UNIVERSITI SAINS MALAYSIA

Second Semester Examination
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MSG 367 – Time Series Analysis
[Analisis Siri Masa]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIXTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all four** [4] questions.

*[**Arahan** : Jawab **semua empat** [4] soalan.]*

1. (a) The first step in the UBJ modeling procedure involves identification of the appropriate model. Explain how the identification process for an ARIMA model differs from the identification process for a SARIMA model.

[20 marks]

(b) When a tentative time series model is going through a diagnostic checking, two of the important elements are to check for adequacy of the order of the fitted model and also the possible existence of ARCH effect. Explain the tests for the two elements. How the results or the outcome of the diagnostic checking can be used to derive a new and better model?

[30 marks]

(c) Let $X_t = a + bt + ct^2 + \cos\left(\frac{\pi t}{2}\right) + Z_t$, for $t = 0, \pm 1, \pm 2, \ldots$ where *a, b, c* and *d* are constants and $\{Z_t\} \sim WN\left(0, \sigma_\varepsilon^2\right)$.

(i) Explain why $U_t = X_t - X_{t-4}$ is not stationary.

(ii) Find the mean, variance and autocovariance function of $V_t = U_t - U_{t-1}$. Is $\{V_t\}$ a stationary process? Briefly explain your reason.

[30 marks]

(d) Rewrite each of the models below using the backward operator *B* and state the form of ARIMA(*p,d,q*) or SARIMA(*p,d,q*)(*P,D,Q*). [*p, d, q, P, D*, and *Q* are positive finite numbers].

(i) $Y_t = Y_{t-1} + \phi_2 Y_{t-2} - \phi_1 Y_{t-3} + \varepsilon_t - \theta_1(\varepsilon_{t-1} - \varepsilon_{t-2})$

(ii) $Y_t = \varepsilon_t - (1-\theta_1)\varepsilon_{t-1} - (1-\theta_1)\varepsilon_{t-2} - \ldots$

(iii) $Y_t = (1+\phi_1)Y_{t-1} - \phi_1 Y_{t-2} + \varepsilon_t + \theta_1\varepsilon_{t-1} - \theta_{12}\varepsilon_{t-12} - \theta_1\theta_{12}\varepsilon_{t-13}$

(iv) $Y_t = \varepsilon_t + 0.7\varepsilon_{t-2} + 0.49\varepsilon_{t-4} + 0.343\varepsilon_{t-6} + \ldots + 0.118\varepsilon_{t-12} + \ldots$

[20 marks]

1. (a) *Langkah pertama dalam prosedur permodelan UBJ melibatkan pengecaman model yang bersesuaian. Terangkan bagaimana proses pengecaman untuk model ARKPB berbeza daripada proses pengecaman untuk model bermusim ARKPB.*

[20 markah]

(b) *Apabila sesuatu model siri masa menjalani penyemakan diagnostik, dua daripada elemen penting adalah untuk memeriksa kecukupan peringkat model yang telah disuai dan juga kemungkinan wujudnya kesan ARCH. Terangkan ujian-ujian untuk dua elemen tersebut. Bagaimana keputusan ataupun hasil dapatan penyemakan diagnostik boleh digunakan untuk membina suatu model yang baru dan lebih baik?*

[30 markah]

(c) *Andaikan* $X_t = a + bt + ct^2 + \cos\left(\frac{\pi t}{2}\right) + Z_t$, *bagi* $t = 0, \pm 1, \pm 2, \ldots$

yang mana a, b, c dan d adalah pemalar dan $\{Z_t\} \sim WN\left(0, \sigma_\varepsilon^2\right)$.

(i) *Terangkan mengapa* $U_t = X_t - X_{t-4}$ *adalah tidak pegun.*

(ii) *Cari min, varians dan fungsi autokovarians bagi* $V_t = U_t - U_{t-1}$. *Adakah* $\{V_t\}$ *suatu proses yang pegun? Terangkan secara ringkas alasan anda.*

[30 markah]

(d) *Tulis semula setiap model di bawah menggunakan pengoperasi anjak kebelakang B dan nyatakan bentuk ARKPB(p,d,q) atau bermusim ARKPB(p,d,q)(P,D,Q). [p, d, q, P, D dan Q adalah nombor-nombor positif terhingga]*

(i) $Y_t = Y_{t-1} + \phi_2 Y_{t-2} - \phi_1 Y_{t-3} + \varepsilon_t - \theta_1(\varepsilon_{t-1} - \varepsilon_{t-2})$

