

# BARGAINING GAME AND EQUITY SHARING SYSTEM IN EMPLOYMENT PROBLEM

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## Abstract

Profit sharing has been proposed as a solution to underemployment resulting from collective bargaining, but the empirical results are ambiguous and the theoretical basis for this claim is weak. This paper shows equity sharing can, in theory, solve the principle shortcomings of profit sharing. First, equity sharing can resolve both underemployment and over employment. Second, the results do not depend on assuming that insiders and the firm are prevented from bargaining over employment level. Third, the system does not require government intervention since both sides can benefit from adopting equity sharing.

Keywords: Bargaining game, profit sharing model, equity sharing.

## 1. Introduction

Weitzman (see e.g. [7]) argued within an insider-outsider framework that profit sharing can resolve the underemployment problem. When a firm offers a profit-sharing package, workers each gain additional income from their fraction of the profit share. In a bargaining model this additional income will not come free, but rather displace wage income. Since profit sharing lowers the wage, and since increasing the number of workers does not increase the total profit share paid to workers, profit sharing lowers the firm's marginal cost of employing workers, inducing higher employment. The Weitzman model has been tested extensively on data from a number of countries. The results are mixed, with some studies finding a positive relationship between profit sharing and employment and others finding no such relationship.

A basic problem with the profit sharing model is it assumes the firm unilaterally determines the employment level. As Anderson and Devereux (see e.g. [1]) show, if employment is a bargaining parameter the bargained employment level is independent of the profit share. Hence the assumption that employment is not bargained over is essential to profit sharing efficacy. Weitzman justifies this assumption by arguing that most union contracts do not specify employment levels, a situation which is consistent with union behavior in the Oswald insider-outsider model when the number of insiders is small. Equity sharing is an alternative mechanism for implementing Weitzman's idea regarding profit sharing.

This paper will reexamine the profit-sharing model and show that insiders, while sometimes willing to cede the employment decision to the firm when there is no profit sharing, will not normally do so with profit sharing, making the assumption difficult to justify. The paper also describes an equity sharing system under which insiders individually own equity in the company, equity which does not give insiders a controlling interest. This distinguishes equity sharing from employee buyouts in which employees do take control of the firm (Beu-Ner and B. Jun, see e.g [2]).

## 2. The Profit Sharing

Oswald (see e.g. [6]) proposed that the union utility function differentiates between "inside" and "outside" workers, with outside workers receiving no weight in the utility function. Assuming there are a total of  $\bar{L}_i$  inside workers with identical utility functions  $U(\cdot)$ , where  $U'(\cdot) > 0$  and  $U''(\cdot) < 0$ , and designating employment of inside workers by  $L_i \leq \bar{L}_i$  and outside workers by  $L_0 \geq 0$ , the union utility function is  $U^u = U(w)L_i + U(\bar{w})(\bar{L}_i - L_i)$  where  $w$  is the bargained wage and  $\bar{w}$  is the market wage. Though the assumption that the union has no concern at all for outside workers might be strong, this utility function captures the essence of the insider-outsider division resulting from the political nature of unions.

Adjusting the union utility function to incorporate a profit share  $s$ , and assuming the firm is interested in maximizing profits net of the profit share,  $(1-s)\Pi = (1-s)(R - wL)$

Where total employment is  $L = L_i + L_0$  and revenue is  $R = R(L)$ , the feasible set in the bargaining game is

$$S = \{(U^u, U^f) \in \mathbb{R}^2 \mid U^u \leq U(w + (s/L)\Pi)L_i + U(\bar{w})(\bar{L}_i - L_i), U^f \leq (1-s)\Pi\}.$$

