problem by employing new five techniques based on the combination (or bagging) of EMD, which is an efficient deal with nonlinear nonstationary time series data, and some traditional models. The first, second, third, and fourth models combine EMD with MA, HW, RW,

and EXP, respectively. The fifth model involves the bootstrapping of EMD and HW. This study proves that the forecasting of the stock market series by the five proposed methods is better than that by existing methods for stock market data of 10 countries in obtaining forecasting results with more accuracy based on five error measurements. These results contribute to the body of knowledge on time series analysis in terms of forecasting financial time series data.

1.6 Thesis Organization

The structure of the thesis is as follows: Chapter 2 presents a review of related literature and a background of EMD and introduces the decomposition in literature with the forecasting methods that are used in this study. Chapter 3 presents the proposed forecasting methods (i.e. hybrid of EMD-HW, EMD-MA, EMD-RW, and EMD-EXP methods, and the bagging EMD-HW method). The root mean square error (RMSE), mean absolute error (MAE), mean percentage error, mean absolute percentage error (MAPE), mean absolute scaled error (MASE), and Theil's U-statistic (TheilU) are also presented in Chapter 3. Chapter 4 presents an analysis of the daily stock market data used in this study. Chapter 5 discusses the results obtained from the application of the proposed methodology. Chapter 6 concludes the research work by presenting the summary of the findings and future work.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter contains six sections. The second section offers a brief theoretical explanation of a number of time series decomposition techniques in the literature. The third section reviews a description of the EMD algorithm for signal decomposition and compares its relative merits with other decomposition methods. This section also presents an overview of developments in the field of EMD methodology and its application in different areas, as presented in the literature. The list of recent studies that apply EMD in forecasting time series is also presented at the end of this section. The fourth section describes the various time series forecasting techniques that are either used or compared with the proposed forecasting methods in this study. The fifth section presents the MBB method with bootstrapping, as presented in the literature. This section also presents a number of recent studies that apply the bootstrapping technique in point forecasting methods.

2.2 Time Series Decomposition Methods

Time series decomposition is one of the important areas of time series, and it has been an area of research for a long time (Johansson et al., 2006). In this field, many methods have been developed so far in the literature, but until this day the time series decomposition still gets researchers' attention. In this section, we will mention the methods most commonly used in the literature by practitioners for ultimate task of process-

ing data. These methods are short-time Fourier transform (STFT), Fourier Transform (FT), wavelet transform (WT), Seasonal-Trend Decomposition Using Loess (STL), and Hilbert-Huang Transform (or Hilbert Transform with empirical mode decomposition). This often comprises several consecutive steps of solving a statistical decision problem (detection, estimation, classification, recognition, etc.).

2.2.1 Fourier Analysis

Fourier analysis (or harmonic analysis) is a technique to decompose time series into a sum of trigonometric series (that is, it is an infinite series consisting of sine and cosine terms) with frequencies in order to determine the frequency content (Bloomfield, 2004). In the literature, the Fourier analysis appears in two types, these are Fourier series and Fourier transform (Kido, 2015). Fourier transform is mostly used for non-periodic time series, while periodic time series tend to use Fourier series.

In Fourier series, the periodic time series $x(t)$ of period T , i.e. $x(t + T) = x(t)$, is depicted by Lestrel (2008) as Equation (2.1).

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t), \quad (2.1)$$

where the independent variable $\omega_0 = \frac{2\pi}{T}$ represents the fundamental angular frequency in radians per second. The sine and cosine term coefficients are determined by using the following formula (Lestrel, 2008):

$$a_0 = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) dt,$$

$$a_k = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \cos(k\omega_0 t) dt,$$

$$b_k = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \sin(k\omega_0 t) dt;$$

where $k = 1, 2, \dots, \infty$, t represents time domain and ω represents frequency domain.