(ii) $Y_t = \varepsilon_t - (1-\theta_1)\varepsilon_{t-1} - (1-\theta_1)\varepsilon_{t-2} - \ldots$

(iii) $Y_t = (1+\phi_1)Y_{t-1} - \phi_1 Y_{t-2} + \varepsilon_t + \theta_1\varepsilon_{t-1} - \theta_{12}\varepsilon_{t-12} - \theta_1\theta_{12}\varepsilon_{t-13}$

(iv) $Y_t = \varepsilon_t + 0.7\varepsilon_{t-2} + 0.49\varepsilon_{t-4} + 0.343\varepsilon_{t-6} + \ldots + 0.118\varepsilon_{t-12} + \ldots$

[20 markah]

2. (a) Given an ARMA(1,1) process for zero-mean series:

$$Y_t = \phi_1 Y_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1}.$$

It is known that the given process is stationary for $|\phi_1| < 1$. Show that the process can be rewritten as:

$$Y_t = \phi_1^j Y_{t-j} + \varepsilon_t + (\phi_1 - \theta_1)\varepsilon_{t-1} + (\phi_1 - \theta_1)\phi_1 \varepsilon_{t-2} + \ldots + (\phi_1 - \theta_1)\phi_1^{j-3}\varepsilon_{t-(j-2)}$$
$$+ (\phi_1 - \theta_1)\phi_1^{j-2}\varepsilon_{t-(j-1)} - \phi_1^{j-1}\theta_1 \varepsilon_{t-j}$$

Explain using the new representation why:

(i) the process is stationary when $|\phi_1| < 1$

(ii) the process is non-stationary if $|\phi_1| \geq 1$

[30 marks]

(b) Once again consider the ARMA(1,1) process as in part (i). Show that:

(i) $\gamma_0 = \dfrac{\left(1 - 2\theta_1\phi_1 + \theta_1^2\right)}{1 - \phi_1^2}$ (ii) $\rho_k = \dfrac{(1 - \theta_1\phi_1)(\phi_1 - \theta_1)}{1 - 2\theta_1\phi_1 + \theta_1^2}\phi_1^{k-1}$

[20 marks]

(c) Given an ARMA(2,1) process:

$$\left(1 - \phi_1 B - \phi_2 B^2\right)Y_t = (1 - \theta_1 B)\varepsilon_t$$

Find a general formula for autocovariance, autocorrelation and partial autocorrelation functions of the process.

(i) Mr. Kid was informed that a series of Ringgit-US Dollar exchange rate returns with 200 observations has been adequately fitted by ARMA(2,1) model with the following coefficients: $\phi_1 = -0.7$, $\phi_2 = 0.18$ and $\theta_1 = 0.15$.

Assist Mr. Kid to calculate the values of autocorrelation for lag k = 1, 2, 3, 4, 5, 6, and partial autocorrelation for lag k = 1 and 2.

[Given the values of acf at lag 6 through to 10 are 0.561, -0.485, 0.440, -0.295 and 0.350 respectively, and pacf at lag 3 through to 8 are -0.030, -0.051, -0.081, 0.140, 0.127 and 0.062 respectively].

2. (a) *Diberi suatu proses* $ARPB(1,1)$ *bagi siri dengan min bukan sifar:*

$$Y_t = \phi_1 Y_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1}.$$

Adalah diketahui bahawa proses yang diberi adalah pegun apabila $|\phi_1| < 1$. *Tunjukkan bahawa proses tersebut boleh ditulis semula sebagai:*

$$Y_t = \phi_1^j Y_{t-j} + \varepsilon_t + (\phi_1 - \theta_1)\varepsilon_{t-1} + (\phi_1 - \theta_1)\phi_1 \varepsilon_{t-2} + \ldots + (\phi_1 - \theta_1)\phi_1^{j-3}\varepsilon_{t-(j-2)} + (\phi_1 - \theta_1)\phi_1^{j-2}\varepsilon_{t-(j-1)} - \phi_1^{j-1}\theta_1 \varepsilon_{t-j}$$

Terangkan dengan menggunakan rumus terbaru mengapa:

(i) *proses tersebut adalah pegun apabila* $|\phi_1| < 1$

(ii) *proses tersebut adalah tidak pegun jika* $|\phi_1| \geq 1$

[30 markah]

(b) *Sekali lagi pertimbangkan proses* $ARPB(1,1)$ *seperti dalam bahagian (i). Tunjukkan bahawa:*

(i) $\gamma_0 = \dfrac{(1 - 2\theta_1\phi_1 + \theta_1^2)}{1 - \phi_1^2}$ (ii) $\rho_k = \dfrac{(1 - \theta_1\phi_1)(\phi_1 - \theta_1)}{1 - 2\theta_1\phi_1 + \theta_1^2}\phi_1^{k-1}$

[20 markah]

(c) *Diberi suatu proses* $ARMA(2,1)$:

$$(1 - \phi_1 B - \phi_2 B^2)Y_t = (1 - \theta_1 B)\varepsilon_t$$

Dapatkan rumus umum bagi fungsi-fungsi autokovarians, autokorelasi dan autokorelasi separa bagi proses tersebut.