Assuming that in disagreement inside workers earn the market wage  $\bar{w}$  and the firm has zero profits, the disagreement point is  $d = (\bar{w}, 0)$ . With union bargaining power  $\gamma$  the weighted Nash bargaining solution to this game  $(S, d)$  is the solution to

$$\max_{w, L_i, L_0} \ln \left( U \left( w + \frac{s}{L} \Pi \right) L_i + U(\bar{w})(\bar{L}_i - L_i) - U(\bar{w})\bar{L}_i \right)^\gamma ((1-s)\Pi)^{1-\gamma} - \mu(L_i - \bar{L}_i) + \sigma L_0 \quad (1)$$

where firm revenue  $R$  is assumed to satisfy  $R_{LL} < 0$ ,  $R_L(0) > 0$  and  $R(0) = 0$ , and,  $\mu$  and  $\sigma$  are the non-negative Kuhn-Tucker multipliers for the constraints  $L_i \leq \bar{L}_i$  and  $L_0 \geq 0$  respectively. The constraint  $L_i \geq 0$  is never binding and is omitted for simplicity. Maximizing with respect to the wage gives

$$w = \frac{R}{L} - \frac{1-\gamma}{\gamma} \frac{U(w + \frac{s}{L}\Pi) - U(\bar{w})}{(1-s)U'(w + \frac{s}{L}\Pi)}, \quad (2)$$

which implies the income of employed workers in agreement,  $Y = w + \frac{s}{L}\Pi$ , is

$$Y = \frac{R}{L} - \frac{1-\gamma}{\gamma} \frac{U(Y)-U(\bar{w})}{U'(Y)}, \quad (3)$$

Maximizing with respect to  $L_i$ , substituting  $w$  from (Eq. 2), and noting that if  $L_i \leq \bar{L}_i$  then  $L_0 = 0$  since the union will always want an insider to be hired first and the firm is indifferent, the first order condition is

$$R_L = Y - \frac{U(Y)-U(\bar{w})}{U'(Y)} + \mu \frac{U(Y)-U(\bar{w})}{\gamma U'(Y)} L. \quad (4)$$

The first order condition maximizing with respect to  $L_0$  is

$$R_L = Y - \sigma \frac{U(Y)-U(\bar{w})}{\gamma U'(Y) \frac{s\Pi}{L} + (1-\gamma)(U(Y)-U(\bar{w}))} \quad (5)$$

and the complementary slackness conditions are  $\mu(L_i - \bar{L}_i) = 0$  and  $\sigma L_0 = 0$ . From the employment constraints three possibilities exist.

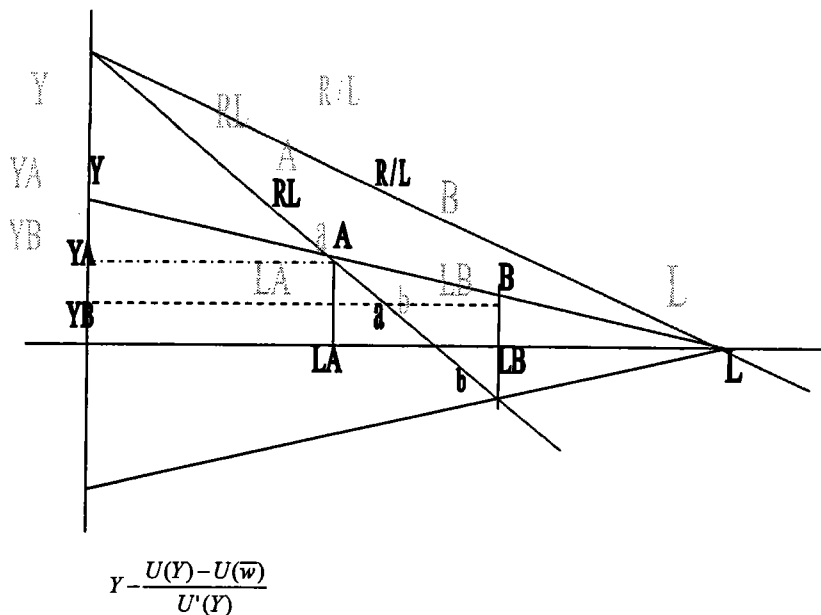
Case I : All insiders are employed ( $L_i = \bar{L}_i$ ) and some outsiders are employed ( $L_0 > 0$ ).

