UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2007/2008

April 2008

MSS 212 – Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all five** [5] questions.

*[**Arahan:** Jawab **semua lima** [5] soalan.]*

1. (a) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & {}^2C_1 & {}^3C_1 & {}^4C_1 \\ 1 & {}^3C_2 & {}^4C_2 & {}^5C_2 \\ 1 & {}^4C_3 & {}^5C_3 & {}^6C_3 \end{bmatrix}$. Show that $\det A = 1$.

[70 marks]

(b)
$$x + 2y + 3z = -1$$
$$w - 2x - y - z = \frac{1}{2}$$
$$w + y - z = \frac{-1}{2}$$
$$2w + 3x + z = 0$$

Solve the above simultaneous equations using Cramer's Rule

[50 marks]

2. Let $V = \{p(x) \in P_4(\mathbb{C}) | p(0) = 0\}$.

Let $W = \{(a_1, a_2, a_3, a_4, a_5, a_6) \in \mathbb{C}^6 | a_3 = a_4 - a_5,\ a_1 = a_2 = a_6\}$

Show that V is isomorphic to W over $\mathbb{C}$ by constructing an isomorphism.

[140 marks]

3. Let $\mathbb{F}$ be a field. A sequence $\{a_n\}$ in $\mathbb{F}$ is a function
$$\sigma: \mathbb{Z}^+ \longrightarrow \mathbb{F}$$
such that $\sigma(n) = a_n$

Let $V = \{\text{all sequences } \{a_n\} \text{ in } \mathbb{R}\}$

For any $\{a_n\}, \{b_n\} \in V, r \in \mathbb{R}$, define

$$\{a_n\} + \{b_n\} = \{a_n + b_n\}$$

$$r \cdot \{a_n\} = \{ra_n\}$$

Show that with these $'+'$ and $'\cdot'$, V is a vector space over $\mathbb{R}$.

[100 marks]

1. (a) *Biar* $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & {}^{2}C_1 & {}^{3}C_1 & {}^{4}C_1 \\ 1 & {}^{3}C_2 & {}^{4}C_2 & {}^{5}C_2 \\ 1 & {}^{4}C_3 & {}^{5}C_3 & {}^{6}C_3 \end{bmatrix}$. *Tunjukkan* $\det A = 1$.

[70 markah]

(b)
$$\begin{aligned} x + 2y + 3z &= -1 \\ w - 2x - y - z &= \tfrac{1}{2} \\ w + y - z &= \tfrac{-1}{2} \\ 2w + 3x + z &= 0 \end{aligned}$$

Selesaikan persamaan serentak di atas dengan menggunakan petua Cramer.

[50 markah]

2. *Biar* $V = \{p(x) \in P_4(\mathbb{C}) | p(0) = 0\}$.

Biar $W = \{(a_1, a_2, a_3, a_4, a_5, a_6) \in \mathbb{C}^6 | a_3 = a_4 - a_5,\ a_1 = a_2 = a_6\}$

Tunjukkan V adalah isomorfisma dengan W atas $\mathbb{C}$ *dengan membina suatu isomorfisma.*

[140 markah]

3. *Biar* $\mathbb{F}$ *ialah suatu medan. Suatu jujukan di* $\mathbb{F}$, $\{a_n\}$, *adalah suatu fungsi*

$$\sigma: \quad \mathbb{Z}^+ \to \mathbb{F}$$

sedemikian hingga $\sigma(n) = a_n$

Biar $V = \{\text{semua jujukan } \{a_n\} \text{ di } \mathbb{R}\}$

Bagi setiap $\{a_n\}, \{b_n\} \in V, r \in \mathbb{R}$, *takrif*

$$\{a_n\} + \{b_n\} = \{a_n + b_n\}$$

$$r \cdot \{a_n\} = \{ra_n\}$$

Tunjukkan dengan '+' *dan* '·' *ini, V is adalah suatu ruang vector atas* $\mathbb{R}$.

[100 markah]

4. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{pmatrix}$

(a) Determine whether A can be diagonalised or not

[100 marks]

(b) Find JCF(A)

[20 marks]

5. Let $M_{2\times2}(\mathbb{R})$ be an inner product space over $\mathbb{R}$ with the following defined inner product:

For any $A = (a_{ij}), B = (b_{ij}) \in M_{2\times2}(\mathbb{R})$

$$< A, B >= \sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij}b_{ij}$$

Define a linear transformation $T: M_{2\times2}(\mathbb{R}) \rightarrow M_{2\times2}(\mathbb{R})$

such that $(A)T = A^T$ (the transpose of A)

(a) Show that T is self-adjoint.

[40 marks]

(b) Find an orthonormal basis β of $M_{2\times2}(\mathbb{R})$ such that $T_{\beta,\beta}$ is a diagonal matrix.

[80 marks]

4. *Biar* $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{pmatrix}$

(a) *Tentukan samada A boleh diperpenjurukan atau tidak*

[100 markah]

(b) *Cari JCF(A)*

[20 markah]

5. *Biar* $M_{2\times2}(\mathbb{R})$ *ialah suatu ruang hasil darap terkedalaman atas* $\mathbb{R}$ *dengan penakrifan hasil darab terkedalaman seperti berikut :*

Bagi setiap $A = (a_{ij}), B = (b_{ij}) \in M_{2\times2}(\mathbb{R})$

$$< A, B >= \sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij}b_{ij}$$

Takrif suatu tranformasi linear $T: M_{2\times2}(\mathbb{R}) \rightarrow M_{2\times2}(\mathbb{R})$

sedemikian hingga $(A)T = A^T$ *(transposisi bagi A)*

(a) *Tunjukkan T adalah swadampingan.*

[40 markah]

(b) *Cari suatu asas ortonormal* β bagi $M_{2\times2}(\mathbb{R})$ *sedemikian hingga* $T_{\beta,\beta}$ *adalah suatu matrik pepenjuru.*

[80 markah]

- 000 O 000 -