A principal analysis tool in many of today's scientific challenges is the Fourier transform (FT). The FT technique is a method to decompose the time series from the time domain into the frequencies domain. In this method, t represents time domain and ω represents frequency domain. If the function (or time series) $x(t)$ is defined on $-\infty < t < \infty$, then the FT of $x(t)$ will be denoted by \mathcal{X} . The integral representation of FT is given by Equation (2.2) by Butz (2006), when $x(t)$ is an integrable function $x(t) : \mathbb{R} \rightarrow \mathbb{C}$, where $i = \sqrt{-1}$.

$$\mathcal{X}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt \quad (2.2)$$

On the other hand, the inverse FT of the function \mathcal{X} is given by Butz (2006) as Equation (2.3), where $\mathcal{X}(\omega)$ integrable function $\mathcal{X} : \mathbb{C} \rightarrow \mathbb{R}$.

$$x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{X}(\omega) e^{i\omega t} d\omega, \quad (2.3)$$

The FT analyzes the time series by using spectral analysis. This is the standard method of obtaining information regarding period signals in the literature. The FT is an extension of the Fourier series that results when the period of the represented function is lengthened and allowed to approach infinity.

There is still another FT to be considered. A type of computational procedure for the Finite FT algorithm is presented, that has been used extensively in the literature because real-life applications produce only finite sequences (Carmona et al., 1998).

2.2.1(a) Fast Fourier Transform

The Fast Fourier transform (FFT) is a computational procedure for calculating the discrete Fourier transform algorithm (DFT) (or finite Fourier transform) of a time series (Cooley et al., 1967). In other words, FFT is a fast computation for DFT, which reduces the number of computations needed for N points from $2N^2$ to $2N\log_2 N$ (Oran Brigham, 1988). Moreover, the DFT is very efficiently calculated by way of the FFT technique (Ahmed and Rao, 2012). The DFT is defined as follows: Let x_1, x_2, \dots, x_{N-1} be a time series of complex numbers. The DFT is given in Equation (2.4) by Johnson and Frigo (2007).

$$\mathcal{X}(\omega) = \sum_{n=0}^{N-1} x_n e^{-i2\pi\omega n/N}; \quad \omega = 0, \dots, N-1. \quad (2.4)$$

The DFT can be computed faster by determining the coefficients using the butterfly algorithm presented by Cooley and Tukey (1965). FFT is widely used in signal-processing and in analysis concept (Oran Brigham, 1988). Moreover, the FT can be extended using DFT.

Unfortunately, researchers have found that there were problems in FT method. These problems are, the FT does not reveal the temporal location of frequencies and can therefore only be used for stationary signals (Karlsson et al., 2000). FT does not recognize the boundaries and discontinuities and will, therefore, create higher order harmonics to complement the waveform. To overcome these problems, a new analysis method was developed. This method was named Short-Time Fourier Transform (STFT). The STFT method will be presented in the next subsection.

2.2.1(b) Short-Time Fourier Transform

The original signal is broken down into smaller duration particles using Short-Time Fourier Transform (STFT) (or short-term Fourier transform (Allen, 1977)). The STFT is known as the spectral window whilst the window in a time-domain is referred to as the time window (Cohen, 1995). A signal is multiplied with window functions $g(t - b)$, where $g(t)$ depicts the functional form and is non-zero in the finite region at time b . FT of $x(t)g(t - b)$ should be determined followed by the movement of the window to a new position and the repetition of the process (Cohen, 1995). Mathematically, STFT is written as in Equation (2.5) as presented in Allen (1977).

$$\mathcal{X}(\omega, b) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t)g(t - b)e^{-i\omega t} dt. \quad (2.5)$$

To reconstruct the function (time series) $x(t)$ from $\mathcal{X}(\omega, b)$ we utilise Equation (2.6) (Allen, 1977).

$$x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{X}(\omega, b)e^{i\omega t} d\omega db, \quad (2.6)$$

Chui (2016) presented some problems for STFT. He argues that because the frequency is proportional to the number of cycles in a specific time interval, locating high-frequency phenomena requires a narrow window, whereas, in order to investigate low-frequency phenomena, a wide time window is necessary. Thus, for signals with both high and low frequencies, STFT is not an ideal solution. Moreover, STFT (FT in general) cannot be used for analyzing signals in a joint time and frequency domain. To overcome these limitations in STFT, the researchers presented a new analysis method, which is the wavelet transforms (WT) in 1982 (Debnath and Shah, 2015).