Case II : All insiders are employed ( $L_i = \bar{L}_i$ ) and no outsiders are employed ( $L_0 = 0$ ).

Case III : Some insiders are unemployed ( $L_i < \bar{L}_i$ ) and no outsiders are employed ( $L_0 = 0$ ).

Fig. 1 depicts employment and income levels in the three cases for given bargaining power ( $\gamma = \frac{1}{2}$ ) when the revenue function is quadratic, the outside wage  $\bar{w}$  is zero, and each union member has a constant relative risk aversion utility function with relative risk aversion  $-\frac{U''(Y)Y}{U'(Y)} = \frac{1}{2}$ . This figure differs from most representations of the union-firm

bargaining problem which concentrate on the shape of the contract curve, usually under the case III assumption of some insiders being unemployed. Rather than allowing bargaining power to vary as is true along a contract curve, this figure assumes that bargaining power is fixed and looks at the bargaining outcomes as a function of the number of insiders. The line segment AB in Figure 1 represents the set of all possible bargaining outcomes, a set composed of single points on each different contract curve corresponding to each different number of insiders.



Note:

Case I :  $\bar{L}_i < L_A \Rightarrow L_i = L_A$

Case II :  $L_A \leq \bar{L}_i \leq L_B \Rightarrow L_i = \bar{L}_i$

Case III:  $\bar{L}_i > L_B \Rightarrow L_i = L_B$

Fig. 1 Underemployment and Over employment

In case I the number of insiders is less than  $L_A$  and insiders agree to the hiring of outsiders up to the point where the total number of worker is  $L_A$ . This is the point where worker income  $Y$  equals the marginal revenue of labor from (Eq. 5). Since the extra workers push income down insiders would prefer not to allow this employment of outsiders, but the revenue gains are so large that the firm can successfully bargain for the extra employment. Employment is still inefficiently low, as seen from the triangle a representing welfare loss at employment level  $L_A$ . Note that if insider bargaining power  $\gamma$  is large then outside employment is small and this loss is large. If insiders have no bargaining power ( $\gamma = 0$ ) then worker income is pushed down to the opportunity cost of labor and there is no efficiency loss.

Case II represents the intermediary situation where the firm does not hire any outside workers but does hire all the insiders. The employment level is therefore fixed at  $\bar{L}_i$ ,

resulting in either underemployment or over employment depending on the number of insiders. Since worker income is still given by (Eq. 3) and is above the marginal product of labor, the firm would prefer to lower employment but is unable to do so in negotiations.

Case III represents the opposite situation in which the number of insiders is so large and the losses of excessive employment so high that the firm not only hires no outsiders but refuses to hire all insiders. For any number of insiders greater than  $L_B$  only  $L_B$  of them are hired. Still this represents some over employment as represented by the welfare loss of triangle  $b$ . Since workers receive income from (Eq. 3) above the marginal product of labor, the firm would prefer to lower employment. Risk averse workers are particularly anxious to avoid unemployment though, and can successfully bargain for some excess employment of insiders as determined by (Eq. 5). Comparable to the underemployment case, over employment will be higher is insider bargaining power is higher, and the problem is eliminated if insiders have no bargaining power.

Weitzman showed that if the wage but not employment were bargained over profit sharing would increase employment by lowering the wage. This can be seen from (Eq. 2), which implies  $\frac{dw}{ds} < 0$ . If we ignore (Eq. 4) and (Eq. 5) and assume the firm can unilaterally hire workers, the lower wage will increase employment. This decrease can be excessive though. From (Eq. 2) when,

$$s = 1 - \frac{1-\gamma}{\gamma} \frac{L}{\pi} \frac{U(Y) - U(\bar{w})}{U'(Y)} \quad (6)$$

the bargained wage falls to the market wage of  $\bar{w}$ , leading to efficient employment. But for profit shares above this level the market wage is pushed below  $\bar{w}$  and demand for labor becomes excessive. Since worker income is still greater than  $\bar{w}$  from (Eq. 3) for  $\frac{R}{L} > \bar{w}$ , workers will accept employment at this wage and over employment results.