(i) *Encik Kid telah dimaklumkan bahawa suatu siri masa pulangan kadar pertukaran Ringgit-US Dollar dengan* 200 *cerapan telah disuaikan dengan cukup oleh model* $ARPB(2,1)$ *dengan koefisien-koefisien:* $\phi_1 = -0.7$, $\phi_2 = 0.18$ *and* $\theta_1 = 0.15$.

Bantu Encik Kid untuk menghitung nilai-nilai autokorelasi bagi susulan k = 1, 2, 3, 4, 5 *dan* 6 *dan autokorelasi separa bagi susulan* k = 1 *dan* 2.

[*Diberi nilai-nilai fak bagi susulan* 6 *hingga* 10 *masing-masing adalah* 0.561, -0.485, 0.440, -0.295 *dan* 0.350, *and faks pada susulan* 3 *hingga* 8 *masing-masing adalah* -0.030, -0.051, -0.081, 0.140, 0.127 *dan* 0.062].

(ii) Based on the calculated values, Mr. Kid is surprised to observe the following phenomena:

$$|\rho_k| < |\rho_{k-1}| \text{ for most } k,$$
$$\text{sign}(\rho_k) = -\text{sign}(\rho_{k-1}) \text{ and } \rho_k \to 0 \text{ as } k \to 10$$

Meanwhile, you are not surprised by such phenomena. Explain to Mr. Kid on the logic for such findings.

(d) Although you are not surprised by such phenomena in (ii), you believe that the model has been over-parameterized and have informed Mr. Kid that the series should be fitted by a more parsimonious model. Mr. Kid responds by saying "of course, it should be fitted with an AR(2) model since $\theta_1 = 0.15$ is small, it can be neglected and therefore it is an AR(2)".

Explain to Mr. Kid for his incorrect reasoning and suggest to him a more appropriate model for the exchange rate return series.

[50 marks]

(ii) *Berdasarkan nilai-nilai yang dihitung, Encik Kid hairan apabila mendapati fenomena-fenomena berikut:*

$|\rho_k| < |\rho_{k-1}|$ *untuk kebanyakan* k

tanda$(\rho_k) = -$*tanda* (ρ_{k-1}) *dan* $\rho_k \to 0$ *apabila* $k \to 10$

Sementara itu, anda tidak merasa hairan terhadap fenomena-fenomena sebegitu. Terangkan kepada Encik Kid mengapa dapatan-dapatan tersebut adalah logik.

(d) *Walaupun anda tidak merasa hairan terhadap fenomena sebegitu dalam bahagian (ii), anda percaya bahawa model yang terhasil mempunyai lebihan parameter dan telah memaklumkan kepada Encik Kid bahawa siri tersebut patut disuaikan dengan model yang lebih parsimoni. Encik Kid memberikan respon dengan berkata "sudah tentu, ia sepatutnya disuaikan dengan model* $AR(2)$ *kerana* $\theta_1 = 0.15$ *adalah kecil, ia boleh diabaikan dan oleh itu adalah* $AR(2)$*".*

Terangkan kepada Encik Kid bagi alasan beliau yang tidak betul dan cadangkan kepada beliau satu model yang lebih sesuai bagi siri pulangan kadar pertukaran wang asing tersebut.

[50 markah]

3. (a) Consider an AR(2) process as given by:

$$\left(1-\phi_1 B-\phi_2 B^2\right)Y_t=\varepsilon_t$$

Using method of moments, show that the estimates of ϕ_1 and ϕ_2 are given by:

$$\hat{\phi}_1=\frac{\hat{\rho}_1(1-\hat{\rho}_2)}{1-\rho_1^2} \quad \text{and} \quad \hat{\phi}_2=\frac{\hat{\rho}_2-\rho_1^2}{1-\rho_1^2}$$

[20 marks]

(b) A series of Kuala Lumpur Composite Index stock for the period between July 2005 and December 2007 has been obtained. ARMA and/or GARCH model are used to fit the return and conditional variance of the series. Three models have been fitted and the results are shown in Appendix A. Discuss the suitability of each model and give reason for the chosen best model.

