UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2007/2008

Jun 2008

MAT 122 – Differential Equations I
[Persamaan Pembezaan I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

1. (a) The differential equation $(x^4+y^4)dx - xy^3dy = 0$ has an integrating factor of the form $\frac{1}{x^k}$. Determine the value of k and hence solve the differential equation.

   (b) (i) Show that on the interval $[0,\pi]$, both the functions $y_1(x) \equiv 1$ and $y_2(x) = \cos x$ satisfy the initial value problem $\frac{dy}{dx} + \sqrt{1-y^2} = 0, \quad y(0) = 1.$

   (ii) Does the above fact contradict the existence and uniqueness theorem? Explain your answer.

   (c) Solve the following differential equation

$$x\frac{dy}{dx} + 6y = 3xy^{4/3}$$

[25 marks]

2. (a) Solve the following differential equations:

   (i) $y'' + y' - 2y = 2x - 40\cos 2x$

   (ii) $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$

   (b) A model for the competition between two species with population densities x and y leads to the differential equation

$$\frac{dx}{dt} = ax - by, \quad \frac{dy}{dt} = -cx + dy$$

   where a, b, c and d are positive constants.

   (i) Show that x satisfies

$$\frac{d^2x}{dt^2} - (a+d)\frac{dx}{dt} + (ad - bc)x = 0$$

   (ii) Show that x has a solution of the form

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

   with at least one α_i positive.

   (iii) Find the solution for y.

   (iv) Using the values $a=d=4$, $b=1$, $c=4$ and $x(0)=700$, $y(0)=3400$, determine the time t when one of the species becomes extinct.

[25 marks]

1. (a) *Persamaan pembezaan* $(x^4+y^4)dx - xy^3dy = 0$ *mempunyai suatu faktor pengamir dalam bentuk* $\frac{1}{x^k}$.

Tentukan nilai k *dan seterusnya selesaikan persamaan pembezaan tersebut.*

(b) (i) *Tunjukkan bahawa pada selang* $[0,\pi]$, *kedua-dua* $y_1(x)\equiv 1$ $y_2(x)=\cos x$ *memenuhi masalah nilai awal* $\frac{dy}{dx}+\sqrt{1-y^2}=0,\quad y(0)=1.$

(ii) *Adakah hal ini bercanggah dengan Teorem Kewujudan dan Keunikan? Jelaskan jawapan anda.*

(c) *Selesaikan persamaan pembezaan berikut:*

$$x\frac{dy}{dx}+6y=3xy^{4/3}$$

[25 markah]

2. (a) *Selesaiakan setiap persamaan pembezaan berikut:*

(i) $y''+y'-2y=2x-40\cos 2x$

(ii) $y''-3y'+2y=\dfrac{1}{1+e^{-x}}$

(b) *Perkembangan bagi dua species yang bersaing untuk suatu sumber makanan diwakili oleh dua persamaan*

$$\frac{dx}{dt}=ax-by,\quad \frac{dy}{dt}=-cx+dy$$

dengan x,y *sebagai populasi kedua – dua species dan* a,b,c,d *ialah pemalar – pemalar positif.*

(i) *Tunjukkan bahawa x memenuhi*

$$\frac{d^2x}{dt^2}-(a+d)\frac{dx}{dt}+(ad-bc)x=0$$

(ii) *Tunjukkan bahawa x mempunyai penyelesaian di dalam bentuk*

$$x=Ae^{\alpha_1 t}+Be^{\alpha_2 t}$$

dengan sekurang – kurangnya satu α_i *positif.*

(iii) *Cari penyelesaian bagi y.*

(iv) *Dengan menggunakan nilai a=d=4, b=1, c=4 dan x(0)=700, y(0)=3400, tentukan bilakah satu species menjadi pupus.*

[25 markah]

3. (a) Deduce Euler's method for the solution of a first order differential equation $y' = f(t, y), y(t_0) = y_0$.

(b) Apply Euler's method to the initial value problem

$$y' = t^2 + y, y(2) = 1.$$

Use $t_1 = 2.1$, $t_2 = 2.2$, and $t_3 = 2.3$. Generate approximations y_1 to y (2.1), y_2 to y (2.2), and y_3 to y (2.3).

(c) Use the improved Euler's method to obtain the approximate value of y (1.5) for the solution of the initial value problem

$$y' = 2xy, y(1) = 1.$$

Compare the results for $h = 0.1$ and $h = 0.05$.

[25 marks]

4. (a) Find the power series solution of the differential equation

$$y'' + xy' + (x^2 + 2)y = 0$$

in powers of x (that is, about $x_0 = 0$).

(b) Solve the following system of equations:

$$X' = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} X.$$

[25 marks]

3. (a) *Deduksikan kaedah Euler untuk memperolehi penyelesaian bagi persamaan pembezaan peringkat pertama*
$y' = f(t, y), y(t_0) = y_0.$

(b) *Gunakan kaedah Euler bagi masalah nilai awal*

$$y' = t^2 + y, y(2) = 1.$$

Guna $t_1 = 2.1$, $t_2 = 2.2$, *and* $t_3 = 2.3$. *Dapatkan penghampiran* y_1 *bagi* $y(2.1)$, y_2 *bagi* $y(2.2)$, *dan* y_3 *bagi* $y(2.3)$.

(c) *Gunakan kaedah Euler diperbaiki untuk memperolehi nilai hampiran bagi* $y(1.5)$ *untuk penyelesaian masalah nilai awal*

$$y' = 2xy, y(1) = 1.$$

Banding keputusan anda bagi $h = 0.1$ *dan* $h = 0.05$.

[25 markah]

4. (a) *Dapatkan penyelesaian siri kuasa bagi persamaan pembezaan*

$$y'' + xy' + (x^2 + 2)y = 0$$

dalam kuasa x (iaitu sekitar $x_0 = 0$*).*

(b) *Selesaikan sistem persamaan berikut:*

$$X' = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} X.$$

[25 markah]

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