### 3. The Equity Sharing Solution

As a solution to inefficient employment resulting from collective bargaining, profit sharing was shown to have several problems. This section considers equity sharing as an alternative system that differs from profit sharing in two essential ways: first, individual workers own equity stakes rather than workers as a group claiming a profit share; second, workers retain their equity stakes even if they leave the firm. Equity sharing can resolve underemployment because of the first difference and can resolve over employment because of the second difference.

When all insiders have identical equity stakes summing to a fraction  $e$  of outstanding equity, the feasible set  $S$  of the bargaining game  $(S, d)$  become

$$S = \{(U^u, U^f) \in \mathfrak{R}^2 \mid U^u \leq U(w + \left(\frac{e}{L_i}\right)\Pi)L_i + U(\bar{w} + \left(\frac{e}{L_i}\right)\Pi)(\bar{L}_i - L_i), U^f \leq (1-e)\Pi\}.$$

Since firm profits are zero in disagreement the disagreement point remains unchanged at  $d = (\bar{w}, 0)$

Note that the  $\frac{s}{L}$  term in the profit sharing model has been changed to  $\frac{e}{\bar{L}_i}$  incorporate the first difference between equity sharing and profit sharing. The second difference is reflected by unemployed insiders  $\bar{L}_i - L_i$  also receiving an equity stake. The weighted Nash bargaining problem is

$$\max_{w, L_i, L_0} \ln \left( U \left( w + \frac{e}{\bar{L}_i} \Pi \right) L_i + U \left( \bar{w} + \frac{e}{\bar{L}_i} \Pi \right) (\bar{L}_i - L_i) - U(\bar{w}) \bar{L}_i \right)^{\gamma} ((1-e)\Pi)^{1-\gamma} \quad (7)$$

$$- \mu(L_i - \bar{L}_i) + \sigma L_0$$

where the complementary slackness conditions are as before.

**Proposition 1.** *Equity sharing can eliminate the underemployment problem.*

**Proof:** Underemployment occurs under case I and under case II when  $R_L(\bar{L}_i) > \bar{w}$ . Case I:

Noting that  $L_i = \bar{L}_i$  and  $L_0 > 0$ , differentiating (Eq. 7) with respect to  $w$  and  $L_0$  gives

$$w = \frac{R}{L} - \frac{1-\gamma}{\gamma} \frac{\bar{L}_i}{\bar{L}_i - eL} \frac{U(Y) - U(\bar{w})}{U'(Y)}, \quad (8)$$

$$R_L = w - \sigma \frac{U(Y) - U(\bar{w})}{\gamma U'(Y) \frac{e\Pi}{\bar{L}_i} + (1-\gamma)(U(Y) - U(\bar{w}))} \quad (9)$$

From (Eq. 8) when

$$e = \frac{L_i}{L} - \frac{1-\gamma}{\gamma} \frac{\bar{L}_i}{\Pi} \frac{U(Y) - U(\bar{w})}{U'(Y)} \quad (10)$$

the bargained wage is  $\bar{w}$ , implying from (Eq. 9) and the complementary slackness condition  $\sigma L_0 = 0$  that the bargained employment level is such that workers are employed up until the marginal revenue product of labor equals the opportunity cost of labor.

Case II: Since  $L_i = \bar{L}_i$  (Eq. 8) still applies. The first order condition with respect to  $L_i$  when  $w = \bar{w}$  is

$$R_L = \bar{w} + \mu \frac{U(Y) - U(\bar{w})}{\gamma U'(Y) \frac{e\Pi}{\bar{L}_i} + (1-\gamma)(U(Y) - U(\bar{w}))} \quad (11)$$

By the non negativity of the Khun-Tucker multipliers (Eq. 9) and (Eq. 11) are both satisfied only when  $R_L(L_i) = \bar{w}$ , indicating case II occurs only when employment is efficient.