2.2.2 Wavelet Transform

The wavelet transform (WT) of time series $x(t)$ is defined as a decomposition of $x(t)$ into a linear combination of scaled (stretched or compressed) and shifted functions known as wavelets. This difference from the FT makes WT more flexible than the FT (Al Wadia and Tahir Ismail, 2011). FT provides a signal which is localized only in the frequency domain, while WT functions are localized in both time and frequency domains (Misiti et al., 2013). Mathematically, the continuous wavelet transform of a function $x(t) \in L^2(\mathbb{R})$ at a scale $a > 0$ and translational value $b \in \mathbb{R}$ is defined by Equation (2.7) as presented in Debnath and Shah (2015).

$$\mathcal{W}_w(x(t), a, b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} x(t) \overline{\psi} \left(\frac{t-b}{a} \right) dt \quad (2.7)$$

where $\psi(t)$ represents a continuous function in both the time and frequency domains, called the mother wavelet function (e.g. Daubechies-n, Morel, Mexican Hat, etc.) and the overline represents operation of complex conjugate. The continuous wavelet transform is an extension of the STFT where multiple scales (i.e. window sizes) are used to analyze the signal.

The main purpose of the mother wavelet is to provide a source function to generate the father wavelets which are simply the translated and scaled versions of the mother wavelet. To recover the original signal $x(t)$, the first inverse continuous wavelet transform can be exploited. The inverse of the continuous wavelet transform presented in Debnath and Shah (2015) is given in Equation (2.8).

$$x(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{W}_\psi(a, b) \psi_{a,b}(t) \frac{da}{a^2} db, \quad (2.8)$$

provided C_ψ satisfies the so-called admissibility condition

$$C_\psi = 2\pi \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega.$$

where $\hat{\psi}(t)$ is the Fourier transform operator of $\psi(t)$. The wavelet analysis was designed for linear but nonstationary (Gröchenig, 2013) and it is non-adaptive technique (Huang, 2014). Recently, an adaptive time series analysis method has designed for nonlinear and nonstationary time series named Hilbert-Huang transform (HHT) in 1998 (Huang, 2014). The HHT will be presented in the following section.

2.2.3 Hilbert-Huang Transform

The Hilbert-Huang transform (HHT) was presented in 1998 (Huang, 2014) as a integrate of empirical mode decomposition (EMD) and Hilbert transform analysis (HT). The strength of HHT is its ability to process non-stationary and non-linear data. Moreover, HHT does not move from the time domain into the frequency domain - Information is maintained in the time domain (Ayenu-Prah et al., 2005) while the HT is applied on intrinsic mode functions (IMF) along with a residual, and obtain instantaneous frequency data. This means, to apply the HT, the components decomposition of signals must be performed beforehand. Mathematically, the HT of a time series $x(t)$ is given by Equation 2.9 in Huang (2014).

$$h(t) = \mathcal{H}[x(t)] = \frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau, \quad (2.9)$$

in which the PV indicates the Cauchy principal value of the singular integral. With this definition, $x(t)$ and $h(t)$ form a complex conjugate pair, so that we can have an analytic

signal, $z(t)$; as

$$z_a(t) = x(t) + iy(t) = a(t) \cdot e^{i\theta(t)}, \quad (2.10)$$

where $a(t) = [x^2(t) + y^2(t)]^{1/2}$ and $\theta(t) = \arctan(y(t)/x(t))$. Here, $a(t)$ is the instantaneous amplitude of $x(t)$ and θ is the phase function. The instantaneous frequency of Hilbert transform is given by the Equation 2.11 as in Huang (2014).

$$\omega = \frac{d\theta}{dt}. \quad (2.11)$$

In the next section, we will present a different type of time series analysis methods from the previous methods that have been dealt with. This method focuses on other components such as seasonal, trend, and remainder since the previous methods do not focus on these components. Furthermore, it does not use the integral transform technique in its algorithm.

2.2.4 A Seasonal-Trend Decomposition Using Loess (STL)

In the early 1920s, the decomposition model along with seasonal adjustment was the major research focus (Zhang and Qi, 2005). Persons (1919) was the first person that developed decomposition techniques for identifying and isolating the salient features of a time series (seasonal, trend and residue). He proposed this technique to decompose time series into four components: the trend, the cycle, seasonality, and a purely accidental hazard.