[30 marks]

(c) A non-stationary seasonal time series $\{S_t\}$ has 500 observations and follows an invertible SARIMA $(0,0,1)(1,1,0)_{12}$ model given by:

$$S_t=\phi_{12}S_{t-1}+S_{t-12}-\phi_{12}S_{t-13}+\varepsilon_t-\theta_1\varepsilon_{t-1}$$

(i) Show that $\{S_t\}$ has variance and autocorrelation function given by:

$$\gamma_0=\frac{1+\theta_1^2}{1-\phi_{12}^2}\sigma_\varepsilon^2 \qquad \rho_{12k}=\phi_{12}^k \text{ for } k=1,2,\ \ldots$$

$$\rho_{12k-1}=\rho_{12k+1}=-\frac{\theta_1}{1+\theta_1^2}\phi_{12}^k \text{ for } k=0,1,2,\ \ldots$$

(ii) Table 1 through to Table 4 in the Appendix B show the sample acf and sample pacf of $\{S_t\}$, $\{\nabla S_t\}$, $\{\nabla_{12}S_t\}$ and $\{\nabla\nabla_{12}S_t\}$. The mean and standard deviation for the original and differenced series are also given.

Discuss the appropriateness of the SARIMA $(0,0,1)(1,1,0)_{12}$ model for the series based on the given sample acf and sample pacf. Calculate the estimate for ϕ_{12}, θ_1 and σ_ε^2.

[50 marks]

3. (a) *Pertimbangkan suatu proses* $AR(2)$ *yang diwakili oleh:*

$$\left(1-\phi_1 B-\phi_2 B^2\right)Y_t=\varepsilon_t$$

Menggunakan kaedah momen, tunjukkan bahawa anggaran bagi ϕ_1 *and* ϕ_2 *diberikan oleh:*

$$\hat{\phi}_1=\frac{\hat{\rho}_1\left(1-\hat{\rho}_2\right)}{1-\rho_1^2} \quad dan \quad \hat{\phi}_2=\frac{\hat{\rho}_2-\rho_1^2}{1-\rho_1^2}$$

[20 markah]

(b) *Suatu siri saham Indeks Komposit Kuala Lumpur bagi jangkamasa di antara Julai 2005 dan Disember 2007 telah diperoleh. Model ARPB dan/atau GARCH telah digunakan untuk menyuaikan pulangan dan varians bersyarat bagi siri tersebut. Tiga model telah disuaikan dan keputusannya ditunjukkan dalam Lampiran A. Bincang kesesuaian bagi setiap model dan beri alasan bagi model terbaik yang dipilih.*

[30 markah]

(c) *Suatu siri masa bermusim tak pegun* $\{S_t\}$ *mempunyai* 500 *cerapan dan mengikuti model boleh songsang ARKPB* $(0,0,1)(1,1,0)_{12}$ *yang diberikan oleh:*

$$S_t=\phi_{12}S_{t-1}+S_{t-12}-\phi_{12}S_{t-13}+\varepsilon_t-\theta_1\varepsilon_{t-1}$$

(i) *Tunjukkan bahawa* $\{S_t\}$ *mempunyai varians dan fungsi autokorelasi yang diwakili oleh:*

$$\gamma_0=\frac{1+\theta_1^2}{1-\phi_{12}^2}\sigma_\varepsilon^2 \qquad \rho_{12k}=\phi_{12}^k \text{ bagi } k=1,2,\ldots$$

$$\rho_{12k-1}=\rho_{12k+1}=-\frac{\theta_1}{1+\theta_1^2}\phi_{12}^k \text{ bagi } k=0,1,2,\ldots$$

(ii) *Jadual* 1 *hingga Jadual* 4 *dalam Lampiran B menunjukkan sampel fak dan faks bagi* $\{S_t\}$, $\{\nabla S_t\}$, $\{\nabla_{12}S_t\}$ *dan* $\{\nabla\nabla_{12}S_t\}$. *Min dan sisihan piawai bagi siri asal dan siri-siri yang telah dibezakan juga diberikan.*

Bincang kesesuaian model ARKPB $(0,0,1)(1,1,0)_{12}$ *bermusim bagi siri tersebut berdasarkan sampel fak dan faks yang diberi. Hitung anggaran bagi* ϕ_{12}, θ_1 *dan* σ_ε^2.