**Proposition 2.** *Equity sharing can eliminate the over employment problem if the feasible set is convex.*

**Proof:** Over employment occurs under case III and under case II when  $R_L(\bar{L}_i) < w$ . Case III: Noting that  $L = L_i$  since  $L_0 = 0$ , differentiating (Eq. 7) with respect to  $w$  and then setting  $w = \bar{w}$  gives

$$\bar{w} = \frac{R}{L} \frac{1-\gamma}{\gamma} \frac{\bar{L}_i}{L_i - eL} \frac{U(Y) - U(\bar{w})}{U'(Y)}, \quad (12)$$

indicating that

$$e = \frac{L_i}{L} \frac{1-\gamma}{\gamma} \frac{\bar{L}_i}{\Pi} \frac{U(Y) - U(\bar{w})}{U'(Y)} \quad (13)$$

is consistent with  $w = \bar{w}$ . Since  $\mu = 0$  when  $L_i < \bar{L}_i$ , equation (11) implies  $R_L = \bar{w}$  when  $w = \bar{w}$ , indicating that the employment choice is efficient. Note that the given value of  $e$  might also be consistent with other stationary points with different values of  $w$ . Since  $w = \bar{w}$  implies efficient employment and equal distribution of income among inside workers, this solution is pareto efficient. No other value of  $w$  can be pareto efficient and a solution if the feasible set is convex.

Case II: By the same argument as in Proposition 1, when the equity share is set such that  $w = \bar{w}$  this case only occurs when  $R_L = \bar{w}$ , indicating efficient investment.

In the above analysis the equity share is exogenous. An exogenously set share is necessary in the profit sharing model, but not in the equity sharing model. Because inside workers need not share their dividends with outside workers, they can benefit from efficiency gains induced by equity sharing.

**Proposition 3.** *If insiders and the firm bargain over the wage, employment, the equity share, and the equity price, (i) employment is efficient and (ii) insiders are better off than without equity sharing.*

**Proof:**

(i) Assuming that insiders pay current stockowners directly for their shares and designating the equity price by  $q$ , the feasible set becomes

$$S = \{(U^u, U^f) \in \mathfrak{R}^2 \mid U^u \leq U(w + \left(\frac{e}{L_i}\right)(\Pi - q)L_i + U(\bar{w} + \left(\frac{e}{L_i}\right)(\Pi - q)(\bar{L}_i - L_i)), U^f \leq (1-e)\Pi + eq\}$$

and the disagreement point is unchanged. Rather than solving the bargaining game directly with this feasible set, consider the set

$$S' = \{(U^u, U^f) \in \mathfrak{R}^2 \mid U^u \leq U(\beta \frac{R^*}{L_i} \bar{L}_i), U^f \leq (1-\beta)R^*, \text{ for } 0 \leq \beta \leq 1 \} \quad \text{where}$$

$R^* = R(L^*)$  and  $L^*$  is the efficient employment level. In this set the monetary surplus is at its largest and each insider can attain an equal share of this surplus.  $S'$  therefore contains all the points in  $S$  since if employment is inefficient  $R < R^*$ , and if the distribution of

income among insiders is not equal  $U^u < U(\beta \frac{R}{L_i}) \bar{L}_i$ . Moreover  $S$  also contains all the

points in  $S'$ . Setting  $w = \bar{w}$  and  $L = L^*$  in set  $S$ , letting  $\beta = \bar{w} \bar{L}_i + e \frac{(R^* - \bar{w} L^* - q)}{R^*}$

indicates that any distribution of firm revenue between the firm and insiders can be attained by adjusting  $e$  and  $q$ . Since  $S = S'$  the solution to the game  $(S, d)$  is the solution to the game  $(S', d)$ . This is simply