Presently, there are several methods for decomposing time series of this type. The most prominent methods are X-11 ARIMA/88 by Dagum (1988), Seasonal Adjustment at Bell Laboratories (SABL) by Cleveland et al. (1979), and Seasonal-Trend decompo-

sition based on Loess smoothing (STL) by Cleveland et al. (1990). *STL* is a robust nonparametric time series decomposition method known as the Seasonal-Trend decomposition filtering procedure based on Loess was introduced by Cleveland et al. (1990). The *STL* technique can divide the time series data into three additive main components: Seasonal $S(t)$, Trend or long-term $T(t)$, and Random $R(t)$ components. These components can be written as in Equation (3.14) (Cleveland et al., 1990).

$$x(t) = S(t) + T(t) + R(t). \quad (2.12)$$

Here, the trend component is defined as the general tendency for a time series to increase, decrease or stagnate over a long period of time. The seasonal component in a time series is fluctuating within a year during the season, such as the effect of weather on sales of clothes. Seasonality is always of a fixed and known period. The random part of a time series is caused by unpredictable influences. Also, it is neither regular nor repeated in a particular pattern. Loess method (a smoothing method based on local regressions) is used to decompose the time series into trend and remainder components for non-seasonal time series, as presented by Cleveland et al. (1992). A number of studies have applied STL decomposition in forecasting time series in the literature. These studies used STL in their methodology, such as Bergmeir et al. (2016) and Xiong et al. (2018).

2.3 Empirical Mode Decomposition (EMD)

The Empirical mode decomposition (EMD) is a new decomposition method has been described by Huang et al. (1998). The main idea for EMD is to decompose a non-stationary and nonlinear time series into a nearly orthogonal combination of simple

time series (Moore et al., 2018). These components are known as intrinsic mode functions (IMFs) and residual (r). The EMD methodology analyzes the time series by keeping this time series in time domain. This decomposition method (EMD) is adaptive, intuitive, direct and highly efficient. After this definition and some property of EMD, in the subsection of this section, the EMD algorithm process, several applications of EMD, theoretical developments of EMD, and forecasting methods based on EMD will be presented.

2.3.1 Sifting Process

The algorithm process of EMD will be presented in 6 steps. This process is named the Sifting process of EMD. The main idea of EMD is to decompose time series into IMFs and $r(t)$. So, the time series $x(t)$ can be constructed back as in Equation (2.13). The sifting decomposition process is based on the local characteristic time scale of the data as presented by Huang et al. (1998).

$$x(t) = \sum_{i=1}^n IMF_i(t) + r(t), \quad (2.13)$$

where $x(t)$ represents the original time series, $r(t)$ represents the residue of the original time series data decomposition, and IMF_i represents the i^{th} intrinsic mode function (IMF) series. In order to estimate the IMFs should initiate the steps of sifting process of time series $x(t)$ as presented by Huang et al. (1998). Thus, this is summarized as follows:

Step 1. The first step begins by taking the original time series as a $x(t)$ for the sifting process. We assume that the value of the two repetition indicators are $i = 1$ and $j = 1$.

Step 2. Then evaluating all the local extrema values (local upper and local lower) of the time series $x(t)$. Figure 2.1 shows an example of step 2. Here, the black line is the original time series $x(t)$, the red circle represents the local upper, and the green circle represents the local lower.

Step 3. After that, form the local upper (local maximum) envelope function $e_u(t)$ by connecting all local maxima values using the cubic spline line. In a similar way, form the local lower (local minimum) envelope function $e_l(t)$. Should all observations in $x(t)$ cover between $e_u(t)$ and $e_l(t)$. After that, calculate a new function named the mean envelop denoted by $m_j(t)$ from $e_u(t)$ and $e_l(t)$ by using Equation (2.14). Figure 2.2 shows an example of step 3. Here, the black line represents the original time series $x(t)$, the red line represents the upper envelope line $e_u(t)$, the green line represents the lower envelope line $e_l(t)$, and the blue line represents the mean envelop $m_j(t)$.

$$m_j(t) = \frac{e_u(t) + e_l(t)}{2} \quad (2.14)$$

Step 4. Next, define a new function $h_j(t)$ using the mean envelope $m_j(t)$ and the signal $x(t)$ on Equation (2.15).

$$h_j(t) = x(t) - m_j(t) \quad (2.15)$$

Check if the function $h_j(t)$ which is an IMF or not, according to IMF conditions (will be presented in Section 2.3.2). If the function $h_j(t)$ has satisfied IMF conditions, then go to step 5. If not, renew the value of $x(t)$ such that it becomes $h_j(t)$. Also, the iteration index value j is renewed. Such that it becomes $j = j + 1$, and repeat the

steps again from step 2 until step 4.