[50 markah]

4. Consider an ARIMA(1,d,3) model for a series with non-zero mean:

$$(1-\phi_1 B)(1-B)^d (Y_t-\mu) = (1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)\varepsilon_t$$

(a) Consider a special case with $d=\theta_2=\theta_3=0$.

(i) Show that the MA coefficient is given by: $\varphi_k = (\phi_1-\theta_1)\phi_1^{k-1}$.

Show that the m-step ahead forecasts made at time $t=N$ is given by:

$$\hat{Y}_N(m) = \mu(1-\phi_1)+\phi_1\hat{Y}_N(m-1) \quad \text{for } m \geq 2$$

and that it can be rewritten as:

$$\hat{Y}_N(m) = \mu(1-\phi_1^m)+\phi_1^m Y_N-\phi_1^{m-1}\theta_1\varepsilon_N \quad \text{for } m \geq 1$$

(ii) Show that the corresponding variance of forecast error is given by:

$$Var[\upsilon_N(m)] = \sigma_\varepsilon^2\left(1+(\phi_1-\theta_1)^2\left[\frac{1-\phi_1^{2(m-1)}}{1-\phi_1^2}\right]\right)$$

(iii) Finally, show that as $m\to\infty$:

$$\hat{Y}_N(m)\to\mu \quad \text{and} \quad Var[\upsilon_N(m)]\to\frac{(1+\theta_1^2-2\phi_1\theta_1)}{1-\phi_1^2}\sigma_\varepsilon^2$$

[30 marks]

4. *Pertimbangkan model ARKPB(1,d,3) bagi siri dengan min bukan sifar:*

$$(1-\phi_1 B)(1-B)^d (Y_t-\mu) = (1-\theta_1 B - \theta_2 B^2 - \theta_3 B^3)\varepsilon_t$$

(a) *Pertimbangkan suatu kes khas dengan* $d = \theta_2 = \theta_3 = 0$.

(i) *Tunjukkan bahawa koefisien PB diberikan oleh:* $\varphi_k = (\phi_1 - \theta_1)\phi_1^{k-1}$.

Tunjukkan bahawa telahan m-langkah kehadapan yang dibuat pada waktu t = N diberikan oleh:

$$\hat{Y}_N(m) = \mu(1-\phi_1) + \phi_1 \hat{Y}_N(m-1) \quad bagi \quad m \geq 2$$

dan ia boleh ditulis semula sebagai:

$$\hat{Y}_N(m) = \mu(1-\phi_1^m) + \phi_1^m Y_N - \phi_1^{m-1}\theta_1 \varepsilon_N \quad bagi \quad m \geq 1$$

(ii) *Tunjukkan bahawa varians bagi ralat telahan yang sepadan diberikan oleh:*

$$Var[\upsilon_N(m)] = \sigma_\varepsilon^2 \left(1 + (\phi_1 - \theta_1)^2 \left[\frac{1-\phi_1^{2(m-1)}}{1-\phi_1^2} \right] \right)$$

(iii) *Akhir sekali, tunjukkan apabila* $m \to \infty$:

$$\hat{Y}_N(m) \to \mu \quad dan \quad Var[\upsilon_N(m)] \to \frac{(1+\theta_1^2 - 2\phi_1\theta_1)}{1-\phi_1^2}\sigma_\varepsilon^2$$

[30 markah]

(b) Consider a special case with $d = 0$.

(i) Show that the one-step and two-step ahead forecasts made at $t = N$ are respectively given by:

$$\hat{Y}_N(1) = \mu(1-\phi_1) + \phi_1 Y_N - \theta_1\varepsilon_N - \theta_2\varepsilon_{N-1} - \theta_3\varepsilon_{N-2}$$

$$\hat{Y}_N(2) = \mu(1-\phi_1) + \phi_1\hat{Y}_N(1) - \theta_2\varepsilon_N - \theta_3\varepsilon_{N-1}$$

and also show that the m-step-ahead forecast is given by:

$$\hat{Y}_N(m) = \mu(1-\phi_1) + \phi_1\hat{Y}_N(m-1) \quad \text{for } m \geq 4$$

(ii) Consider N = 250. If estimated values for the coefficients are $\hat{\phi}_1 = 0.8$, $\hat{\theta}_1 = -0.3$, $\hat{\theta}_2 = 0.73$, $\hat{\theta}_1 = 0.315$, $\hat{\mu} = 200$, $s_\varepsilon^2 = 12$ with $Y_{250} = 216$, $\varepsilon_{250} = 4$, $\varepsilon_{249} = 12$ and $\varepsilon_{248} = 8$, obtain value of $\hat{Y}_{250}(m)$ for m = 1, 2, …, 6. Construct a 95% forecast interval for Y_{251}, Y_{252}, Y_{253} and Y_{254}. Give comment on the six forecast values obtained.

