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UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang  
Sidang Akademik 2007/2008

Jun 2008

**MAA 102 – Calculus for Science Students II**  
***[Kalkulus untuk Pelajar Sains II]***

Duration : 3 hours  
*[Masa : 3 jam]*

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Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions:** Answer **all ten** [10] questions.

**Arahan:** Jawab **semua 10** [10] soalan.]

1. Determine whether the sequence  $\left\{ \frac{2n}{n+1} \right\}$  is monotonic.

[5 marks]

2. Test the convergence of the series:

(a)  $\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$

(b)  $\sum_{n=1}^{\infty} (-1)^n \frac{2^{2n}}{3^n}$

[10 marks]

3. (a) Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} n!(2x-1)^n.$$

- (b) Find the power series representation for  $\frac{1}{1-x^2}$ . Hence, find the first three terms of the power series representation of  $f(x) = \frac{1+x^2}{1-x^2}$  which holds for  $|x| < 1$ .

[15 marks]

4. Determine whether the integral  $\int_2^{\infty} \frac{2+\cos x}{x} dx$  is convergent or divergent.

[5 marks]

5. (a) Find the domain of the function  $f(x, y) = \ln(9 - 9y^2 - x^2)$ .

- (b) Show that  $w = \ln(x^2 + y^2 + z^2)$  satisfies the partial differential equation

$$x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} + z \frac{\partial w}{\partial z} = 2.$$

[7 marks]

6. Let  $z = f(x, y)$ , where  $f$  is differentiable,  $x = r + \cos r$  and  $y = e^r$ .

If  $f_x(1, 1) = 5$  and  $f_y(1, 1) = 0$ , find  $\frac{dz}{dr}$  when  $r = 0$ .

[8 marks]

1. Tentukan sama ada jujukan  $\left\{ \frac{2n}{n+1} \right\}$  monotonik.  
[5 markah]
2. Uji penumpuan siri:
- (a)  $\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$                       (b)  $\sum_{n=1}^{\infty} (-1)^n \frac{2^{2n}}{3^n}$   
[10 markah]
3. (a) Dapatkan jejari dan selang penumpuan bagi siri kuasa  
 $\sum_{n=1}^{\infty} n!(2x-1)^n$ .
- (b) Dapatkan perwakilan siri kuasa bagi  $\frac{1}{1-x^2}$ . Seterusnya, dapatkan tiga sebutan pertama perwakilan siri kuasa  $f(x) = \frac{1+x^2}{1-x^2}$  yang sah untuk  $|x| < 1$ .  
[15 markah]
4. Tentukan sama ada kamiran  $\int_2^{\infty} \frac{2+\cos x}{x} dx$  menumpu atau mencapah.  
[5 markah]
5. (a) Dapatkan domain bagi fungsi  $f(x, y) = \ln(9 - 9y^2 - x^2)$ .
- (b) Tunjukkan bahawa  $w = \ln(x^2 + y^2 + z^2)$  memenuhi persamaan pembezaan separa  $x \frac{\partial w}{\partial x} + y \frac{\partial w}{\partial y} + z \frac{\partial w}{\partial z} = 2$ .  
[7 markah]
6. Biar  $z = f(x, y)$ ,  $f$  fungsi yang terbezakan,  $x = r + \cos r$  dan  $y = e^r$ .  
Jika  $f_x(1,1) = 5$  dan  $f_y(1,1) = 0$ , dapatkan  $\frac{dz}{dr}$  bila  $r = 0$ .  
[8 markah]

7. Find the rate of change of  $f(x, y, z) = x^2y + y^2z + z^2x$  at  $P(2, 1, 3)$  in the direction of the vector  $\mathbf{v} = -2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ .

[10 marks]

8. Find the shortest distance from the point  $(2, 1, -1)$  to the plane  $x + y - z = 1$ .

[12 marks]

9. Evaluate the integrals.

(a) 
$$\int_0^4 \int_0^{\sqrt{4y-y^2}} \frac{1}{\sqrt{x^2+y^2}} dx dy.$$

(b) 
$$\iint_D \frac{y}{x^2+1} dA, \text{ where } D \text{ is the region bounded by } y = \sqrt{x}, y = 0 \text{ and } x = 1.$$

[12 marks]

10. (a) Find the integrating factor for the differential equation  $x \frac{dy}{dx} - 3y = x^3$ .  
Hence, find the particular solution when  $y(1) = 1$ .

- (b) Find the orthogonal trajectories of the family of curves  $y = (x + k)^{-1}$ , where  $k$  is an arbitrary constant.

[16 marks]

7. Dapatkan kadar perubahan  $f(x, y, z) = x^2y + y^2z + z^2x$  pada  $P(2, 1, 3)$  dalam arah vektor  $v = -2i - j - 3k$ .

[10 markah]

8. Dapatkan jarak terdekat dari titik  $(2, 1, -1)$  ke satah  $x + y - z = 1$ .

[12 markah]

9. Nilaikan kamiran.

(a) 
$$\int_0^4 \int_0^{\sqrt{4y-y^2}} \frac{1}{\sqrt{x^2+y^2}} dx dy.$$

(b) 
$$\iint_D \frac{y}{x^2+1} dA, D \text{ merupakan rantau yang dibatasi oleh } y = \sqrt{x}, y = 0$$
  
dan  $x = 1$ .

[12 markah]

10. (a) Dapatkan faktor pengamir bagi persamaan pembezaan  $x \frac{dy}{dx} - 3y = x^3$ .

Seterusnya, dapatkan penyelesaian khusus bila  $y(1) = 1$ .

- (b) Dapatkan famili trajektori ortogonal bagi lengkung  $y = (x+k)^{-1}$ ,  
 $k$  pemalar sebarang.

[16 markah]