Step 5. This step has three processes. Firstly, save $h_j(t)$ which obtained from the last step as a IMF_i , where $IMF_i(t) = h_j(t)$. Secondly, define a new function $c(t)$ using the $IMF_i(t)$ and the signal $x(t)$ by the Equation (2.16). Thirdly, renew the iterations index values of i and j . Such that it become $i = i + 1$ and $j = 1$.

$$c(t) = x(t) - IMF_i(t) \quad (2.16)$$

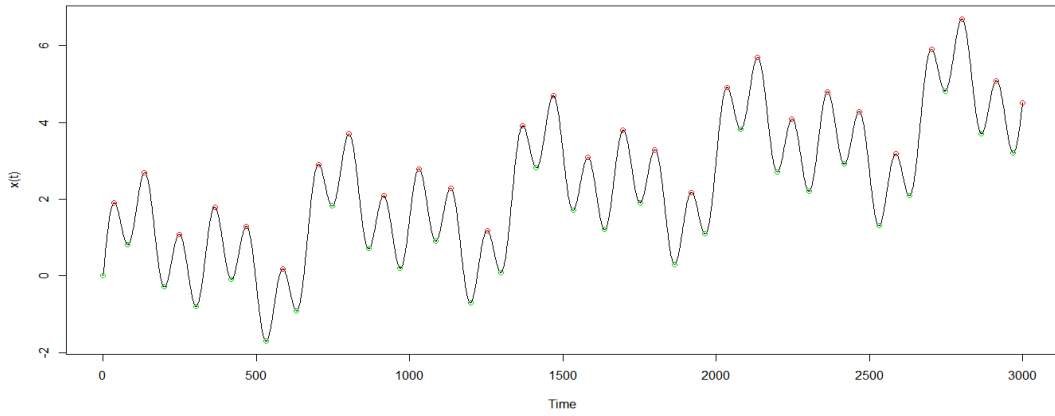


Figure 2.1: The extraction of local extremum values of time series

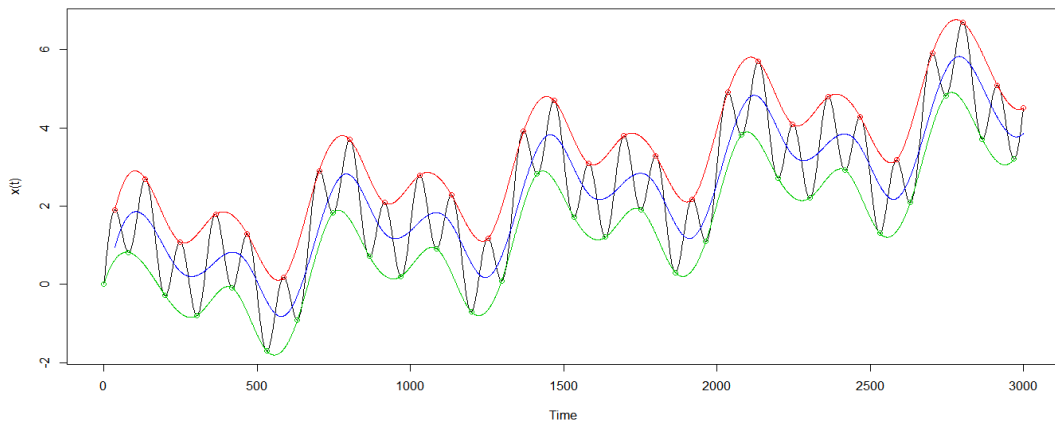


Figure 2.2: The evaluation of upper and lower envelopes of the time series.