What is the most likely forecast value at time t = 400 and its corresponding 95% forecast interval? Give explanation.

(iii) It is now observed at time t = 251 that a new observation is noted as 212. Calculate the updated values for $Y_{252}, \ldots Y_{256}$. Compare these new forecasts with those calculated in (ii) above and discuss.

[45 marks]

(c) Consider a process with $\phi_1 = 1.1$, $d = 1$ and $\theta_1 = \theta_2 = \theta_3 = 0$.

(i) Show that the explicit expression of the MA coefficient is given by:

$$\varphi_k = \frac{1-\phi_1^{k+1}}{1-\phi_1}$$

(ii) Using similar information as in part b(ii), calculate value of $\hat{Y}_{250}(m)$ for m = 1, 2, …, 6. Construct a 95% forecast interval for Y_{251}, Y_{252}, Y_{253} and Y_{254}. Give comment on the four forecast values and its corresponding forecast intervals.

[25 marks]

(b) *Pertimbangkan kes khas dengan* $d = 0$.

(i) *Tunjukkan bahawa telahan satu-langkah dan dua-langkah kehadapan yang dilakukan pada* $t = N$ *masing-masing diberikan oleh:*

$$\hat{Y}_N(1) = \mu(1-\phi_1) + \phi_1 Y_N - \theta_1 \varepsilon_N - \theta_2 \varepsilon_{N-1} - \theta_3 \varepsilon_{N-2}$$

$$\hat{Y}_N(2) = \mu(1-\phi_1) + \phi_1 \hat{Y}_N(1) - \theta_2 \varepsilon_N - \theta_3 \varepsilon_{N-1}$$

dan tunjukkan juga bahawa telahan m-langkah kehadapan diberikan oleh:

$$\hat{Y}_N(m) = \mu(1-\phi_1) + \phi_1 \hat{Y}_N(m-1) \quad \textit{bagi } m \geq 4$$

(ii) *Pertimbangkan* $N = 250$. *Sekiranya nilai-nilai teranggar bagi koefisien-koefisien adalah* $\hat{\phi}_1 = 0.8$, $\hat{\theta}_1 = -0.3$, $\hat{\theta}_2 = 0.73$, $\hat{\theta}_1 = 0.315$, $\hat{\mu} = 200$, $s_\varepsilon^2 = 12$ *dengan* $Y_{250} = 216$, $\varepsilon_{250} = 4$, $\varepsilon_{249} = 12$ *dan* $\varepsilon_{248} = 8$, *dapatkan nilai* $\hat{Y}_{250}(m)$ *bagi* $m = 1, 2, \ldots, 6$. *Bina selang telahan 95% bagi* Y_{251}, Y_{252}, Y_{253} *dan* Y_{254}. *Beri komen bagi enam nilai telahan yang diperoleh.*

Apakah nilai berkemungkinan bagi telahan pada waktu $t = 400$ *dan nilai sepadan selang telahan 95%? Berikan penjelasan.*

(iii) *Sekarang pada waktu* $t = 251$ *satu cerapan baru bernilai* 212 *diperoleh. Hitung nilai telahan kemaskini bagi* $Y_{252}, \ldots Y_{256}$. *Bandingkan nilai telahan terkini dengan nilai telahan yang dihitung dalam (ii) di atas dan bincang.*

[45 markah]

(c) *Pertimbangkan suatu proses dengan* $\phi_1 = 1.1$, $d = 1$ *dan* $\theta_1 = \theta_2 = \theta_3 = 0$.

(i) *Tunjukkan bahawa ungkapan tak tersirat bagi koefisien PB diberikan oleh:*

$$\varphi_k = \frac{1-\phi_1^{k+1}}{1-\phi_1}$$

(ii) *Dengan menggunakan maklumat yang sama seperti di bahagian b(ii), hitung nilai* $\hat{Y}_{250}(m)$ *bagi* $m = 1, 2, \ldots, 6$. *Bina selang telahan 95% bagi* Y_{251}, Y_{252}, Y_{253} *dan* Y_{254}. *Beri komen bagi empat nilai telahan serta selang telahannya.*

[25 markah]

APPENDIX/LAMPIRAN A

Model 1

Dependent Variable: KLCI
Method: Least Squares
Sample(adjusted): 2 622
Included observations: 621 after adjusting endpoints