$$\beta = 1 - \frac{1-\gamma}{\gamma} \frac{U(\beta \frac{R^*}{L_i}) - U(\bar{w})}{U'(\beta \frac{R^*}{L_i})} \frac{\bar{L}_i}{R^*} \quad (14)$$

or, letting  $Y = \beta \frac{R^*}{L_i}$ ,

$$Y = \frac{R^*}{L_i} - \frac{1-\gamma}{\gamma} \frac{U(Y) - U(\bar{w})}{U'(Y)} \quad (15)$$

which is efficient.

(ii) When  $e = 0$  and  $q = 0$  the bargaining problem reduces to the profit sharing bargaining problem when  $s = 0$ . From (Eq. 3) total insider income is

$(\frac{R}{L} - (1-\gamma) \frac{(U(Y) - U(\bar{w}))}{\gamma U'(Y)}) L_i + \bar{w}(\bar{L}_i - L_i)$ . This compares with total insider income

under equity sharing from (Eq. 15). If  $L_i = \bar{L}_i$ , as occurs when there is initially underemployment, then total income of insiders is higher with equity sharing since  $R^* > R$  and  $\bar{L}_i \leq L$ . If  $L_i < \bar{L}_i$ , as occurs when there is initially over employment, then total income of insiders is also higher with equity sharing since  $Y \geq \bar{w}$  and  $R^* > R$ . In each case since income is equally distributed among insiders higher total income implies higher total utility for insiders.

A problem might arise if the bargaining is done in two stages, the first stage covering the equity share and equity price and the second stage covering the wage and employment. Since dividend income displaces wage income in the bargaining solution, insiders can increase their income by selling the stock before the second stage. Anticipating this sell off, the firm will not be willing to sell the stock at a discount in the first stage. This problem can be prevented by locking workers in to their equity stake for a fixed period as occurs with most employee stock ownership plans. Generally, the problem will be lessened when capital market imperfections limit the ability of workers to sell a large block of stock or when workers have particular reasons for holding on to their firm's stock.

It is worthwhile to briefly compare equity sharing with other mechanisms that have been suggested to resolve underemployment and over employment. Regarding underemployment, one possibility is to use a two-tier wage structure in which new workers are paid less than existing workers. Equity sharing mimics this result because inside workers receive a higher income due to their equity holdings. Since all workers still



receive the same wage the result may be more acceptable to insiders and to new workers. It has also been proposed that unions incorporate and require any new members to buy a share in the union. Sales of these shares would compensate existing members for the losses of extra employment, leading to an efficient outcome. Related, the workers can buy the firm and require new workers to also buy stock in the firm (this would correspond to a labor-managed firm with a membership fee. Such a fee can eliminate the problem of underemployment in labor-managed firms, a problem which is closely related to that of underemployment with unions. Note that equity sharing can achieve these same results in a much simpler fashion. Only inside workers need to purchase stock and they need not acquire all of the stock nor acquire control of the firm.

Regarding over employment, cash payments to exiting workers is one solution. The problem is that workers will still press to enter the firm if the wage is above the market wage. This might explain why cash payments are often used for managerial staff but rarely for unionized workers. Equity sharing works as an equivalent method that is self-enforcing. Since the equity share pushes down the wage rate to the market wage, workers are willing to leave the firm voluntarily. Although profit sharing also pushes down the wage rate, workers will not leave because they forfeit their profit share upon exit.

#### 4. Conclusion

Equity sharing success depends on one similarity with and two key differences from profit sharing. The similarity is that in a bargaining model income from the equity share pushes down the wage rate. The first difference is that each insider receives a portion of the firm's profits, rather than workers as a whole receiving a share which is then divided. The second difference is that workers retain a right to their equity share even if they leave the company. As a result insiders are more willing to allow new workers to be hired when employment is inefficiently low, and more willing to exit the firm when employment is inefficiently high.

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