Step 6. In this step, according to the characteristics of the function $c(t)$ it will be decided whether the sifting process is over or not. If $c(t)$ is a monotonic or a constant function from which one cannot extract more IMF or the value of SD_h (standard deviation) is between 0.2 and 0.3 (Huang, 2014), where SD_h is defined in Equation (2.17), then the residue $r(t) = c(t)$ and all the IMFs will be saved, and the sifting process stops. If the function $c(t)$ is not a monotonic or a constant function from which one cannot extract more IMF and the value of SD_h (standard deviation) is between 0.2 and 0.3, the value of $x(t)$ will be renewed, such that it becomes $c(t)$, and go back to step 2. This step is named "Stoppage criteria of the sifting process".

$$SD_h = \sum_{t=0}^T \frac{|h_{k-1}(t) - h_k(t)|^2}{h_{k-1}^2(t)}. \quad (2.17)$$

The steps 1 through 6 which were discussed above allow the sifting process (EMD algorithm) to separate the time-altering signal properties. Figure 2.3 is a flowchart summarizing all the sifting process steps.

2.3.2 Intrinsic Mode Function (IMF)

In the last section, the EMD algorithm (sifting process) was presented. The IMFs are produced as a result of this algorithm. The tree diagram in Figure 2.4 shows the way of reconstruction of the IMFs. The IMFs' produced by the sifting process need to satisfy two conditions, namely

$$1. \quad |Num[extrima] - Num[cross - zero]| \leq 1,$$

where $Num[extrima]$ represents the number of local extreme points and $Num[cross - zero]$ represents the number of cross-zero points.

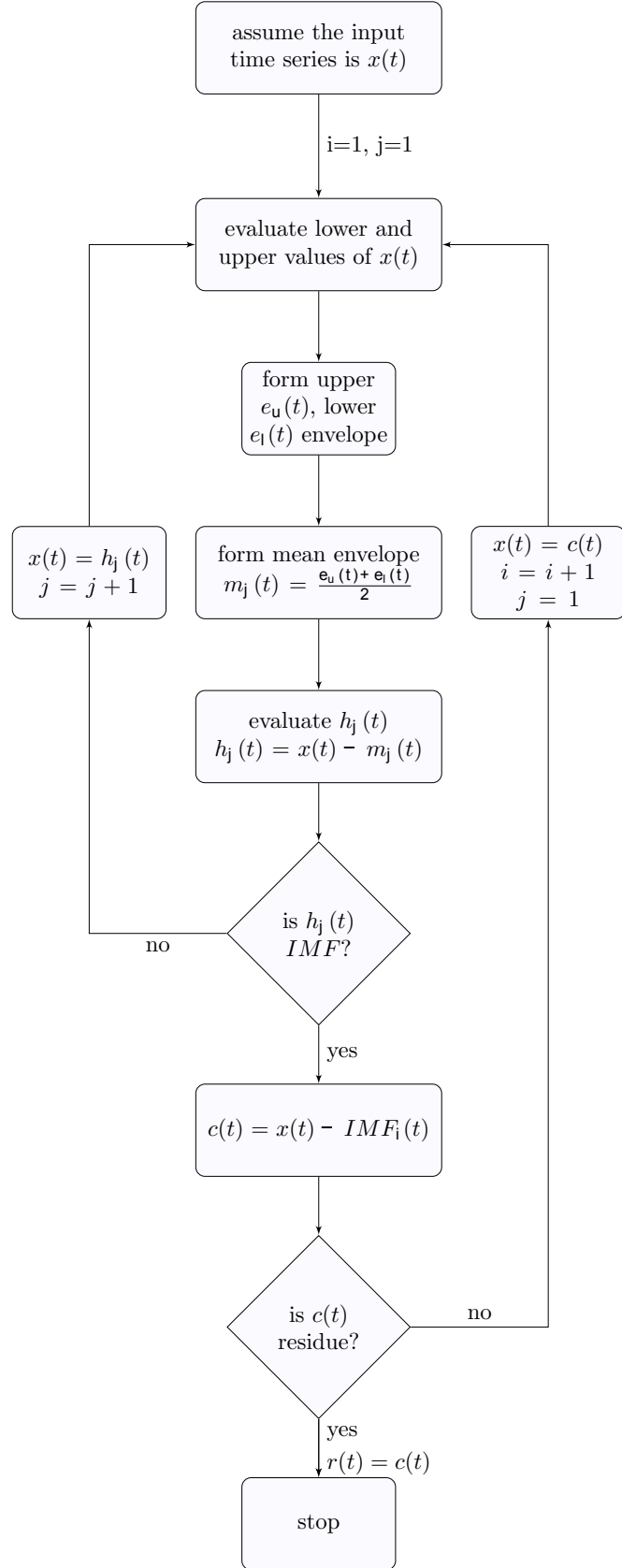


Figure 2.3: Flowchart of EMD estimation process.

