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UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang  
Sidang Akademik 2007/2008

Jun 2008

**MAT 101 – Calculus**  
**[Kalkulus]**

Duration : 3 hours  
[Masa : 3 jam]

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Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions:** Answer **all four** [4] questions.

**Arahan:** Jawab **semua empat** [4] soalan.]

1. (a) Find the following limit:

- (i)  $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^2 + x - 6}$
- (ii)  $\lim_{x \rightarrow 0} \frac{\sin(2x)\sin(4x)}{x}$
- (iii)  $\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 + 1}}$

(b) Prove by using the definition of limit that  $\lim_{x \rightarrow 1} 2x + 1 = 3$ .

- (c) (i) Show that if  $f$  is a differentiable function at  $x = a$ , then  $f$  is continuous at  $x = a$ .
- (ii) Show that the converse statement in (i) is not true, namely give an example of a function  $f$  that is continuous at  $x = a$  but not differentiable at  $x = a$ .

[100 marks]

2. (a) (i) Write the statement of the Intermediate Value Theorem.
- (ii) Show that there is a root of  $\sin x = 1 - x$  in the interval  $(0, 1)$ .

- (b) (i) Write the statement of the Mean Value Theorem.
- (ii) Prove the inequality  $|\sin u - \sin v| \leq |u - v|$  for all real  $u$  and  $v$ .

- (c) For  $f(x) = \begin{cases} cx + 1, & \text{if } x \leq 3 \\ cx^2 - 1, & \text{if } x > 3 \end{cases}$ , find the value of the constant  $c$  such that  $f$  is continuous on  $(-\infty, \infty)$

[100 marks]

3. (a) Find the extremum of  $f(x) = x^3 - 2x^2 + 5$  on the interval  $[0, 6]$ .

(b) Consider the function  $f(x) = \frac{x^3 + 1}{x^3 - 1}$ ,  $x \neq 1$ .

- (i) Find the critical number for  $f$ .
- (ii) Find the interval(s) on which  $f$  is increasing and the interval(s) on which  $f$  is decreasing.
- (iii) Find, if any, the local extremum of  $f$ .
- (iv) Find all the asymptotes of  $f$ .

[100 marks]

1. (a) Cari had berikut:

(i)  $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^2 + x - 6}$

(ii)  $\lim_{x \rightarrow 0} \frac{\sin(2x)\sin(4x)}{x}$

(iii)  $\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 + 1}}$

(b) Bukti dengan menggunakan takrif had bahawa  $\lim_{x \rightarrow 1} 2x + 1 = 3$ .

(c) (i) Tunjukkan bahawa jika  $f$  fungsi terbezakan pada  $x = a$ , maka  $f$  adalah selanjar pada  $x = a$ .

(ii) Tunjukkan bahawa pernyataan akas dalam (i) tidak benar, iaitu beri satu contoh fungsi yang selanjar pada  $x = a$  tetapi tidak terbezakan pada  $x = a$ .

[100 markah]

2. (a) (i) Tulis pernyataan untuk Teorem Nilai Pertengahan.

(ii) Tunjukkan bahawa terdapat suatu punca untuk  $\sin x = 1 - x$  dalam selang  $(0, 1)$ .

(b) (i) Tulis pernyataan untuk Teorem Nilai Min.

(ii) Tunjukkan ketaksamaan  $|\sin u - \sin v| \leq |u - v|$  untuk semua nilai nyata  $u$  dan  $v$ .

(c) Untuk  $f(x) = \begin{cases} cx + 1, & \text{if } x \leq 3 \\ cx^2 - 1, & \text{if } x > 3 \end{cases}$ , cari nilai pemalar  $c$  supaya  $f$  adalah selanjar pada  $(-\infty, \infty)$

[100 markah]

3. (a) Cari ekstremum untuk  $f(x) = x^3 - 2x^2 + 5$  pada selang  $[0, 6]$ .

(b) Pertimbangkan fungsi  $f(x) = \frac{x^3 + 1}{x^3 - 1}$ ,  $x \neq 1$ .

(i) Cari nombor genting untuk  $f$ .

(ii) Cari selang berlakunya  $f$  menokok and selang berlakunya  $f$  menyusut.

(iii) Cari, jika ada, ekstremum tempatan untuk  $f$ .

(iv) Cari semua asimptot untuk  $f$ .

[100 markah]

4. (a) (i) Evaluate the integral  $\int \frac{1}{x\sqrt{4+x^2}} dx$ .

(ii) Find  $g'(1)$  if  $g(t) = \int_1^{2t} \frac{1}{x\sqrt{4+x^2}} dx$ .

(b) Find the volume of the solid generated by revolving the region bounded by the curve  $y = \frac{1}{x^2}$  and the  $x$ -axis, between  $x = 1$  and  $x = 2$  around

(i) the  $x$ -axis

(ii) the line  $x = -1$ .

[100 marks]

4. (a) (i) *Nilaikan kamiran*  $\int \frac{1}{x\sqrt{4+x^2}} dx$ .
- (ii) *Cari*  $g'(1)$  *jika*  $g(t) = \int_1^{2t} \frac{1}{x\sqrt{4+x^2}} dx$ .

(b) *Cari isipadu pepejal yang dijana dengan memutarakan rantau yang dibatasi oleh lengkung*  $y = \frac{1}{x^2}$  *dan paksi-x, di antara*  $x = 1$  *and*  $x = 2$  *sekitar*

- (i) *paksi-x*
- (ii) *garis*  $x = -1$ .

[100 markah]

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