Variable	Coeff.	Std.Err.	t-Stats	Prob.
AR(1)	0.186	0.039	4.72	0.00
R-sq	0.025		AIC	2.2738
Adj R-sq	0.025		BIC	2.2809
Log L	-705.016		D-W	1.9883
Inverted AR Roots			0.19	

Residual analysis

lag	acf	pacf	Q-stats	p-value	ARCH-LM	p-value
2	-0.063	-0.063	2.52	0.112	44.82	0.000
3	0.125	0.126	12.33	0.002	122.71	0.000
5	-0.120	-0.106	22.10	0.000	127.06	0.000
10	0.062	0.016	38.16	0.000	158.61	0.000
15	-0.074	-0.050	56.94	0.000	160.53	0.000
20	0.072	0.053	73.11	0.000	163.49	0.000

Model 2

Dependent Variable: KLCI
Method: ML - ARCH
Sample(adjusted): 3 622
Included observations: 620 after adjusting endpoints

	Coeff.	Std.Err.	z-Stats	Prob.
AR(1)	-0.725	0.056	-12.87	0.00
AR(2)	0.148	0.046	3.22	0.00
MA(1)	0.954	0.027	35.69	0.00
Variance Equation				
C	0.011	0.004	2.99	0.00
ARCH(1)	0.141	0.023	6.24	0.00
GARCH(1)	0.842	0.024	34.80	0.00
R-sq	0.052		AIC	1.8871
Adj R-sq	0.044		BIC	1.9300
Log L	-579.016		D-W	2.0179
Inverted AR Roots		0.17		-0.89
Inverted MA Roots		-0.95		

Residual analysis

lag	acf	pacf	Q-stats	p-value	ARCH-LM	p-value
1	-0.008	-0.008	0.04		0.25	0.618
2	0.003	0.003	0.04		1.21	0.545
5	-0.066	-0.067	6.10	0.047	3.63	0.604
10	0.069	0.063	12.81	0.077	7.41	0.687
15	0.014	0.023	18.92	0.090	12.47	0.643
20	0.061	0.072	23.71	0.128	19.79	0.471

Model 3

Dependent Variable: KLCI
Method: ML - ARCH
Sample(adjusted): 4 622
Included observations: 619 after adjusting endpoints

	Coeff.	Std.Err.	z-Stats	Prob.
AR(1)	-0.105	0.132	-0.80	0.423
AR(2)	-0.760	0.080	-9.53	0.000
AR(3)	-0.036	0.138	-0.26	0.791
MA(1)	0.324	0.117	2.76	0.006
MA(2)	0.782	0.073	10.78	0.000
MA(3)	0.319	0.125	2.56	0.011
Variance Equation				
C	0.012	0.004	3.01	0.003
ARCH(1)	0.164	0.026	6.20	0.000
GARCH(1)	0.823	0.027	30.52	0.000
R-squared	0.060		AIC	1.8846
Adj R-sq	0.047		BIC	1.9490
Log L	-574.284		D-W	2.0352
Inverted AR Roots		-.03 -.87i	-.03+.87i	-0.05
Inverted MA Roots		.04 -.90i	.04+.90i	-0.39

Residual analysis

lag	acf	pacf	Q-stats	p-value	ARCH-LM	p-value
1	-0.005	-0.005	0.02		0.68	0.409
2	0.010	0.010	0.08		2.42	0.298
6	-0.095	-0.095	6.33		4.89	0.557
7	-0.015	-0.016	6.47	0.011	4.97	0.663
10	0.047	0.048	10.88	0.028	6.94	0.731
15	0.019	0.023	13.51	0.141	11.40	0.723
20	0.050	0.048	19.53	0.146	18.77	0.537

APPENDIX/LAMPIRAN B

Table 1: Series $\{S_t\}$, mean = 1.017, std deviation = 17.857

lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ACF	-0.219	-0.591	0.044	0.581	-0.100	-0.420	-0.106	0.581	0.044	-0.592
PACF	-0.219	-0.671	-0.608	-0.136	-0.026	0.088	-0.386	-0.066	0.207	-0.075
lag	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ACF	-0.201	0.970	-0.218	-0.574	0.049	0.558	-0.089	-0.415	-0.101	0.569
PACF	-0.825	0.656	0.161	0.338	0.402	-0.107	-0.070	-0.208	0.012	0.022
lag	**21**	**22**	**23**	**24**	**25**	**26**	**28**	**30**	**32**	**34**
ACF	0.046	-0.590	-0.180	0.936	-0.214	-0.556	0.537	-0.412	0.557	-0.587
PACF	0.164	0.171	0.096	-0.015	-0.063	-0.009	0.072	0.041	0.006	-0.044
lag	**35**	**36**	**37**	**38**	**40**	**42**	**44**	**46**	**47**	**48**
ACF	-0.158	0.898	-0.209	-0.536	0.517	-0.411	0.544	-0.583	-0.136	0.859
PACF	0.032	-0.044	0.026	0.068	0.024	-0.005	0.025	-0.018	0.012	-0.052