2.

$$m(t_i) = \frac{e_u(t_i) + e_l(t_i)}{2} = 0; \quad 1 \leq i \leq N,$$

where $e_u(t)$, $e_l(t)$, and $m(t)$ represent the upper, lower, and mean envelopes functions, respectively, as presented in Section 2.3, i is an integer number, and N the observations number in $x(t)$.

A time series $x(t)$ is taken as an example to show the original time series with its IMFs and residue. The results are displayed in Figure 2.5.

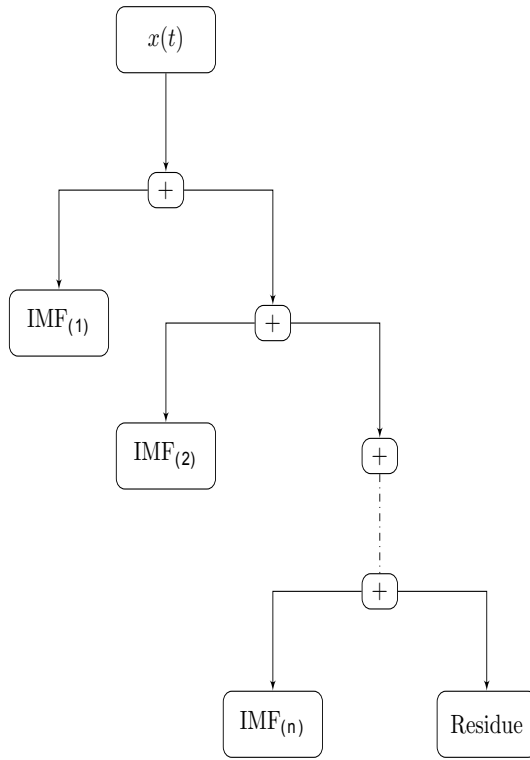


Figure 2.4: Reconstruction of IMFs

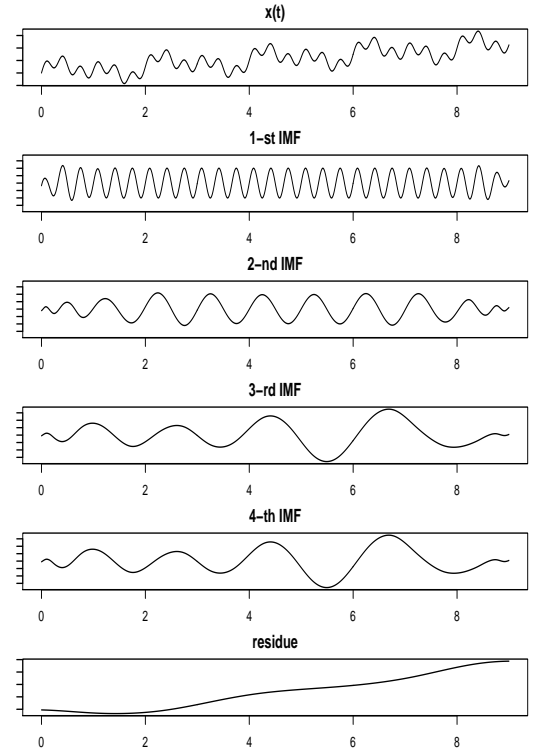


Figure 2.5: A time series $x(t)$ with its IMFs and residue

2.3.3 Limitation, Extension, Comparison, and Applications of EMD

Since its introduction, EMD has been widely used in many research areas, such as in financial time series by Oh et al. (2009) and Jaber et al. (2014); medicine by Mas-selot et al. (2018); mechanical engineering by Zhang et al. (2010), in which the EMD method was employed in the EMD-Golay de-noising algorithm to reduce noise effectively on lidar signals; electronic engineering by Suvasini et al. (2015); sciences, such as climate by Coughlin and Tung (2004), and dynamics by Zhang et al. (2003); civil and construction engineering by OBrien et al. (2017); and traffic by Wang et al. (2016) and Zhang and Pan (2017).