Table 2: Series $\{\nabla S_t\}$, mean = 0.080, std deviation = 27.860

lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ACF	-0.343	-0.417	0.042	0.496	-0.143	-0.258	-0.157	0.500	0.046	-0.424
PACF	-0.343	-0.606	-0.710	-0.441	-0.310	0.169	-0.215	-0.249	0.159	0.554
lag	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ACF	-0.321	0.968	-0.338	-0.405	0.048	0.471	-0.128	-0.260	-0.151	0.488
PACF	-0.808	0.295	0.054	0.053	0.348	0.004	-0.059	-0.235	0.038	-0.061
lag	**21**	**22**	**23**	**24**	**25**	**26**	**28**	**30**	**32**	**34**
ACF	0.052	-0.432	-0.291	0.930	-0.329	-0.391	0.449	-0.265	0.476	-0.440
PACF	0.020	0.022	0.006	-0.062	-0.031	0.167	0.055	-0.106	-0.026	-0.026
lag	**35**	**36**	**37**	**38**	**40**	**42**	**44**	**46**	**47**	**48**
ACF	-0.258	0.888	-0.318	-0.377	0.427	-0.272	0.462	-0.445	-0.225	0.844
PACF	0.092	-0.038	-0.022	0.060	0.013	-0.015	-0.014	-0.013	0.047	-0.013

Table 3: Series $\{\nabla_{12} S_t\}$, mean = 0.035, std deviation = 1.737

lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ACF	-0.466	-0.104	0.102	0.045	-0.143	0.118	-0.110	0.055	0.074	-0.107
PACF	-0.466	-0.410	-0.241	-0.081	-0.188	-0.056	-0.199	-0.142	-0.006	-0.089
lag	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ACF	-0.272	0.614	-0.261	-0.107	0.107	0.048	-0.152	0.098	-0.124	0.123
PACF	-0.597	0.062	0.113	0.090	0.086	0.118	0.091	0.078	-0.084	0.021
lag	**21**	**22**	**23**	**24**	**25**	**26**	**28**	**30**	**32**	**34**
ACF	0.016	-0.065	-0.196	0.420	-0.175	-0.126	0.051	0.026	0.130	-0.024
PACF	0.000	0.061	-0.011	0.035	0.034	-0.088	0.021	-0.055	0.040	-0.055
lag	**35**	**36**	**37**	**38**	**40**	**42**	**44**	**46**	**47**	**48**
ACF	-0.081	0.223	-0.110	-0.058	0.091	-0.021	0.132	-0.025	-0.004	0.100
PACF	0.073	0.006	-0.004	0.116	-0.004	-0.006	0.003	0.041	0.047	-0.007

Table 4: Series $\{\nabla\nabla_{12} S_t\}$, mean = 0.000, std deviation = 2.973

lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ACF	-0.620	0.050	0.090	0.044	-0.150	0.165	-0.132	0.048	0.067	-0.004
PACF	-0.620	-0.543	-0.457	-0.241	-0.295	-0.108	-0.151	-0.204	-0.054	0.308
lag	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ACF	-0.359	0.599	-0.346	-0.023	0.091	0.049	-0.151	0.160	-0.159	0.120
PACF	-0.444	-0.220	-0.032	0.054	0.064	0.104	0.088	0.153	-0.045	-0.038
lag	**21**	**22**	**23**	**24**	**25**	**26**	**28**	**30**	**32**	**34**
ACF	-0.011	0.019	-0.255	0.411	-0.214	-0.078	0.035	0.081	0.124	0.028
PACF	-0.089	-0.013	-0.060	-0.028	0.078	-0.003	-0.025	0.036	0.041	-0.086
lag	**35**	**36**	**37**	**38**	**40**	**42**	**44**	**46**	**47**	**48**
ACF	-0.123	0.216	-0.129	-0.016	0.085	0.032	0.122	0.003	-0.026	0.094
PACF	0.017	0.023	-0.069	0.057	0.003	-0.011	-0.051	0.015	0.063	0.078

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