However, the EMD technology has a number of limitations in its algorithm. First, the theoretical base is not fully established, and most of the steps of the EMD methodology ignore mathematical expressions. Flandrin et al. (2004) even declared that no theory exists for EMD. Subsequently, many studies have proposed theoretical assumptions for EMD that are largely defined as algorithmic steps. A time series of size N usually only needs $\log_2 N$ IMFs (Wu et al., 2001). Moreover, the average period of each IMF can be calculated by $\frac{2 \times N}{(\# \text{ of zero crossings})}$ (Amirthanathan et al., 2005). Wu and Huang (2004) assumed that the IMF components are all normally distributed, and the Fourier spectra of the IMF components are all identical and cover the same area on a semi-logarithmic period scale. Kizhner et al. (2006) proposed a number of theoretical fundamentals for the EMD algorithm by offering three hypotheses on the EMD sifting process. However, the theoretical part of EMD remains poorly explained (Rilling and Flandrin, 2006).

The second limitation is related to the sensitivity toward endpoint treatments (i.e., boundary effect) when using an EMD algorithm (Coughlin and Tung, 2004), and

many studies were developed using the EMD methodology to overcome this limitation. Coughlin and Tung (2014) extended the beginning and end of the time series by adding typical waves. Jaber et al. (2014) accompanied local polynomial quantile regression with a sifting process for automatic boundary correction.

The third limitation is related to mode mixing (Huang et al., 2003). Wu et al. (2010) solved this limitation by increasing the amount of EMD iteration with additional mathematical operators based on differentiation and integration. Niang et al. (2010) introduced the partial differential equation and used the approach as an alternative to the algorithm of sifting processes, and then applied this technique on image analysis to their subsequent work in Niang et al. (2012). To further overcome the third limitation, Tang et al. (2012) introduced a novel method based on the revised blind source separation; Li et al. (2017) applied the differential operation to the separation of IMFs; and Moore et al. (2018) introduced wavelet-bound EMD. The five proposed forecasting methodologies in this study able overcome these limitations with the basic EMD.

Many studies provided an extension of the EMD technology. Xu et al. (2006) generalized the EMD technique in two dimensions. Rehman and Mandic (2010) presented the multivariate extensions of EMD. He et al. (2017) presented an efficient three-dimensional EMD to decompose a volume into three-dimensional IMFs. Dragomiretskiy and Zosso (2014) used variational mode decomposition (VMD) as an alternative decomposition method to EMD, and the VMD was applied to the noise reduction of a diesel engine in the study of Yao et al. (2017). Complex VMD was also developed and applied to complex-valued signals (Wang et al., 2017). The ensemble EMD (EEMD) was presented in Wu and Huang (2009) as an extension of EMD, and this EEMD method was applied by Montalvo et al. (2017) and Henzel (2017). The com-

plete EEMD was also developed by Torres et al. (2011). Moreover, the EMD algorithm was modified by Zeng and He (2004), Deering and Kaiser (2005), and Bagherzadeh and Asadi (2017).

Furthermore, many studies compared EMD technique with other decomposition methods. Wang et al. (2011a) compared EMD with wavelet decomposition (WD) in nonlinear time series analysis, and the accuracy of the EMD decomposition was found better than that of the wavelet decomposition. A difficult problem in WD involves the selection of wavelet basis functions and decomposition levels, but EMD did not present such problems. EMD-based HT was found applicable in linear and nonlinear regime decomposition, and so was WD-based WT for linear regime decomposition.

Lu et al. (2013) applied EMD to ultrasonic signals and compared the method with Chirplet signal decomposition. These methods were applied to ultrasonic signal feature extraction, the aim of which was to further explore feature information. The EMD method exhibited accurate parameter estimation of the time series. Moreover, the EMD method was regarded as a dynamic method for tracking changes of a time series.

Ghosh et al. (2014) applied Fourier transform, short-term Fourier transform, wavelet analysis, and EMD techniques to de-noise an electrocardiogram signal, in which these methods were used as filtration techniques. The purpose of their study was to provide a review and comparison of the above methods, and the authors confirmed that EMD was superior to the three other decomposition techniques in analyzing nonlinear and non-stationary signals. The complications of WT method can be diminished by EMD method.